Modeling the Atmospheric Convective Boundary Layer within a Zero-Order Jump Approach: An Extended Theoretical Framework

EVGENI FEDOROVICH*

Institut für Hydrologie und Wasserwirtschaft, Universität Karlsruhe, Karlsruhe, Germany

(Manuscript received 24 October 1994, in final form 17 January 1995)

ABSTRACT

The paper presents an extended theoretical background for applied modeling of the atmospheric convective boundary layer within the so-called zero-order jump approach, which implies vertical homogeneity of meteorological fields in the bulk of convective boundary layer (CBL) and zero-order discontinuities of variables at the interfaces of the layer.

The zero-order jump model equations for the most typical cases of CBL are derived. The models of nonsteady, horizontally homogeneous CBL with and without shear, extensively studied in the past with the aid of zero-order jump models, are shown to be particular cases of the general zero-order jump theoretical framework. The integral budgets of momentum and heat are considered for different types of dry CBL. The profiles of vertical turbulent fluxes are presented and analyzed. The general version of the equation of CBL depth growth rate (entrainment rate equation) is obtained by the integration of the turbulence kinetic energy balance equation, invoking basic assumptions of the zero-order parameterizations of the CBL vertical structure. The problems of parameterizing the turbulence vertical structure and closure of the entrainment rate equation for specific cases of CBL are discussed. A parameterization scheme for the horizontal turbulent exchange in zero-order jump models of CBL is proposed. The developed theory is generalized for the case of CBL over irregular terrain.

1. Introduction

Convective boundary layer (CBL) developing in the daytime over a heated underlying surface constitutes a substantial period in the diurnal evolution of atmospheric planetary boundary layer (Stull 1988). The principal feature of CBL is strong turbulent mixing occupying the main portion of the layer in the vertical. The turbulence in CBL is primarily of convective origin; therefore, convection is the dominant mechanism defining the structure of the layer. Over the heated surface, the thermals of warm air are created, which effectively transport heat upward. Rising thermals interact with the ambient air, loosing their potential energy. Downdrafts represent another part of convective circulation. They are cooler and weaker than thermals but usually have larger scales in the horizontal. The composition of updrafts and downdrafts in CBL provides for intensive mixing and uniformity of vertical distributions of physical substances. The core of the convective layer, where horizontally averaged meteorological variables like potential temperature, specific humidity, and wind velocity are nearly height constant, is commonly called the mixed layer. This layer is separated from the underlying surface by relatively shallow surface layer within which meteorological variables vary sharply from their near-surface values to the mixed-layer ones. Above the mixed layer, the entrainment zone, which is also called the entrainment layer, or the inversion layer, or the interfacial layer, is located. Within this layer, the energy of thermals is expended for their penetration into the stably stratified atmospheric air aloft and mixing (entraining) it downward. The structure of the free atmosphere is characterized by stable density stratification. Entrainment of the free-atmosphere air is accompanied, therefore, by the downward transport of heat from the free atmosphere to the mixed layer. Thus, one of the definitions of the entrainment zone is the range of heights with negative kinematic heat flux. Typically, the entrainment layer is characterized by strong vertical gradients of horizontally averaged meteorological variables. Its depth is quite variable, but usually smaller than the depth of the mixed layer. Toward the top of the entrainment zone, turbulence decays and turbulent transport becomes negligible. Entrainment of more buoyant air into the mixed layer leads to the increase of CBL depth. The wind shear plays a secondary role in CBL, con-

* Permanent affiliation: Main Geophysical Observatory, St. Petersburg, Russia.

Corresponding author address: Dr. E. Fedorovich, Institut für Hydrologie und Wasserwirtschaft, Universität Karlsruhe, Kaiserstraße 12, 76128 Karlsruhe, Germany.
E-mail: gg07@dkauni2.bitnet

tributing to turbulence generation in the regions with sharp velocity gradients: in the surface layer and in the entrainment zone.

Model studies of the CBL passed through several historical stages. CBL was most thoroughly studied within the so-called zero-order jump approach initiated by the pioneer works of Ball (1960) and Lilly (1968). A zero-order jump parameterization for the vertical structure of the convective boundary layer is presented in Fig. 1. It is based on the observed features of vertical distributions of meteorological variables in CBL, discussed above. During the convection and deepening of CBL the potential temperature within the layer is presumed to be height constant. Its changes with height in the surface layer, and at the top of CBL, in the entrainment layer, are reduced to the zero-order discontinuities of temperature profile. Thus, within the zero-order approach the CBL is represented by the mixed layer with two interfaces, upper (at $z = h$) and lower (at $z = 0$), across which the potential temperature changes in a jumplike way.

The analogous parameterization is employed for the wind velocity profile (Garrat et al. 1982). The velocity is taken to be height constant within the mixed layer. At the mixed-layer upper interface it changes abruptly to the free-atmosphere value. The sharp increase of velocity with height in the surface layer is also represented by step in the profile. The vertical structure of the velocity and temperature fields in the stably stratified free-atmosphere layer above CBL is assumed to be known.

The breakthrough in the zero-order jump modeling of CBL took place in the 1970s when Betts (1973), Carson (1973), Tennekes (1973), Stull (1973, 1976a,b), Carson and Smith (1974), Zilitinkevich (1975a), and Zeman and Tennekes (1977) used the zero-order jump approach to describe the evolution and energetics of one-dimensional shear-free CBL. Later Zilitinkevich (1991) suggested a generalized model for this type of CBL, comprising aforementioned ones as the asymptotic cases. Several applied models for purposes of mesometeorological studies, for example, those of Kraus and Leslie (1982), Brutsaert (1987), Batchvarova and Gryning (1991), and Zilitinkevich et al. (1992), were developed based on the zero-order parameterization of the CBL vertical structure. Most of those models dealt with horizontally homogeneous CBL and used simplifying assumptions concerning the effects of wind shear.

Trying to reproduce the characteristic features of CBL in a more detailed way, higher-order bulk models of CBL were proposed. In the first-order jump model, Betts (1974) introduced the interfacial layer of finite thickness between the mixed layer and free atmosphere. Potential temperature profile was taken to be linear in this layer, undergoing first-order discontinuities at its upper and lower boundaries. The general-structure

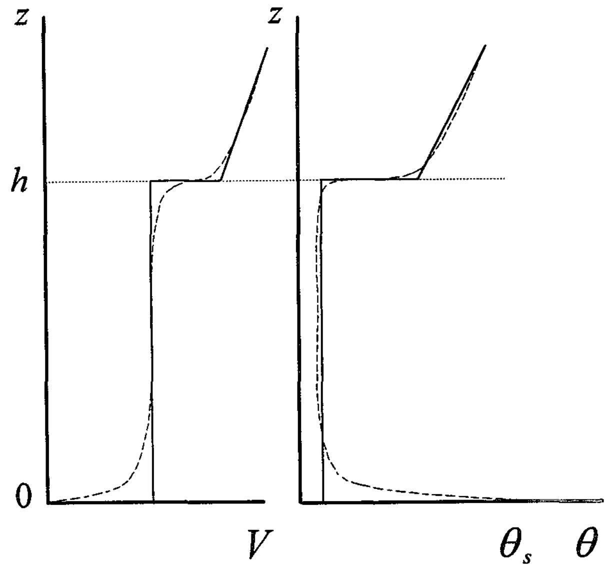

FIG. 1. Actual (dashed lines) and parameterized (solid lines) profiles of wind velocity V and potential temperature θ in the CBL; θ_s is the near-surface value of θ.

CBL bulk models of Deardorff (1979), and Fedorovich and Mironov (1995) provide for realistic representation of temperature–buoyancy profile in the entrainment zone, accounting for nonstationarity of the zone and self-similarity of its buoyant structure. All mentioned models of higher orders were designed for shear-free cases of CBL.

Within the last decade, the attention of modelers dealing with CBL switched almost completely to large-eddy simulations (LES) of atmospheric convection. Though the first activities in this area were at the beginning of the 1970s when Deardorff (1970a, 1972, 1974) performed his first numerical experiments on LES of convection, the progress was slow because of limited computing power and capacity of data storage devices. During the last 10 years the situation has improved, and significant results in LES studies of the CBL have been achieved by Moeng (1984, 1986, 1987), Nieuwstadt and Brost (1986), Moeng and Wyngaard (1988), Mason (1989), Schmidt and Schumann (1989), Nieuwstadt (1990), and Schumann and Moeng (1991), to be mentioned in the first turn. Different types of CBL and various features of its structure were investigated. The results of the comparison of the four best known LES codes applied to a particular case of shear-free CBL were discussed in Nieuwstadt et al. (1993). Combined effects of wind shear and buoyancy forces in the atmospheric boundary layer have been studied by Moeng and Sullivan (1994). A review of the LES technique has been presented recently by Mason (1994).

