LECTURE 37
Planetary Boundary Layer (continued)

Low Level Jet (LLJ)

Blackadar (1957) described the LLJ as an inertial oscillation arising in the PBL **in response to the sudden shut down of friction** (shutdown of mixing by dry-convective thermals) around sunset. ***K*** **changes drastically at sunset**.

Holton (1967) described LLJs over the gentle slope of the Great Plains as arising from the PBL response to **diurnal heating/cooling of the slope** (diurnal oscillation in baroclinic generation of vorticity -- the horiz component). The Holton mechanism can be important, but is often thought to be not as important as Blackadar mechanism.

Blackadar Theory

Consider a **horizontally homogeneous PBL** with **eddy viscosity parameterization** of the Reynolds stresses (but K can vary with z). The horizontal equations of motion under the Boussinesq approximation are then like (35) and (36) (lecture 34) but with provision for local derivative terms, and with K inside $\partial/\partial z$:

(*) $$\frac{\partial \bar{u}}{\partial t} = f\,\bar{v} - \frac{1}{\rho_c}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(K\frac{\partial \bar{u}}{\partial z}\right),$$

(**) $$\frac{\partial \bar{v}}{\partial t} = -f\bar{u} - \frac{1}{\rho_c}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}\left(K\frac{\partial \bar{v}}{\partial z}\right).$$

We assume that in the free atmosphere, the horizontal pgf is balanced by Coriolis force. Neither force varies in x or y (horizontal homogeneity).

As in Ekman derivation, vertical eqn of motion (in combination with horizontal homogeneity) shows that the horiz pgf is indep of z (u_g and v_g are indep of z).

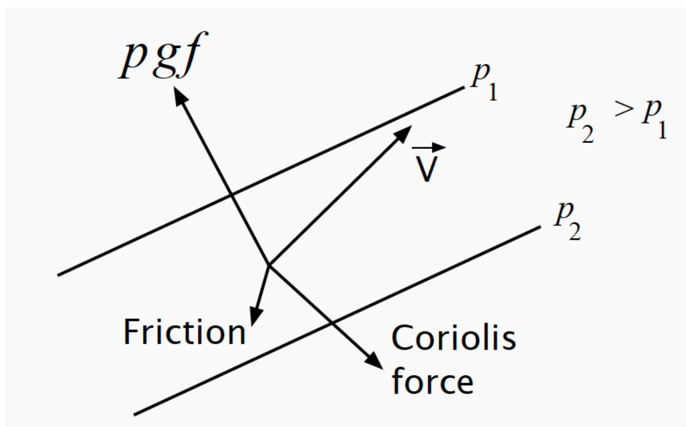
Consider a late afternoon **PBL where turbulence is largely due to dry-convective mixing by thermals** and the boundary layer is (relatively) fully developed. **The PBL flow is (relatively) stationary**. This is basically the Ekman scenario. The force balance (steady state) is a 3-way balance between friction, Coriolis and horiz pgf. Equations (*) and (**) become:

$$0 = f\,\overline{v} - \frac{1}{\rho_c}\frac{\partial p}{\partial x} + \frac{d}{dz}\left(K\frac{d\overline{u}}{dz}\right),$$

$$0 = -f\overline{u} - \frac{1}{\rho_c}\frac{\partial p}{\partial y} + \frac{d}{dz}\left(K\frac{d\overline{v}}{dz}\right).$$

