Lecture 1

Review of some of the pre-requisite math (we'll use <u>repeatedly</u> in this course).

More complete reviews of vectors, vector calc, and ODEs will be given in the Wed afternoon sessions.

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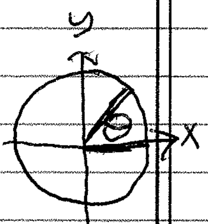
1. $\boxed{\text{Average} = \dfrac{\text{Integral}}{\text{Interval}}}$ (☆)

For example, the average temperature on the edge of a circular ring can be computed as:



$$\overline{T} = \frac{\int_0^{2\pi} T(\theta)\,d\theta}{2\pi}$$

integral is of temp around the ring

ave temp around circular ring of radius R

the interval is $2\pi - 0$ which is 2π

You can rewrite (☆) as:

$$\boxed{\text{Integral} = \text{Average times Interval}}$$

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2. Suppose you have a vector $\vec{L}$. Then the <u>unit vector</u> in the direction of $\vec{L}$ is simply $\vec{L}$ divided by the magnitude of $\vec{L}$:

$$\hat{l} = \frac{\vec{L}}{|\vec{L}|}$$

unit vector

Now multiply this eq^n by $|\vec{L}|$, get:

$$\vec{L} = |\vec{L}|\,\hat{\ell}$$

This deceptively simple eq^n says that <u>any</u> vector ($\vec{L}$) is equal to the <u>product</u> of that vector's magnitude (which is $|\vec{L}|$) and the <u>unit vector</u> in that vector's direction (which is $\hat{\ell}$).

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3. <u>Binomial Approximation</u>

Suppose we have $(1+a)^b$ where $|a| \ll 1$.

<u>Then</u> we can make the binomial approximation:

$$(1+a)^b \approx 1 + ab$$

The a and/or the b can be positive or neg.
But $|a|$ <u>must</u> be $\ll 1$.

For example:

The exact evaluation of $(1-0.1)^2$ is 0.81

close

The binomial-approximated evaluation of $(1 \underline{-0.1})^2$ is $1 - 0.1*2 = 0.8$

here the "a" is -0.1
It's magnitude is $\ll 1$

Another example:

Write the full density ρ as a sum of an average density $\bar{\rho}$ and the perturbation density ρ':

$$\rho = \bar{\rho} + \rho' \quad \text{where } |\rho'| << \bar{\rho}$$

Then:

$$\frac{1}{\rho} = \frac{1}{\bar{\rho} + \rho'}$$

Can't apply binomial approx to this thing yet, but soon we can. Factor out the larger factor (which is $\bar{\rho}$)

$$= \frac{1}{\bar{\rho}\left(1 + \frac{\rho'}{\bar{\rho}}\right)}$$

$$= \frac{1}{\bar{\rho}}\left(1 + \frac{\rho'}{\bar{\rho}}\right)^{-1}$$

Since $|\rho'| << \bar{\rho}$, $\left|\frac{\rho'}{\bar{\rho}}\right| << 1$

So it's legit to now use the binomial approx and get

$$\approx \frac{1}{\bar{\rho}}\left(1 - \frac{\rho'}{\bar{\rho}}\right)$$

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4. Properties of exponentials.

$$e^{a+b} = e^a e^b$$

$$(e^a)^b = e^{ab}$$

$$\frac{e^a}{e^b} = e^a e^{-b} = e^{a-b}$$

$$e^0 = 1$$

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5. <u>Properties of natural logarithms (ln)</u>

$\ln(ab) = \ln a + \ln b$ (a and b must be pos)

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ (a and b must be pos)

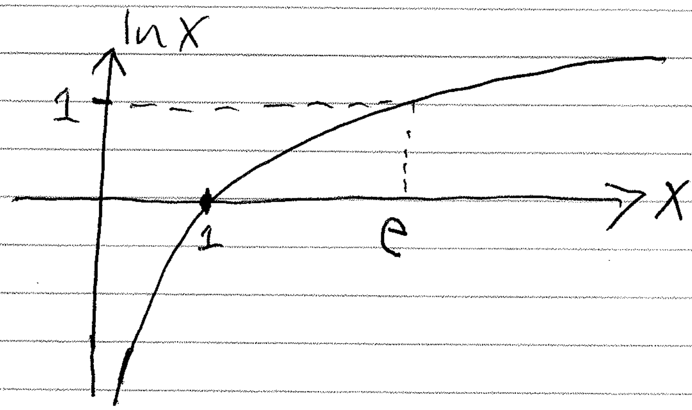
$\ln y^b = b \ln y$

$\ln e^y = y$

$\ln e = 1$

$\ln 1 = 0$

$\ln 0$ blows up (singularity) [negative infinity]



$\boxed{\ln x > 0 \text{ for } x > 1}$

$\boxed{\ln x < 0 \text{ for } x < 1}$

$$\frac{\partial}{\partial x} \ln f = \frac{1}{f}\frac{\partial f}{\partial x}$$

$$\frac{\partial}{\partial \tau} \ln f = \frac{1}{f}\frac{\partial f}{\partial \tau}$$

$$\frac{D}{D\tau} \ln f = \frac{1}{f}\frac{Df}{D\tau}$$

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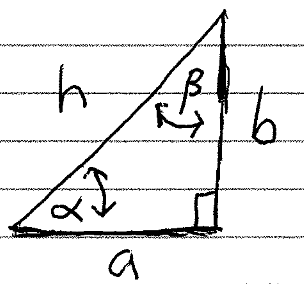
6. Some triggy stuff



$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\therefore \quad a = h\cos\alpha$$

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{h}$$

$$\therefore \quad b = h\sin\alpha$$

Similarly, $b = h\cos\beta$ and $a = h\sin\beta$

Addition/subtraction formulae for cosines and sines:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$