Nevertheless, being a very valuable and efficient tool for the fundamental studies of CBL, and possessing the abilities of high-resolution measurements in reproducing the fine structure of the flow, the LES technique at present can hardly be used in applied CBL models due to the enormous computer resources it needs. The prospects of employment of higher-order bulk models for the same purpose also seem to be quite doubtful, because none of the existing models of this kind are suitable for incorporating meteorological forcings other than of pure convective origin. This explains why the zero-order jump modeling continues to be a popular approach in applied studies of CBL.

The theoretical background of this the zero-order jump modeling was developed only for a few cases of CBL. The shear-free CBL is one of them. For horizontally homogeneous CBL with wind shear, quite a few parameterizations have been proposed (Stull 1988). The effects of baroclinicity (thermal wind), advection, and horizontal diffusion, which can be important in CBL under certain conditions, were also left beyond the framework of previous model considerations. The idea of the present paper is to develop a unified theoretical basis for applied zero-order jump modeling of CBL to account for all these factors. The general forms of CBL momentum and heat budget equations (section 2), expressions of vertical turbulent flux profiles (section 3), and the entrainment rate equation (section 4) will be derived. In section 5 the typical cases of CBL together with alterations of theory will be considered, and problems of determination of the integral parameters of turbulent regime for particular cases of CBL will be discussed. A parameterization of horizontal turbulent diffusion in a zero-order jump model for the spatially nonhomogeneous CBL will be suggested in section 6. In section 7 the ways of prescribing values of external parameters, and of practical implementation of the developed theory, will be outlined.

2. Integral budgets of momentum and heat in the convective boundary layer

The following initial equations written in the Reynolds form will be used to describe the flow in the nonstationary, horizontally nonhomogeneous CBL.

The momentum balance equations

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} + \frac{\partial wu}{\partial z} = f(v - v_{g0} - \Gamma_v z) + \frac{\partial \tau_x}{\partial z}, \quad (1)$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} + \frac{\partial wv}{\partial z} = -f(u - u_{g0} - \Gamma_u z) + \frac{\partial \tau_y}{\partial z}, \quad (2)$$

where u, v, and w are the components of mean wind velocity along axes x, y, and z, respectively; $\tau_x = -\langle w'u' \rangle$ and $\tau_y = -\langle w'v' \rangle$ are components of the turbulent shear stress normalized by density, the angle brackets denote the operation of Reynolds averaging; f is the Coriolis parameter; and u_{g0} and v_{g0} are the near-surface values of the geostrophic wind components, where Γ_u and Γ_v are the vertical gradients of these components. The last four characteristics are presumed to be given functions of time and horizontal coordinates. In section 7 we shall show the simplest method for determining them from pressure and temperature spatial distributions. It is assumed in Eqs. (1) and (2) that contributions of the horizontal shear to the momentum balance are negligible compared to the effects of the vertical shear, and conditions of hydrostatic equilibrium in CBL are satisfied (Qi et al. 1994). These assumptions are quite reasonable when horizontal scales in the flow dominate over the vertical ones, which is typical for the majority of convective boundary layer cases observed in nature and simulated in the laboratory. Nevertheless, it will be shown later how the effects of horizontal turbulent diffusion can be accounted for within the zero-order jump model approach.

The mass conservation equation is employed in the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (3)$$

The heat transfer equation

$$\frac{\partial \theta}{\partial t} + \frac{\partial u\theta}{\partial x} + \frac{\partial v\theta}{\partial y} + \frac{\partial w\theta}{\partial z} = -\frac{\partial Q}{\partial z}, \quad (4)$$

where θ is the mean potential temperature and $Q = \langle w'\theta' \rangle$ is the turbulent kinematic heat flux, implies that advection and vertical turbulent transport of heat dominate over its transfer by horizontal turbulent fluctuations; this assumption is analogous to the one used in momentum balance equations. We also neglect the contributions of molecular and radiative heat transfer to the heat balance of CBL. The first one is very small in the atmosphere compared to turbulent heat transfer. The radiation effects are omitted for simplicity. Principally, they can be incorporated in the zero-order jump model of CBL, as is done, for instance, in Zilitinkevich et al. (1992).

The above equations represent the general case of the convective boundary layer over the flat underlying surface. To obtain the equations of the momentum and heat budget of the layer, we integrate Eqs. (1), (2), and (4) over the boundary layer depth, that is, over z from 0 to h, taking into account the zero-order jump representations of the temperature and velocity profiles (see Fig. 1). Integrating the equations, we shall be including the zero-order discontinuity surface in the integration domain, thus regarding the upper side of this surface as the CBL edge.

Calculating the integrals from each term of the first equation of motion, we come to the equation of the integral budget of momentum along the x axis (see appendix A),

$$h\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) = \Delta u \frac{Dh}{Dt} + h\left[f(\bar{v} - v_{g0}) - \frac{h}{2} f \Gamma_v\right] - \tau_{xs}, \quad (5)$$

where $\bar{u}$ is the average value of the x component of wind velocity within the mixed layer; $\Delta u = u_h - \bar{u}$ is the increment of this component across the mixed-layer upper interface; u_h is the value of u in the stable layer, at the upper side of the interface represented by the surface of zero-order discontinuity; $Dh/Dt \equiv \partial h/\partial t + \partial \bar{u}h/\partial x + \partial \bar{v}h/\partial y$ is the substantial (total) variation of h. The operation of averaging over the mixed-layer depth is defined here as $\overline{(\ )} = h^{-1}\int_0^h (\ )dz$.

The variation Dh/Dt represents, actually, the combined effect of three factors determining the evolution of the CBL depth. The first of them, $\partial h/\partial t$, is associated with local changes of h (nonstationarity). Horizontal advection by mean wind, which is the second factor, contributes to the variation by $\bar{u}(\partial h/\partial x) + \bar{v}(\partial h/\partial y)$. The third mechanism is the subsidence related to the horizontal divergence of the flow in CBL (Stull 1988), which gives $h[(\partial \bar{u}/\partial x) + (\partial \bar{v}/\partial y)] = -w_{hs}$, where w_{hs} is the subsidence velocity at the CBL top. It can be seen from the expression of vertical velocity at $z = h$ [Eq. (A5) of appendix A] that w_{hs} constitutes a part of w_h.

Integration procedures analogous to that given in appendix A can be applied to the second equation of motion (2), and to the heat balance equation (4). In the first case, the result is the equation of the momentum balance along the y axis:

$$h\left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y}\right) = \Delta v \frac{Dh}{Dt} - h\left[f(\bar{u} - u_{g0}) - \frac{h}{2} f \Gamma_u\right] - \tau_{ys}. \quad (6)$$

Integration of Eq. (4) gives the expression of the integral budget of heat in the convective boundary layer

$$h\left(\frac{\partial \bar{\theta}}{\partial t} + \bar{u}\frac{\partial \bar{\theta}}{\partial x} + \bar{v}\frac{\partial \bar{\theta}}{\partial y}\right) = \Delta \theta \frac{Dh}{Dt} + Q_s, \quad (7)$$

where Q_s is the near-surface value of the kinematic heat flux. The notation in Eqs. (6) and (7) corresponds to that introduced in Eq. (5).

So far, we shall consider Q_s, as well as the components of the near-surface shear stress, τ_{xs} and τ_{ys}, to be known functions of time and horizontal coordinates. Below, in section 7, some approaches toward specifying these characteristics in the zero-order jump models of the convective boundary layer will be presented.