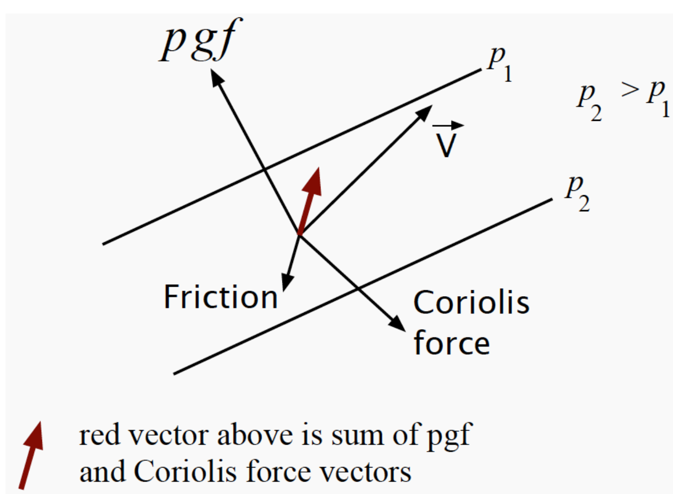
The vector force balance appears graphically as:

Late afternoon force balance in PBL



Can also think of this 3 way balance as: **friction vector exactly opposes sum of horiz pgf and Coriolis force vectors:**

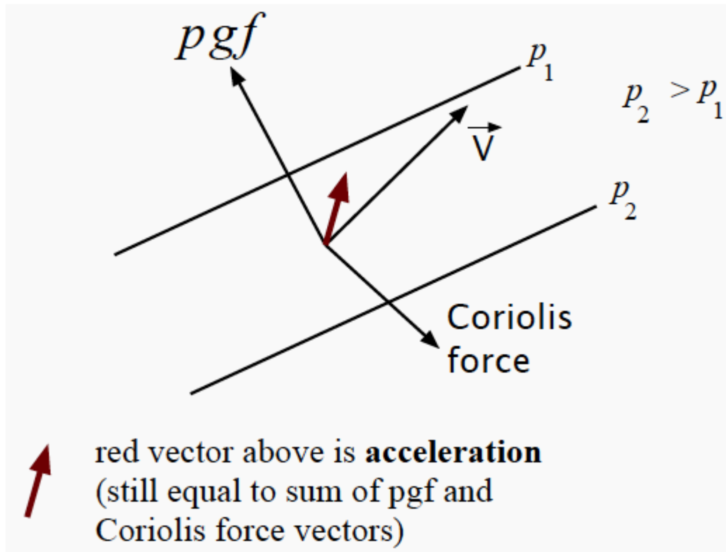
Same late afternoon force balance in PBL



red vector above is sum of pgf and Coriolis force vectors

As sun sets, surface heating ends and **turbulent mixing by thermals gets shut down** ($K \to 0$ or near 0). Friction drops out. Friction used to oppose sum of pgf and Coriolis force (red vector above). Now **sum of pgf and Coriolis is unopposed** and the flow **accelerates** (below):

Force imbalance and acceleration in PBL at sunset:



Examine response of the boundary layer to this sudden "kick" at sunset. Starting at sunset, and extending through the night, K is negligible, and (*) and (**) become:

$$\frac{\partial \bar{u}}{\partial t} = f\bar{v} - \frac{1}{\rho_c}\frac{\partial p}{\partial x},$$
$$\frac{\partial \bar{v}}{\partial t} = -f\bar{u} - \frac{1}{\rho_c}\frac{\partial p}{\partial y}.$$

There are no z-derivatives in these equations (friction has dropped out). Solve them at any height z independently of any other height z. The **initial conditions needed to solve these equations will depend on the height you choose!**

Rewrite horizontal pgf in terms of geostrophic winds ($-fv_g = -\frac{1}{\rho_c}\frac{\partial p}{\partial x}$, and $fu_g = -\frac{1}{\rho_c}\frac{\partial p}{\partial y}$), then work with ageostrophic winds $\bar{u}_a \equiv \bar{u} - u_g$, $\bar{v}_a \equiv \bar{v} - v_g$ (similar to Lecture 34 for the Ekman layer). The night-time equations then become:

(***) $\dfrac{\partial \bar{u}_a}{\partial t} = f\,\bar{v}_a,$

(****) $\dfrac{\partial \bar{v}_a}{\partial t} = -f\,\bar{u}_a.$

Apart from notational differences, we've already solved these equations [pgs 4 and 5 of handout "2nd order linear const coeff ODEs"]. The general solution for $\bar{v}_a$ is:

$$\bar{v}_a(t) = d_1 \cos ft + d_2 \sin ft.$$

Applying this in (****) (careful with signs) yields $\bar{u}_a$ as

$$\bar{u}_a = d_1 \sin ft - d_2 \cos ft.$$

These solutions are oscillatory (in time) with a period of $T = 2\pi/f$ (~21 hrs in Oklahoma). Evaluating these at sunset (t = 0), when $\bar{v}_a$ is $\bar{v}_a(0)$ and $\bar{u}_a$ is $\bar{u}_a(0)$, we get $d_1 = \bar{v}_a(0)$ and $d_2 = -\bar{u}_a(0)$ (**actual values depend on *z***). So we can write

$$\boxed{\bar{u}_a(t) = \bar{u}_a(0)\cos ft + \bar{v}_a(0)\sin ft}$$

$$\boxed{\bar{v}_a(t) = \bar{v}_a(0)\cos ft - \bar{u}_a(0)\sin ft}$$

We'll plot the evolution of the ageostrophic wind $\vec{V}_a(t) = \bar{u}_a(t)\hat{i} + \bar{v}_a(t)\hat{j}$ as a hodograph. It will help to first evaluate the magnitude of $\vec{V}_a(t)$ as a function of time:

$$\left|\vec{V}_a(t)\right| = \sqrt{[\bar{u}_a(t)]^2 + [\bar{v}_a(t)]^2}$$ plug in the above solutions for $\bar{u}_a(t)$ and $\bar{v}_a(t)$.

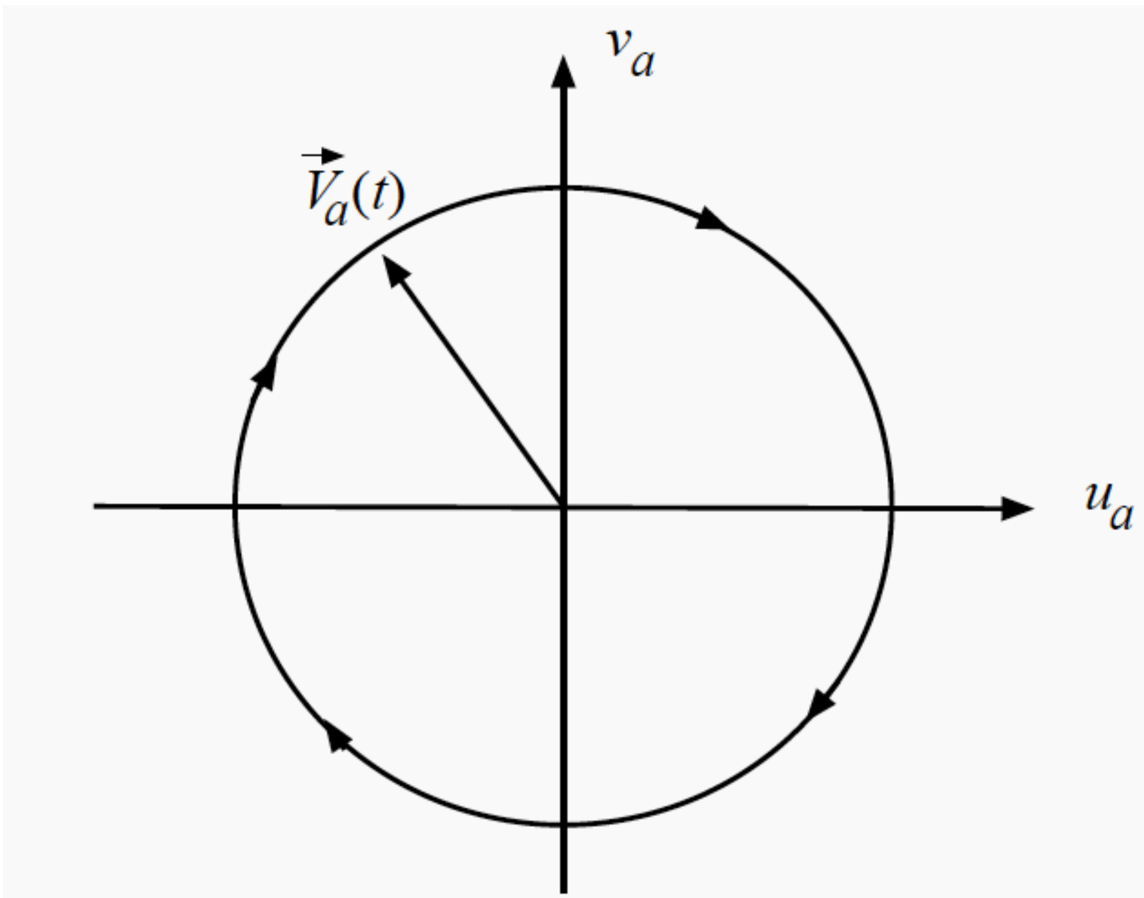
$$= \sqrt{\bar{u}_a^2(0)\cos^2 ft + 2\bar{u}_a(0)\bar{v}_a(0)\cos ft \sin ft + \bar{v}_a^2(0)\sin^2 ft + \bar{v}_a^2(0)\cos^2 ft - 2\bar{u}_a(0)\bar{v}_a(0)\cos ft \sin ft + \bar{u}_a^2(0)\sin^2 ft}$$