3. Profiles of turbulent fluxes

The expressions of the momentum and heat flux profiles are derived by integrating Eqs. (1), (2), and (4) over the vertical coordinate from 0 to z. From Eq. (1), we have for the x component of the momentum flux

$$\tau_x = \tau_{xs} + \int_0^z \left(\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} + \frac{\partial wu}{\partial z}\right)dz - \int_0^z f(v - v_{g0} - \Gamma_v z)dz.$$

Using the assumptions concerning the shape of the velocity profile in the simulated CBL, and evaluating w by the integration of the continuity equation from 0 to z, it is easy to show that τ_x is the quadratic function of height:

$$\tau_x = \tau_{xs} + z\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) + z\left[f(\bar{v} - v_{g0}) + \frac{z}{2} f \Gamma_v\right],$$

which can be written, using the following substitution [see Eq. (5)]:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \frac{\Delta u}{h}\frac{Dh}{Dt} + f(\bar{v} - v_{g0}) - \frac{h}{2} f \Gamma_v - \frac{\tau_{xs}}{h},$$

in the form

$$\tau_x = \tau_{xs}(1 - \zeta) + \Delta u \frac{Dh}{Dt}\zeta + \frac{h^2}{2} f \Gamma_v \zeta(\zeta - 1), \quad (8)$$

where $\zeta = z/h$ is the dimensionless height.

Similarly, the expressions of the y component of the momentum flux

$$\tau_y = \tau_{ys}(1 - \zeta) + \Delta v \frac{Dh}{Dt}\zeta - \frac{h^2}{2} f \Gamma_u \zeta(\zeta - 1), \quad (9)$$

and the heat flux profile

$$Q = Q_s(1 - \zeta) - \Delta \theta \frac{Dh}{Dt}\zeta, \quad (10)$$

as functions of dimensionless height, can be obtained. It is seen from (10) that kinematic heat flux in the simulated CBL is linear with height.

Equations (8), (9), and (10) show that both components of the momentum flux, as well as the heat flux, undergo the discontinuities at the mixed-layer interface. Just below this surface they reach the values $\Delta u(Dh/Dt)$, $\Delta v(Dh/Dt)$, and $-\Delta\theta(Dh/Dt)$, respectively. From the upper side of the discontinuity surface all turbulent fluxes are equal to zero.

4. Entrainment rate equation

Depth of the convective layer h is one of the most important variables characterizing the process of convection. Due to the penetration of thermals into the stably stratified flow above the mixed layer, the heat and momentum from the stable region are entrained, or mixed down, into the bulk of the turbulized convective layer. Its depth is growing with convection. To describe the variations of h in time and space, we shall depart from the turbulent kinetic energy (TKE) balance equation. In the case under consideration, it has the form

$$\frac{\partial e}{\partial t}+\frac{\partial ue}{\partial x}+\frac{\partial ve}{\partial y}+\frac{\partial we}{\partial z} = \tau_x\frac{\partial u}{\partial z}+\tau_y\frac{\partial v}{\partial z}+\beta Q-\frac{\partial \Phi}{\partial z}-\epsilon, \quad (11)$$

where e is the turbulence kinetic energy per unit mass, ϵ is its dissipation rate, and Φ is the vertical transport of kinetic energy due to turbulent exchange and pressure fluctuations.

To obtain the entrainment rate equation, we integrate Eq. (11) over z from 0 to h.

The integration of the left-hand side, representing the temporal variations of e, and its transformations due to advection, yields

$$\int_0^h\left(\frac{\partial e}{\partial t}+\frac{\partial ue}{\partial x}+\frac{\partial ve}{\partial y}+\frac{\partial we}{\partial z}\right)dz = \frac{\partial}{\partial t}\bar{e}h+\frac{\partial}{\partial x}\overline{eu}h+\frac{\partial}{\partial y}\overline{ev}h.$$

While deriving the above expression, we set $e_h = 0$, since it is assumed that turbulence vanishes at $z = h$.

The integration of the shear production terms in the right-hand side of (11) cannot be carried out directly, because we have to integrate the products of the shear stress components (which are discontinuous at $z = h$) and vertical derivatives of the velocity components. (They are infinite at $z = 0$ and $z = h$ within the framework of parameterization used.) The following integration approach can be used. (We shall consider it by the example of the first term.)

We isolate in the vicinity of h a thin layer with depth δh. Then we approximate velocity derivative within this layer by $\Delta u/\delta h$, and the increment of the x component of the shear stress by the linear function $\Delta u(Dh/Dt)(h - z/\delta h)$, multiply them, and carry out the integration over the layer δh. This yields $0.5\Delta u^2(Dh/Dt)$, the value of the integral being independent on δh. Therefore, it holds true when δh tends to zero. Calculating the integral over the rest of the mixed layer we should take into account that velocity is constant with height in the bulk of the layer; therefore, there is no shear production of turbulent kinetic energy in this region. The contribution of the shear in the thin near-surface layer where velocity sharply increases from zero to the value characteristic of the mixed layer, and shear stress variation with height is negligibly small, can be evaluated with a method analogous to that used in the vicinity of h. It is easy to show that such integration results in $\bar{u}\tau_{xs}$.

Thus, for the integral shear production of TKE we obtain

$$\int_0^h\left(\tau_x\frac{\partial u}{\partial z}+\tau_y\frac{\partial v}{\partial z}\right)dz = \bar{u}\tau_{xs}+\bar{v}\tau_{ys}+\frac{1}{2}(\Delta u^2+\Delta v^2)\frac{Dh}{Dt}.$$

The integral production of the TKE by the buoyancy forces is expressed as

$$\int_0^h \beta Q dz = \beta\frac{h}{2}\left(Q_s-\Delta\theta\frac{Dh}{Dt}\right).$$

We assume that there is no transport of energy through the underlying surface. Therefore, the integral of the transport term yields $-\Phi_h$, the negative flux of energy from the CBL top, associated presumably with the wavy motions since the decay of turbulence at $z = h$ is postulated in the zero-order jump model (Zilitinkevich 1991). It was noted by Stull (1976c) that for typical atmospheric conditions the energy drain from the boundary layer top due to radiation of waves is relatively small. On the other hand, Fedorovich and Mironov (1995) discovered the pronounced effect of wave transport in the evolving CBL for the cases when the Richardson number $\mathrm{Ri}_N = 0.5(\beta Q_s)^{-2/3}h^{4/3}N^2$, characterizing the interaction between the growing CBL and turbulence-free stably stratified layer aloft (N is the buoyancy frequency within it), was in the range from 20 to 100. These Ri_N values should be regarded as quite normal for the atmospheric conditions, as one may conclude from considerations of Schmidt and Schumann (1989), who aimed to reproduce a realistic atmospheric convective situation.

Summarizing the above expressions of the different components of the integral TKE balance, we come to the entrainment equation

$$\left[\bar{e}-\frac{1}{2}(\Delta u^2+\Delta v^2-\beta h\Delta\theta)\right]\frac{Dh}{Dt}+\left(\frac{\partial \bar{e}}{\partial t}+\bar{u}\frac{\partial \bar{e}}{\partial x}+\bar{v}\frac{\partial \bar{e}}{\partial y}-\frac{\beta}{2}Q_s+\bar{\epsilon}\right)h = \bar{u}\tau_{xs}+\bar{v}\tau_{ys}-\Phi_h. \quad (12)$$

To solve this equation, one should specify the way of evaluating the mixed-layer means $\bar{e}$ and $\bar{\epsilon}$, and calculating the transport of energy at the mixed-layer top, Φ_h. Within the zero-order approach these variables are commonly determined using parameterizations based on the similarity arguments. Below, while discussing the particular cases of the convective boundary layer, we shall consider some of them. Still, a similarity theory that allows the determination of the above characteristics in the general case has not yet been developed.

5. Particular cases of convective boundary layer

a. Nonsteady shear-free convective boundary layer

This case, characterized by the absence of the shear turbulence, and mean transport, is usually the subject of laboratory experiments in water tanks. In the atmosphere and in the ocean, this type of boundary layer is observed when the buoyant turbulence ultimately dominates over the shear one, which happens in the atmosphere, for example, during strong radiative heating of the underlying surface, accompanied by light wind velocities. In this case, the budget of momentum drops off the consideration, and the equation of the heat budget (7) acquires the form

$$h\frac{d\bar{\theta}}{dt} - \Delta\theta\frac{dh}{dt} = Q_s, \tag{13}$$

where $\bar{\theta}$ is the mean value of temperature within the mixed layer, Q_s is the near-surface value of the turbulent heat flux, $h = h(t)$ is the convective layer depth, $\Delta\theta = \theta_h - \bar{\theta}$ is the temperature increment across the mixed-layer upper interface, and θ_h is the temperature value in the stable layer, at the upper side of the interface.

The heat flux profile (10) in this case keeps the linear form and is presented by the expression

$$Q = Q_s\left(1 - \frac{z}{h}\right) - \frac{z}{h}\Delta\theta\frac{dh}{dt}. \tag{14}$$

At the top of the mixed layer, the kinematic heat flux has the zero-order discontinuity, being equal to $-\Delta\theta(dh/dt)$ beneath the mixed-layer interface, and 0 above it.

For this type of convective layer the entrainment rate equation (12) reduces to the form

$$\left(\bar{e} + \frac{\beta h\Delta\theta}{2}\right)\frac{dh}{dt} + \left(\frac{d\bar{e}}{dt} + \bar{\epsilon} - \frac{\beta}{2}Q_s\right)h = -\Phi_h. \tag{15}$$

The most widely used parameterizations for $\bar{e}$ and $\bar{\epsilon}$, corresponding to the case, are based on the Deardorff's (1970) hypothesis of self-similarity, stating that profiles of e and ϵ, being normalized using h as the height scale, and $w_* = (\beta Q_s h)^{1/3}$ as the velocity scale, are universal functions of dimensionless height $\zeta = z/h$. This allows representations: $\bar{e} = C_e(\beta Q_s h)^{2/3}$ for the mean value of TKE, and $\bar{\epsilon} = C_\epsilon \beta Q_s$ for the mean dissipation rate, where C_e and C_ϵ are universal constants. Based on the data of laboratory and atmospheric measurements, Zilitinkevich (1991) found these constants to be 0.5 and 0.4, respectively. The later analysis by Fedorovich and Mironov (1995), for which they employed additionally the data of LES, has shown that these estimates are exaggerated approximately by 25% due to the disturbing presence of shear during field measurements and contribution of horizontal velocity fluctuations in the laboratory tank, induced by the large-scale bottom temperature variations in the horizontal.

For the vertical transport of energy from the CBL top, the two most known parameterizations were suggested so far, the first of Kantha (1977), and the second of Zilitinkevich (1991), both based on the relationship of Thorpe (1973), who expressed the energy drain from the mixed layer through the parameters of the waves propagating in the nonturbulent fluid aloft the layer. Parameterization of Kantha, being accompanied by the geometric formula of Stull (1976b), relating the so-called entrainment coefficient $A = Q_s^{-1}\Delta\theta(dh/dt)$ to the ratio $(\Delta h/2)(h - \Delta h/2)^{-1}$, where Δh is the entrainment zone depth, results in

$$\Phi_h = C_N N^3 h^3\left(\frac{A}{1+A}\right)^2,$$

whereas Zilitinkevich's parameterization gives

$$\Phi_h = C'_N N^3 h^3\left(\frac{A}{1+A}\right)^3.$$

In the above expressions, C_N and C'_N are dimensionless constants. Zilitinkevich (1991) estimated C'_N to be about 0.02. From the experiments with general-structure CBL model, Fedorovich and Mironov (1995) found that the value of C_N is about one order of magnitude smaller than C'_N.

b. Nonsteady horizontally homogeneous convective boundary layer with wind shear

This type of CBL is typical for the atmospheric conditions when horizontal variations of the boundary layer structure are negligibly small, but the flow velocity shear contributes to the turbulence production. For the budgets of the momentum components, we obtain from Eqs. (5) and (6)

$$h\frac{d\bar{u}}{dt} - \Delta u\frac{dh}{dt} = fh\left(\bar{v} - v_{g0} - \frac{\Gamma_v h}{2}\right) - \tau_{xs}, \tag{16}$$

$$h\frac{d\bar{v}}{dt} - \Delta v\frac{dh}{dt} = -fh\left(\bar{u} - u_{g0} - \frac{\Gamma_u h}{2}\right) - \tau_{ys}. \tag{17}$$

The equation of the heat budget in this case has the same form as it has in the shear-free case, see Eq. (13).

The profiles of the turbulent fluxes are expressed as follows:

$$\tau_x = \tau_{xs}\left(1 - \frac{z}{h}\right) + \frac{z}{h}\Delta u \frac{dh}{dt} + \frac{f\Gamma_v}{2} h^2 \frac{z}{h}\left(\frac{z}{h} - 1\right), \quad (18)$$

$$\tau_y = \tau_{ys}\left(1 - \frac{z}{h}\right) + \frac{z}{h}\Delta v \frac{dh}{dt} - \frac{f\Gamma_u}{2} h^2 \frac{z}{h}\left(\frac{z}{h} - 1\right), \quad (19)$$

$$Q = Q_s\left(1 - \frac{z}{h}\right) - \frac{z}{h}\Delta\theta \frac{dh}{dt}. \quad (20)$$

As can be seen from Eqs. (18) and (19), in the case considered, the components of shear stress are quadratic functions of height. At the mixed-layer top the increments of the momentum flux components are $\Delta u(dh/dt)$ and $\Delta v(dh/dt)$, while the discontinuity of the heat flux has the same value as in the shear-free case: $-\Delta\theta(dh/dt)$.

The equation describing the temporal variations of the convective boundary layer depth h is obtained from the general-case entrainment equation (12) by neglecting the terms responsible for the spatial variations of the boundary layer characteristics:

$$\left[\bar{e} - \frac{1}{2}(\Delta u^2 + \Delta v^2 - \beta h \Delta\theta)\right]\frac{dh}{dt} + \left(\frac{d\bar{e}}{dt} - \frac{\beta}{2}Q_s + \bar{\epsilon}\right)h = \bar{u}\tau_{xs} + \bar{v}\tau_{ys} - \Phi_h. \quad (21)$$

To close the problem, it is still left to define the way of parameterizing the energy flux at the mixed-layer top, Φ_h, and the characteristics of the turbulent regime, $\bar{e}$ and $\bar{\epsilon}$.

For Φ_h in the case with shear, no parameterizations have been proposed yet. To use Kantha's (1977) or Zilitinkevich's (1991) formulations, one has to determine the value of the entrainment-zone depth Δh, which cannot be done in this case as easily as in the model of shear-free CBL because geometric formula does not work any more. One of possible solutions is to use the diagnostic relationship between the normalized entrainment zone depth $\Delta h/h$ and dimensionless parameter of entrainment $\mathrm{Ri}_E = \beta\Delta\theta(dh/dt)^{-2}h$, suggested by Gryning and Batchvarova (1994) and utilized later in their applied model for the height of the daytime mixed layer and the entrainment zone (Batchvarova and Gryning 1994): $\Delta h/h = 3.3\ \mathrm{Ri}_E^{-1/3} + 0.2$.

In the applied models of CBL, based on bulk approach, the TKE balance equation is employed usually in stationary form (Stull 1976a; Zilitinkevich et al. 1992; Gryning and Batchvarova 1994). As seen from Eqs. (11) and (21), this removes the problem of parameterizing $\bar{e}$, but leaves open the problem of defining appropriate scales to evaluate mean dissipation rate. In most cases, $\bar{\epsilon}$ is set merely equal to a combination of components of TKE integral production, each taken with empirical coefficient of proportionality (Stull 1976a; Tennekes and Driedonks 1981; Driedonks 1982; Driedonks and Tennekes 1984; Batchvarova and Gryning 1991). The shear forcings in these models are parameterized in terms of u_*^3 (at the CBL bottom), and of $(\Delta\bar{V})^3$ (at the upper interface), where $u_* = (\tau_{xs}^2 + \tau_{ys}^2)^{1/2}$ is the friction velocity, and $\Delta\bar{V}$ is the wind shear across the top of CBL. The direct integration of the TKE balance equation, invoking the assumptions of zero-order jump approach, see (21), yields quite different expressions of shear production: $\bar{u}\tau_{xs} + \bar{v}\tau_{ys}$ (at the surface), and $0.5(\Delta u^2 + \Delta v^2)dh/dt$ (at the mixed-layer top). Generally, it is not so evident even for average conditions whether $\bar{u}\tau_{xs} + \bar{v}\tau_{ys}$ can be taken proportional to u_*^3, and $0.5(\Delta u^2 + \Delta v^2)dh/dt$ to $(\Delta\bar{V})^3$. It is easy to notice also that the first of the last two terms always remains positive during the convection, whereas the second has the sign of $\Delta\bar{V}$ and thus can be a source of negative production of TKE, which is physically senseless.

c. *Atmospheric convective boundary layer over the irregular terrain*

Under realistic atmospheric conditions, the convective boundary layer develops over irregular underlying surfaces whose topography and aerodynamic properties vary in space. If these variations are not very sharp, it is possible to generalize the above theory for the case when the upper interface of CBL is a function of time and horizontal coordinates: $h = h(t, x, y)$, and the lower interface is represented by some known function describing the topography: $H = H(x, y)$ (see Fig. 2).