$$= \sqrt{\bar{u}_a^2(0)\cos^2 ft + \bar{v}_a^2(0)\sin^2 ft + \bar{v}_a{}^2(0)\cos^2 ft + \bar{u}_a^2(0)\sin^2 ft}$$ after some cancellation

$$= \sqrt{\bar{u}_a^2(0)\left[\cos^2 ft + \sin^2 ft\right] + \bar{v}_a^2(0)\left[\cos^2 ft + \sin^2 ft\right]}$$ now use $\cos^2 ft + \sin^2 ft = 1$

$= \sqrt{\overline{u}_a^2(0) + \overline{v}_a^2(0)} = \left|\vec{V}_a(0)\right|$ a constant!

Since the magnitude of $\vec{V}_a(t)$ is constant, the $\overline{u}_a$, $\overline{v}_a$ hodograph is circular and centered on $\overline{u}_a = 0$, $\overline{v}_a = 0$:



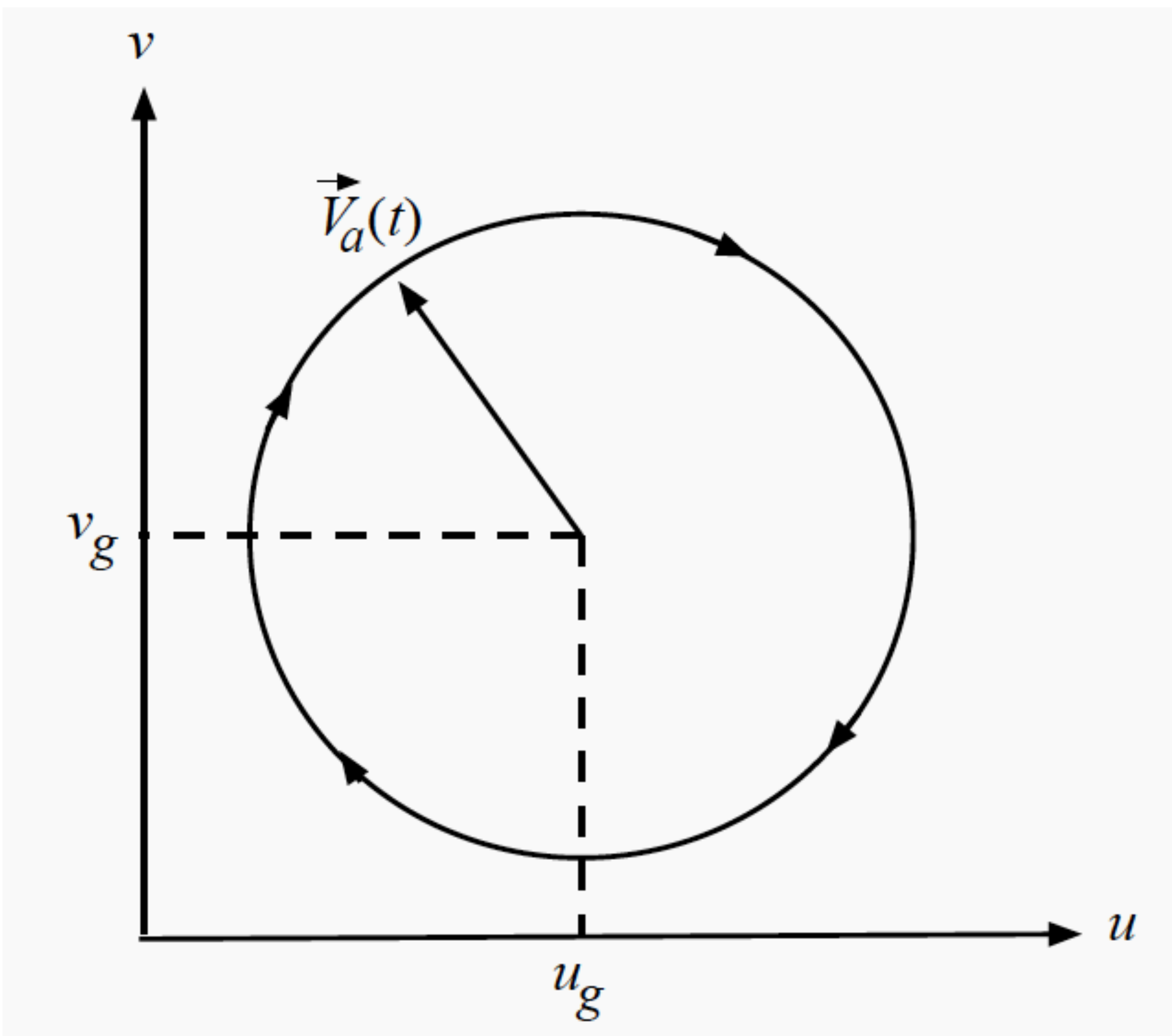
Can show that the ageostrophic wind vector rotates in the clockwise sense in N. hemisphere (also makes sense from point of view of Coriolis force trying to deflect parcels to right of where they're going). As in Ekman spiral, the hodograph curvature does NOT mean that streamlines are curved (the streamlines are all straight since the velocity field at any *z* and *t* is unidirectional). However the velocity field does change in time (as indicated above), so the trajectories are curved (circular).

Using $\overline{u}_a \equiv \overline{u} - u_g$, $\overline{v}_a \equiv \overline{v} - v_g$ we obtain the full wind components as:

$$\boxed{\overline{u}(t) = u_g + \overline{u}_a(0)\cos ft + \overline{v}_a(0)\sin ft}$$