The initial equations for the case under consideration differ slightly from the basic equations (1)–(4) rep-

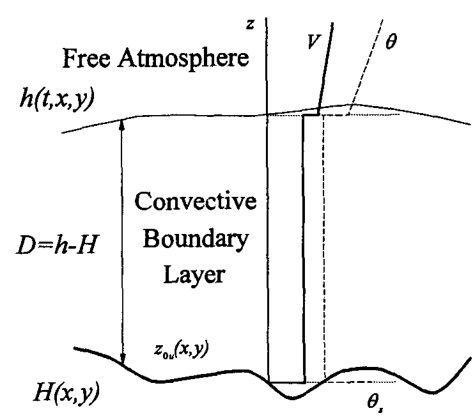

FIG. 2. Schematic of the atmospheric CBL over the irregular terrain.

resenting the general case of the boundary layer over the flat surface. We still assume that mass consistency and heat balance can be expressed using the relationships (3) and (4), respectively. In the momentum balance equations, the first terms in the right-hand sides have to be modified in the following way [cf. the corresponding terms of Eqs. (1) and (2)]:

$$f[v - v_{g0} - \Gamma_v(z - H)],$$
$$-f[u - u_{g0} - \Gamma_u(z - H)],$$

since now the near-surface values of the geostrophic wind are prescribed at the level $H(x, y)$. Functions Γ_v and Γ_u become the internal parameters of the model. We shall outline later the method for evaluating them from the temperature field.

Let us transform the coordinate system, introducing new time and space variables in the following manner:

$$t_n = t, \quad x_n = x, \quad y_n = y,$$
$$z_n = \frac{z - H(x, y)}{h(t, x, y) - H(x, y)},$$

where the new variables are denoted by n subscript.

In the transformed coordinates, the first equation of motion can be integrated over z_n, see appendix B, yielding the momentum budget equation

$$D\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) = \Delta u\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right) + D\left[f(\bar{v} - v_{g0}) - \frac{D}{2}f\Gamma_v\right] - \tau_{xs}, \quad (22)$$

which is very similar to Eq. (5) presenting the integral balance of the x component of momentum in the convective boundary layer over the flat surface. In Eq. (22), $D(t, x, y) = h - H$ is the relative CBL depth, and averaging is defined as integration over the dimensionless vertical coordinate.

In an analogous method, the equation of the momentum balance along the y axis

$$D\left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y}\right) = \Delta v\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right) - D\left[f(\bar{u} - u_{g0}) - \frac{D}{2}f\Gamma_u\right] - \tau_{ys} \quad (23)$$

and the integral heat balance equation

$$D\left(\frac{\partial \bar{\theta}}{\partial t} + \bar{u}\frac{\partial \bar{\theta}}{\partial x} + \bar{v}\frac{\partial \bar{\theta}}{\partial y}\right) = \Delta\theta\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right) + Q_s \quad (24)$$

can be obtained.

Integration of the momentum balance, and heat transfer equations over the normalized vertical coordinate from 0 to z_n yields the representations of the profiles of the shear stress components and turbulent heat flux as functions of z_n:

$$\tau_x = \tau_{xs}(1 - z_n) + \Delta u\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right)z_n + \frac{D^2}{2}f\Gamma_v z_n(z_n - 1), \quad (25)$$

$$\tau_y = \tau_{ys}(1 - z_n) + \Delta v\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right)z_n - \frac{D^2}{2}f\Gamma_u z_n(z_n - 1), \quad (26)$$

$$Q = Q_s(1 - z_n) - \Delta\theta\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right)z_n. \quad (27)$$

Substituting the above expressions into the TKE balance equation (14) written in the new coordinate system, and carrying out the termwise integration of this equation in the method similar to that employed in section 4, we obtain the version of the entrainment rate equation for the convective boundary layer over the irregular terrain:

$$\left[\bar{e} + \frac{1}{2}(\Delta u^2 + \Delta v^2 - \beta h\Delta\theta)\right]\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}D}{\partial x} + \frac{\partial \bar{v}D}{\partial y}\right) + \left(\frac{\partial \bar{e}}{\partial t} + \bar{u}\frac{\partial \bar{e}}{\partial x} + \bar{v}\frac{\partial \bar{e}}{\partial y} - \frac{\beta}{2}Q_s + \bar{\epsilon}\right)D = \bar{u}\tau_{xs} + \bar{v}\tau_{ys} - \Phi_h, \quad (28)$$

which has a form quite similar to that of the entrainment rate equation for the basic case, see Eq. (12).

6. Parameterization of horizontal turbulent diffusion

Contribution of horizontal diffusion to the momentum and heat budgets of convective boundary layer can be taken into account by adding the divergence components of corresponding turbulent fluxes to the right-hand sides of balance equations (1), (2), and (4). The terms $\partial\tau_{xx}/\partial x$ and $\partial\tau_{xy}/\partial y$ appear in the first equation of motion (1); the second equation is complemented by $\partial\tau_{yx}/\partial x$ and $\partial\tau_{yy}/\partial y$, where $\tau_{x_ix_j} = -\langle u_i'u_j'\rangle$ are components of the turbulent flux of momentum, normalized by density. Additional influxes of heat in the heat balance equation are expressed as $-\partial Q_x/\partial x$ and $-\partial Q_y/\partial y$, where $Q_{x_i} = \langle u_i'\theta'\rangle$ are the components of horizontal kinematic heat flux. As seen from the above expressions, horizontal diffusion plays an important part when variability of turbulent fluxes in the horizontal is enhanced. This may occur, for example, when the convective boundary layer develops over the irregular underlying surface.

To parameterize the horizontal diffusion, we assume the values of the fluxes to be proportional to the horizontal gradients of mean characteristics of the flow:

$$\tau_{x_i x_j} = K_{uj}\frac{\partial u_i}{\partial x_j}, \quad Q_{x_j} = -K_{\theta j}\frac{\partial \theta}{\partial x_j}, \tag{29}$$

where K_{uj} and $K_{\theta j}$ are the coefficients of turbulent transfer over coordinate x_j for momentum and heat, respectively. Such relationships between fluxes and gradients of corresponding substances are widely used in mesoscale atmospheric modeling (see, e.g., Pielke 1984). Within the zero-order jump approach, coefficients K_{uj} and $K_{\theta j}$ (turbulent diffusivities) can be taken constant with height in the mixed layer. In the turbulence-free layer they equal zero. With due regard to the employed representations of velocity and temperature profiles this allows us to express integral contributions of horizontal diffusion to the budgets of momentum and heat in the following way:

$$\int_0^h \frac{\partial \tau_{x_i x_j}}{\partial x_j} dz = \frac{\partial}{\partial x_j} K_{uj} \int_0^h \frac{\partial u_i}{\partial x_j} dz - \left(K_{uj}\frac{\partial u_i}{\partial x_j}\right)_h \frac{\partial h}{\partial x_j} = \frac{\partial}{\partial x_j}\left(hK_{uj}\frac{\partial \bar{u}_i}{\partial x_j}\right), \tag{30}$$

$$\int_0^h \frac{\partial Q_{x_j}}{\partial x_j} dz = -\frac{\partial}{\partial x_j} K_{\theta j} \int_0^h \frac{\partial \theta}{\partial x_j} dz + \left(K_{\theta j}\frac{\partial \theta}{\partial x_j}\right)_h \frac{\partial h}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(hK_{\theta j}\frac{\partial \bar{\theta}}{\partial x_j}\right). \tag{31}$$

It can be shown that horizontal diffusion changes the profiles of vertical fluxes of momentum and heat as well. If we integrate momentum and heat balance equations from 0 to z, taking into account horizontal diffusion terms, and make the substitutions analogous to those we have done in the basic case (see section 3), we shall obtain additional terms in the expressions of τ_x, τ_y, and Q, [cf. Eqs. (8)–(10)]:

$$\tau_x = \tau_{xs}(1-\zeta) + \left[\Delta u\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}h}{\partial x} + \frac{\partial \bar{v}h}{\partial y}\right) + K_{ux}\frac{\partial \bar{u}}{\partial x}\frac{\partial h}{\partial x} + K_{uy}\frac{\partial \bar{u}}{\partial y}\frac{\partial h}{\partial y}\right]\zeta + \frac{h^2}{2} f\Gamma_v \zeta(\zeta - 1), \tag{32}$$

$$\tau_y = \tau_{ys}(1-\zeta) + \left[\Delta v\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}h}{\partial x} + \frac{\partial \bar{v}h}{\partial y}\right) + K_{ux}\frac{\partial \bar{v}}{\partial x}\frac{\partial h}{\partial x} + K_{uy}\frac{\partial \bar{v}}{\partial y}\frac{\partial h}{\partial y}\right]\zeta - \frac{h^2}{2} f\Gamma_u \zeta(\zeta - 1), \tag{33}$$

$$Q = Q_s(1-\zeta) - \left[\Delta\theta\left(\frac{\partial h}{\partial t} + \frac{\partial \bar{u}h}{\partial x} + \frac{\partial \bar{v}h}{\partial y}\right) + K_{\theta x}\frac{\partial \bar{\theta}}{\partial x}\frac{\partial h}{\partial x} + K_{\theta y}\frac{\partial \bar{\theta}}{\partial y}\frac{\partial h}{\partial y}\right]\zeta. \tag{34}$$

As seen from the above expressions, the profiles of momentum flux components retain the quadratic shapes, and kinematic heat flux is presented by linear function, as in the case without horizontal diffusion. The values of increments of the fluxes across the mixed-layer interface can be obtained from the expressions of profiles in a usual way, by setting $\zeta = 1$.

We can use similar parameterization for the horizontal turbulent transport of TKE. The terms describing this transport appear as additional two terms $-\partial\Phi_x/\partial x - \partial\Phi_y/\partial y$ in the right-hand side of TKE balance equation (11). Integrating them over z from 0 to h, and assuming that $\Phi_{x_j} = -K_{ej}(\partial e/\partial x_j)$, where K_{ej} are the turbulence diffusivities for TKE, we obtain

$$-\int_0^h \left(\frac{\partial \Phi_x}{\partial x} + \frac{\partial \Phi_y}{\partial y}\right) dz = \frac{\partial}{\partial x}\left(hK_{ex}\frac{\partial \bar{e}}{\partial x}\right) + \frac{\partial}{\partial y}\left(hK_{ey}\frac{\partial \bar{e}}{\partial y}\right). \tag{35}$$

The above terms represent the direct modification of the TKE balance by horizontal turbulent transfer of energy. Additional changes of turbulent regime are induced by effects of horizontal diffusion on profiles of vertical turbulent fluxes of momentum and heat, see Eqs. (32)–(34). These effects will modify the integral shear production of TKE:

$$\int_0^h \left(\tau_x\frac{\partial u}{\partial z} + \tau_y\frac{\partial v}{\partial z}\right) dz = \bar{u}\tau_{xs} + \bar{v}\tau_{ys} + \frac{1}{2}\left[(\Delta u^2 + \Delta v^2)\frac{Dh}{Dt} + \Delta u K_{ux}\frac{\partial \bar{u}}{\partial x}\frac{\partial h}{\partial x} + \Delta v K_{uy}\frac{\partial \bar{u}}{\partial y}\frac{\partial h}{\partial y}\right], \tag{36}$$

as well as its integral buoyant production

$$\int_0^h \beta Q dz = \beta\frac{h}{2}\left(Q_s - \Delta\theta\frac{Dh}{Dt} + K_{\theta x}\frac{\partial \bar{\theta}}{\partial x}\frac{\partial h}{\partial x} + K_{\theta y}\frac{\partial \bar{\theta}}{\partial y}\frac{\partial h}{\partial y}\right), \tag{37}$$

which can be noticed by comparing Eqs. (36) and (37) with their counterparts for the basic case (see section 4).

The formulations of horizontal diffusion contributions to the integral budgets of momentum and heat, to the flux profiles representations, and to the integral balance of TKE can be easily generalized for parameterizing the horizontal turbulent transfer in the convective boundary layer over irregular terrain by replacing h by D in terms containing h or its horizontal partial derivatives as multipliers (see section 5).

The values of turbulent exchange coefficients K_{ux}, K_{uy}, $K_{\theta x}$, $K_{\theta y}$, K_{ex}, and K_{ey} complement the list of model parameters to be defined. For the purposes of the applied modeling, these coefficients can be presumed equal to each other, and constant in space: $K_{ux} = K_{uy} = K_{\theta x} = K_{\theta y} = K_{ex} = K_{ey} = K = \text{const}$. The detailed recipes of defining coefficients of horizontal turbulent exchange in the mesoscale models can be found in Pielke (1984).

7. Prescribing the values of external parameters

In the atmospheric modeling, the value of the near-surface heat flux Q_s is commonly derived from the surface heat balance equation, provided the methods of evaluating the other components of the balance are known. The simple algorithm for this purpose was proposed in Zilitinkevich et al. (1992), which incorporates the expression relating $\bar{\theta}$ to the near-surface value of temperature θ_s, and formula for calculating the roughness parameter for temperature.

Departing from the value of Q_s, and assuming that the roughness parameter of the underlying surface relative to the wind is a given function of x and y, the components of the near-surface shear stress in the atmospheric convective boundary layer, τ_{xs} and τ_{ys}, can be evaluated. In the above mentioned paper of Zilitinkevich et al. (1992), the following relationships between the components of the stress and the components of velocity in the mixed layer were used:

$$\bar{V} = u_* \frac{a_u + \ln(|L| z_{0u}^{-1})}{k},$$

$$\bar{u} = \bar{V} \cos\alpha_h, \quad \bar{v} = \bar{V} \sin\alpha_h,$$

$$\sin(\alpha_s - \alpha_h) = \frac{a_\alpha}{k}\left(\frac{h}{|L|}\right)^{-1/3} \frac{u_*}{\bar{V}} \operatorname{sgn} f,$$

$$\tau_{xs} = u_*^2 \cos\alpha_s, \quad \tau_{ys} = u_*^2 \sin\alpha_s,$$

where α_h is the angle between the x axis and the velocity vector $\bar{\mathbf{V}}$ with components $\bar{u}$ and $\bar{v}$, α_s is the angle between the x axis and the shear stress vector at the surface, $L = -u_*^2(\beta Q_s)^{-1}$ is the Monin–Obukhov length scale, z_{0u} is the roughness parameter relative to wind, $k = 0.4$ is the von Kármán constant, and a_u and a_α are dimensionless parameters for which Zilitinkevich (1975b) obtained estimates 1 and 3, respectively.

The above expressions constitute the closed set of equations for determination of the components of the near-surface momentum flux, provided Q_s, $\bar{u}$, $\bar{v}$, and z_{0u} are known.

The near-surface values of the geostrophic wind components in the atmospheric boundary layer, u_{g0} and v_{g0}, can be expressed through the horizontal gradients of the near-surface pressure field p_s by the well-known formulas:

$$u_{g0} = -\frac{1}{f\rho_s}\frac{\partial p_s}{\partial y}, \quad v_{g0} = \frac{1}{f\rho_s}\frac{\partial p_s}{\partial x},$$

where ρ_s is the near-surface density. Both p_s and ρ_s can be evaluated from the weather forecast data, or from the output of some larger-scale atmospheric model.

The vertical variations of the geostrophic wind are usually related to the effects of the so-called thermal wind, which is one of the manifestations of the baroclinicity in the atmosphere. In CBL, the thermal wind appears as a result of the thermal nonuniformity of the layer in the horizontal. Under these conditions, the vertical gradients of the geostrophic wind components can be represented in the first approximation as

$$\Gamma_u = -\frac{\beta}{f}\frac{\partial\bar{\theta}}{\partial y}, \quad \Gamma_v = \frac{\beta}{f}\frac{\partial\bar{\theta}}{\partial x},$$

following traditional thermal wind relationship given, for example, in Holton (1972), and using the CBL bulk potential temperature values.