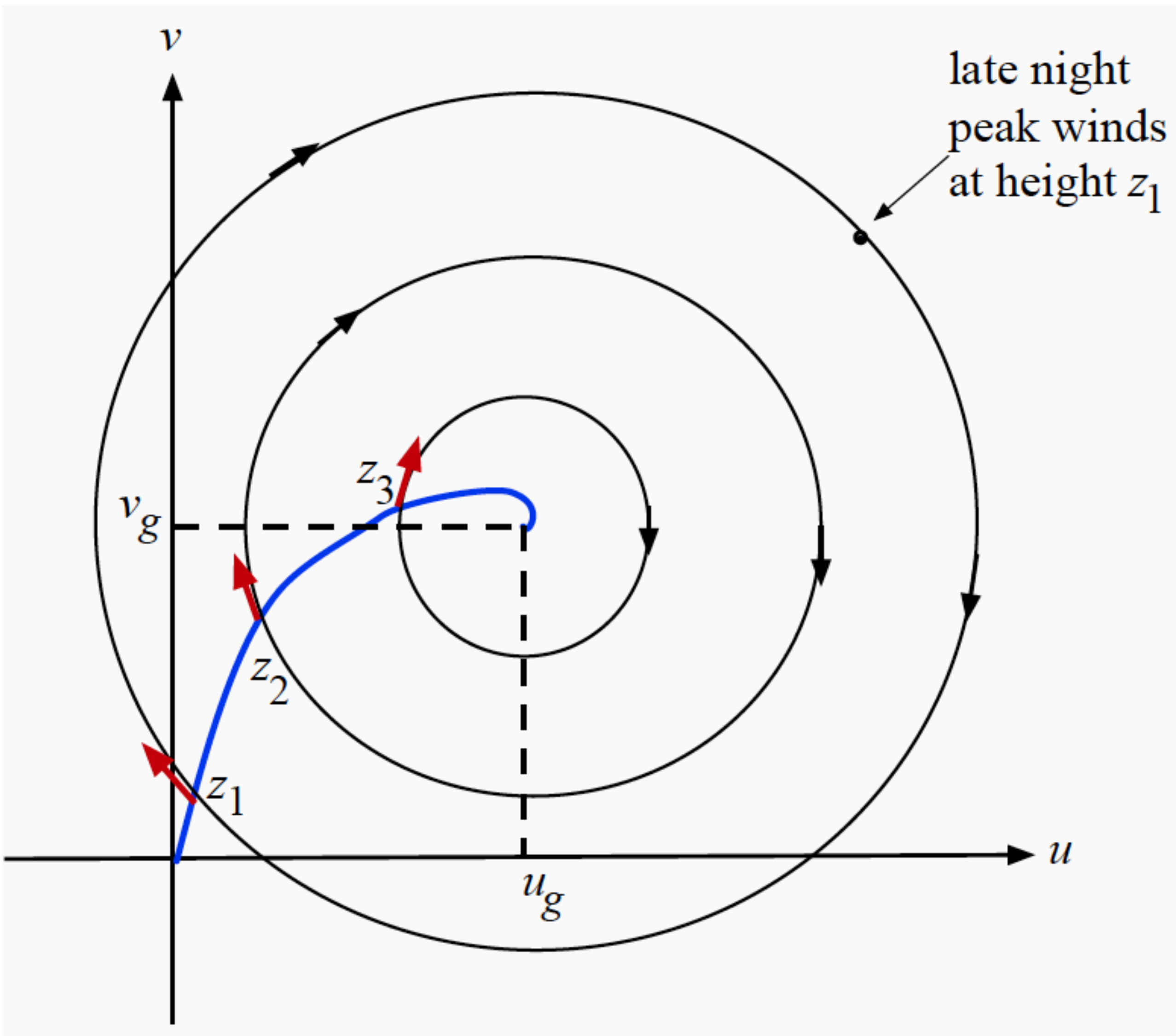
$$\boxed{\overline{v}(t) = \overline{v}_g + \overline{v}_a(0)\cos ft - \overline{u}_a(0)\sin ft}$$

The **hodograph of the full wind vector looks like the above hodograph, but with the circle centered on the geostrophic wind components**. The radius of the circle is still the magnitude of the ageostrophic wind:



Now consider how winds vary in height and time. Consider the initial (sunset) profile to be an Ekman-like spiral (blue curve below). Hodographs are shown for 3 heights. **Greatest late-night wind speed is found at heights where the largest ageostrophic winds were found at sunset. These largest sunset ageostrophic winds correspond to smallest actual winds -- weak winds near ground**.

$[\bar{u}_a \equiv \bar{u} - u_g, \ \bar{v}_a \equiv \bar{v} - v_g]$



Typically, the greatest LLJ winds occur at low levels (a few 100 m above the ground) a few hrs after midnight.