The theory presented in the previous sections of the paper deals with different cases of dry CBL. To account for the effects of air humidity in the unsaturated atmosphere, one should complement the model with equation of moisture transfer. It is shown in Zilitinkevich et al. (1992) that within the zero-order jump approach the forms of CBL integral humidity budget equation, and profile of moisture turbulent flux are analogous to their potential temperature counterparts. In the terms of model equations, representing the buoyancy effects, the values of potential temperature θ should be replaced by corresponding values of the virtual potential temperature, $\theta_v = \theta(1 + 0.61q)$, where q is the specific air humidity, and the buoyancy flux taken in the form $\beta Q + 0.61 gE$, where E is the kinematic turbulent flux of moisture.

8. Summary

Zero-order jump approach proves to be an effective tool for applied modeling of the atmospheric convective boundary layer. Within this model framework a variety of physical mechanisms determining the temporal and spatial structure of CBL can be accounted for with limited consumption of computer resources. An advantageous opportunity the zero-order jump modeling presents for applied studies of the wind field in the planetary boundary layer, when mass-consistent models (Dickerson 1978; Sherman 1978), used widely for this purpose, fail to describe appropriately perturbations of the velocity field induced by thermal effects during convection. The pollutants dispersion modeling can be noted as one more prospective area of application of the zero-order jump approach.

In the present paper, the zero-order jump model equations for the most typical cases of CBL were derived. The models of nonsteady, horizontally homogeneous CBL with and without shear, extensively studied in the past with the aid of zero-order jump models, were shown to be particular cases of the general zero-order jump theoretical framework. The integral budgets of momentum and heat were considered for different types of dry CBL. The profiles of vertical turbulent fluxes were presented and analyzed. The general version of the equation of CBL depth growth rate (entrainment rate equation) was obtained by the integration of the TKE balance equation, invoking basic assumptions of the zero-order parameterizations of the CBL vertical structure. The problems of parameterizing the turbulence vertical structure and closure of the entrainment rate equation for specific cases of CBL were discussed, the parameterization scheme for the horizontal turbulent exchange in zero-order jump models of CBL was proposed, and the theory was generalized for the case of CBL over irregular terrain.

The progress in the applied modeling of CBL within the framework of the zero-order jump approach depends essentially on the success in the studies aimed at defining the appropriate scalings for parameterization of CBL turbulent structure in the general case. Large-eddy simulations of convection are expected to provide the main information background for such studies. Additional experiments in the laboratory (wind-tunnel studies) and in nature are also of vital importance for the promotion of knowledge on CBL structure under combined effects of buoyancy and wind shear.

Acknowledgments. The author gratefully acknowledges the support from the Alexander von Humboldt Foundation, Germany, in the course of the presented study.

APPENDIX A

Derivation of the Momentum Budget Equation

Integration of the nonstationarity term of the first equation of motion is performed in the following way:

$$\int_0^h \frac{\partial u}{\partial t} dz = \frac{\partial}{\partial z} \int_0^h u dz - u_h \frac{\partial h}{\partial t}$$
$$= \frac{\partial}{\partial t} \bar{u} h - u_h \frac{\partial h}{\partial t} = -\Delta u \frac{\partial h}{\partial t} + h \frac{\partial \bar{u}}{\partial t}. \quad \text{(A1)}$$

Carrying out the integration, we employed the rule of differentiating the integrals with variable limits, since in our case $h = h(t, x, y)$. Applying this rule to the other terms in the left-hand side of Eq. (1), taking into account that the adopted representation of the velocity profile implies $\overline{uu} = \bar{u}^2$, $\overline{vu} = \bar{v}\bar{u}$, and using condition $u = v = w = 0$ at $z = 0$, we obtain

$$\int_0^h \frac{\partial uu}{\partial x} dz = \frac{\partial}{\partial x} \int_0^h uudz - u_h^2 \frac{\partial h}{\partial x}$$
$$= (\bar{u}^2 - u_h^2) \frac{\partial h}{\partial x} + h \frac{\partial \bar{u}^2}{\partial x}, \quad \text{(A2)}$$

$$\int_0^h \frac{\partial vu}{\partial y} dz = \frac{\partial}{\partial y} \int_0^h vudz - v_h u_h \frac{\partial h}{\partial y}$$
$$= (\bar{v}\bar{u} - v_h u_h) \frac{\partial h}{\partial y} + h \frac{\partial \bar{v}\bar{u}}{\partial y}, \quad \text{(A3)}$$

$$\int_0^h \frac{\partial wu}{\partial z} dz = w_h u_h. \quad \text{(A4)}$$

Now we can integrate the mass conservation equation (3) to find the value of vertical velocity at the mixed-layer top as follows:

$$w_h = -\int_0^h \frac{\partial u}{\partial x} dz - \int_0^h \frac{\partial v}{\partial y} dz$$
$$= -\frac{\partial \bar{u} h}{\partial x} + u_h \frac{\partial h}{\partial x} - \frac{\partial \bar{v} h}{\partial y} + v_h \frac{\partial h}{\partial y}. \quad \text{(A5)}$$

Integration of the first term in the right-hand side of Eq. (1) yields

$$\int_0^h f(v - v_{g0} - \Gamma_v z) dz = fh\left(\bar{v} - v_{g0} - \frac{\Gamma_v h}{2}\right). \quad \text{(A6)}$$

For the last term of Eq. (1), we have

$$\int_0^h \frac{\partial \tau_x}{\partial z} dz = -\tau_{xs}, \quad \text{(A7)}$$

where τ_{xs} is the near-surface value of the x component of the shear stress. It is assumed that there is no turbulence at the upper side of the mixed-layer interface, and hence we set $\tau_{xh} = 0$.

APPENDIX B

Momentum Budget in CBL over the Irregular Terrain

The relationships between the temporal and spatial derivatives in the initial and transformed coordinate systems are as follows:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_n} + \frac{\partial z_n}{\partial t} \frac{\partial}{\partial z_n} = \frac{\partial}{\partial t_n} - \frac{z_n h_t}{D} \frac{\partial}{\partial z_n}, \quad \text{(B1)}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_n} + \frac{\partial z_n}{\partial x} \frac{\partial}{\partial z_n}$$
$$= \frac{\partial}{\partial x_n} - \frac{z_n h_x + (1 - z_n) H_x}{D} \frac{\partial}{\partial z_n}, \quad \text{(B2)}$$

$$\frac{\partial}{\partial y}=\frac{\partial}{\partial y_n}+\frac{\partial z_n}{\partial y}\frac{\partial}{\partial z_n}$$
$$=\frac{\partial}{\partial y_n}-\frac{z_nh_y+(1-z_n)H_y}{D}\frac{\partial}{\partial z_n}, \qquad \text{(B3)}$$
$$\frac{\partial}{\partial z}=\frac{\partial z_n}{\partial z}\frac{\partial}{\partial z_n}=\frac{1}{D}\frac{\partial}{\partial z_n}, \qquad \text{(B4)}$$

where $D(t,x,y)\equiv h-H$, $(\ )_t=\partial/\partial t$, $(\ )_x=\partial/\partial x$, $(\ )_y=\partial/\partial y$.

Substituting the above expressions into the first of the momentum balance equations (1), integrating the resulting equation over z_n from 0 to 1, that is, over the normalized boundary layer depth, and taking into account the adopted representation of the velocity field, we obtain

$$\frac{\partial\bar u}{\partial t}-\frac{\Delta u h_t}{D}+\frac{\partial\bar u^2}{\partial x}+\frac{1}{D}[h_x(\bar u^2-u_h^2)-H_x\bar u^2]$$
$$+\frac{\partial\bar v\bar u}{\partial y}+\frac{1}{D}[h_y(\bar v\bar u-v_hu_h)-H_y\bar v\bar u]$$
$$+\frac{w_hu_h}{D}=f(\bar v-v_{g0})-\frac{D}{2}f\Gamma_v, \qquad \text{(B5)}$$

where the notation for the variables is equivalent to their designations in the basic case (section 3, appendix A), and n subscripts at t, x, and y are omitted hereafter for simplicity.

The expression of the vertical velocity at the boundary layer top can be derived in the usual way, by the integration of the mass conservation equation (3). This gives

$$w_h=-D\left(\frac{\partial\bar u}{\partial x}+\frac{\partial\bar v}{\partial y}\right)+h_xu_h-D_x\bar u+h_yv_h-D_y\bar v. \qquad \text{(B6)}$$

Combining the above expression with (B5), we come to the equation of the integral momentum balance along the x axis in CBL over the irregular terrain, that is Eq. (22) of section 5.

REFERENCES

Ball, F. K., 1960: Control of inversion height by surface heating. *Quart. J. Roy. Meteor. Soc.*, **86**, 483–494.

Batchvarova, E., and S.-E. Gryning, 1991: Applied model for the growth of the daytime mixed layer. *Bound.-Layer Meteor.*, **56**, 261–274.

——, and ——, 1994: An applied model for the height of the daytime mixed layer and the entrainment zone. *Bound.-Layer Meteor.*, **71**, 311–323.

Betts, A. K., 1973: Non-precipitating cumulus convection and its parameterization. *Quart. J. Roy. Meteor. Soc.*, **99**, 178–196.

——, 1974: Reply to comment on the paper "Non-precipitating cumulus convection and its parameterization." *Quart. J. Roy. Meteor. Soc.*, **100**, 469–471.

Brutsaert, W., 1987: Nearly steady convection and the boundary layer budgets of water vapor and sensible heat. *Bound.-Layer Meteor.*, **39**, 283–300.

Carson, D. J., 1973: The development of dry inversion-capped convectively unstable boundary layer. *Quart. J. Roy. Meteor. Soc.*, **99**, 450–467.

——, and F. B. Smith, 1974: Thermodynamic model for the development of a convectively unstable boundary layer. *Advances in Geophysics*, Vol. 18A, Academic Press, 111–124.

Deardorff, J. W., 1970a: Preliminary results from numerical integration of the unstable boundary layer. *J. Atmos. Sci.*, **27**, 1209–1211.

——, 1970b: Convective velocity and temperature scales for the unstable planetary boundary layer and for Raleigh convection. *J. Atmos. Sci.*, **27**, 1211–1213.

——, 1972: Numerical investigation of neutral and unstable planetary boundary layers. *J. Atmos. Sci.*, **29**, 91–115.

——, 1974: Three dimensional numerical study of turbulence in an entraining mixed layer. *Bound.-Layer Meteor.*, **7**, 199–226.

——, 1979: Prediction of convective mixed-layer entrainment for realistic capping inversion structure. *J. Atmos. Sci.*, **36**, 424–436.

Dickerson, C. A., 1978: MASCON—A mass-consistent model atmospheric flux model for regions with complex terrain. *J. Appl. Meteor.*, **17**, 241–253.

Driedonks, A. G. M., 1982: Models and observations of the growth of the atmospheric boundary layer. *Bound.-Layer Meteor.*, **23**, 283–306.

——, and H. Tennekes, 1984: Entrainment effects in the well-mixed atmospheric boundary layer. *Bound.-Layer Meteor.*, **30**, 75–103.

Fedorovich, E. E., and D. V. Mironov, 1995: A model for shear-free convective boundary layer with parameterized capping inversion structure. *J. Atmos. Sci.*, **52**, 83–95.

Garrat, J. R., J. C. Wyngaard, and R. J. Francey, 1982: Winds in the atmospheric boundary layer—Prediction and observation. *J. Atmos. Sci.*, **39**, 1307–1316.

Gryning, S.-E., and E. Batchvarova, 1994: Parameterization of the depth of the entrainment zone above the daytime mixed layer. *Quart. J. Roy. Meteor. Soc.*, **120**, 47–58.

Holton, J. R., 1972: *An Introduction to Dynamic Meteorology*. Academic Press, 319 pp.

Kantha, L. H., 1977: Note on the role of internal waves in thermocline erosion. *Modelling and Predictions of the Upper Layer of the Ocean*, E. B. Kraus, Ed., Pergamon Press, 173–177.

Kraus, E. B., and L. D. Leslie, 1982: The interactive evolution of the oceanic and atmospheric boundary layers in the source regions of the trades. *J. Atmos. Sci.*, **39**, 2760–2772.

Lilly, D. K., 1968: Models of cloud-topped mixed layers under a strong inversion. *Quart. J. Roy. Meteor. Soc.*, **94**, 292–309.

Mason, P. J., 1989: Large-eddy simulation of the convective atmospheric boundary layer. *J. Atmos. Sci.*, **46**, 1492–1516.

——, 1994: Large-eddy simulation: A critical review of the technique. *Quart. J. Roy. Meteor. Soc.*, **120**, 1–26.

Moeng, C.-H., 1984: A large-eddy-simulation model for the study of planetary boundary layer turbulence. *J. Atmos. Sci.*, **41**, 2052–2062.

——, 1986: Large-eddy simulation of a stratus-topped boundary layer. Part I: Structure and budgets. *J. Atmos. Sci.*, **43**, 2886–2900.

——, 1987: Large-eddy simulation of a stratus-topped boundary layer. Part II: Implications for mixed-layer modeling. *J. Atmos. Sci.*, **44**, 1606–1614.

——, and J. C. Wyngaard, 1988: Spectral analysis of large-eddy simulations of the convective boundary layer. *J. Atmos. Sci.*, **45**, 3574–3587.

——, and P. P. Sullivan, 1994: A comparison of shear- and buoyancy-driven planetary boundary layer flows. *J. Atmos. Sci.*, **51**, 999–1022.

Nieuwstadt, F. T. M., and R. A. Brost, 1986: Decay of convective turbulence. *J. Atmos. Sci.*, **43**, 532–546.

——, P. J. Mason, C.-H. Moeng, and U. Schumann, 1993: Large-eddy simulation of the convective boundary layer: A comparison of four computer codes. *Turbulent Shear Flows 8*, F. Durst et al., Eds., Springer-Verlag, 343–367.

Pielke, R. A., 1984: *Mesoscale Meteorological Modeling.* Academic Press, 612 pp.
Qi, Y., J. Zhou, and B. Fu, 1994: Airflow over a mountain and the convective boundary layer. *Bound.-Layer Meteor.,* **68,** 301–318.
Schmidt, H., and U. Schumann, 1989: Coherent structures of the convective boundary layer derived from large-eddy simulations. *J. Fluid. Mech.,* **200,** 511–562.
Schumann, U., and C.-H. Moeng, 1991: Plume fluxes in clear and cloudy convective boundary layers. *J. Atmos. Sci.,* **48,** 1746–1757.
Sherman, C. A., 1978: A mass-consistent model for wind fields over complex terrain. *J. Appl. Meteor.,* **17,** 312–319.
Stull, R. B., 1973: Inversion rise model based on penetrative convection. *J. Atmos. Sci.,* **30,** 1092–1099.
——, 1976a: The energetics of entrainment across a density interface. *J. Atmos. Sci.,* **33,** 1260–1267.
——, 1976b: Mixed-layer depth model based on turbulent energetics. *J. Atmos. Sci.,* **33,** 1268–1278.
——, 1976c: Internal gravity waves generated by penetrative convection. *J. Atmos. Sci.,* **33,** 1279–1286.
——, 1988: *An Introduction to Boundary Layer Meteorology.* Kluwer Academic, 666 pp.
Tennekes, H., 1973: A model for the dynamics of the inversion above a convective boundary layer. *J. Atmos. Sci.,* **42,** 558–567.
——, and A. J. M. Driedonks, 1981: Basic entrainment equations for the atmospheric boundary layer. *Bound.-Layer Meteor.,* **20,** 515–531.
Thorpe, S. A., 1973: Turbulence in stably stratified fluids: A review of laboratory experiments. *Bound.-Layer Meteor.,* **5,** 95–119.
Zeman, O., and H. Tennekes, 1977: Parameterization of the turbulent energy budget at the top of the daytime atmospheric boundary layer. *J. Atmos. Sci.,* **34,** 111–123.
Zilitinkevich, S. S., 1975a: Comments on "A model for the dynamics of the inversion above a convective boundary layer." *J. Atmos. Sci.,* **32,** 991–992.
——, 1975b: Resistance laws and prediction equations for the depth of the planetary boundary layer. *J. Atmos. Sci.,* **32,** 741–752.
——, 1991: *Turbulent Penetrative Convection.* Avebury Technical, 179 pp.
——, E. E. Fedorovich, and M. V. Shabalova, 1992: Numerical model of a non-steady atmospheric planetary boundary layer, based on similarity theory. *Bound.-Layer Meteor.,* **59,** 387–411.