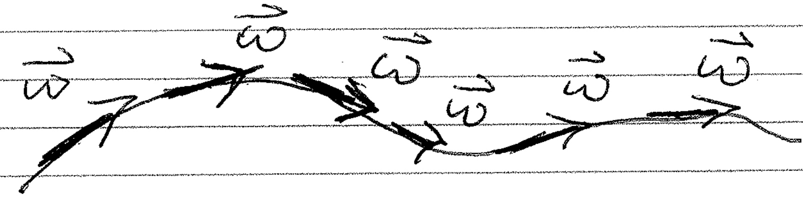
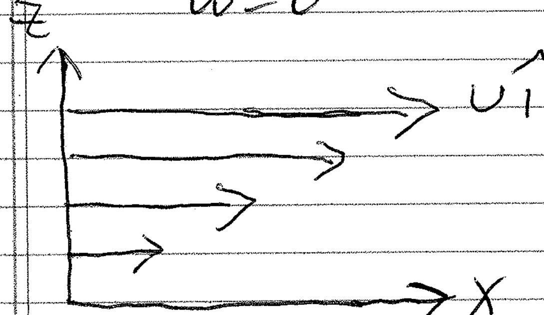
Lecture 17 (1)

<u>Vortex line</u>: a line (curve) that's <u>everywhere tangent to local vorticity vectors</u>. Constructed at a fixed time.



e.g. <u>consider unidirectional shear flow of the form,</u>

$u = \gamma z$ where $\gamma > 0$ is a constant
$v = 0$
$w = 0$

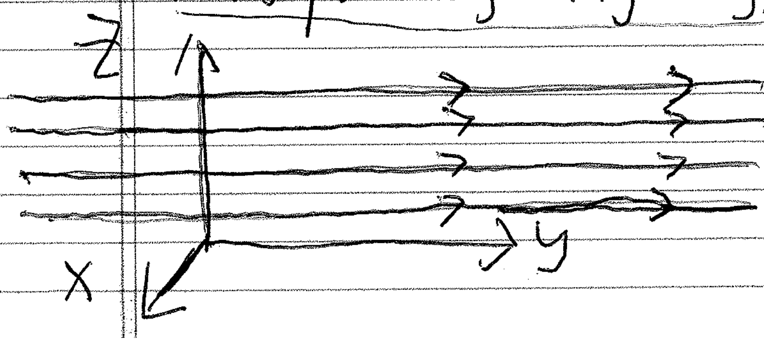


The vorticity vector in this case is $\vec{\omega} = \hat{j}\,\frac{\partial u}{\partial z}$

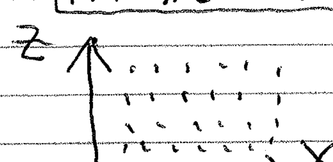
So all vorticity is in the $+\hat{j}$ direction.

Could also infer that $\vec{\omega}$ is in the $+\hat{j}$ direction from <u>right hand rule for shear vorticity</u>. Get <u>into the page</u> vorticity in this case. And that's the y-direction. [Review meaning of right-hand Cartesian coord system]

So <u>vortex lines in this case are straight lines pointing in $+\hat{j}$ (y) direction.</u>



$\boxed{\text{in xz view}}$:



vortex lines go into page (like bristles of a brush).

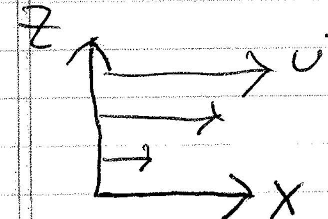
Now back to our interpretation of the vertical vorticity equation.

Tilting Terms

$$\frac{\partial \zeta}{\partial t} = \ldots \boxed{\frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}} + \ldots$$

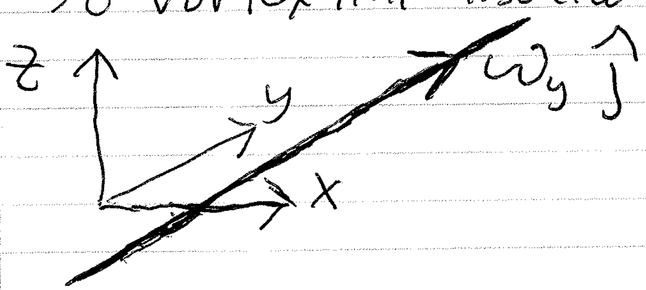
examine these two terms separately

$\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}$ e.g. suppose u increases with z and w increases with y

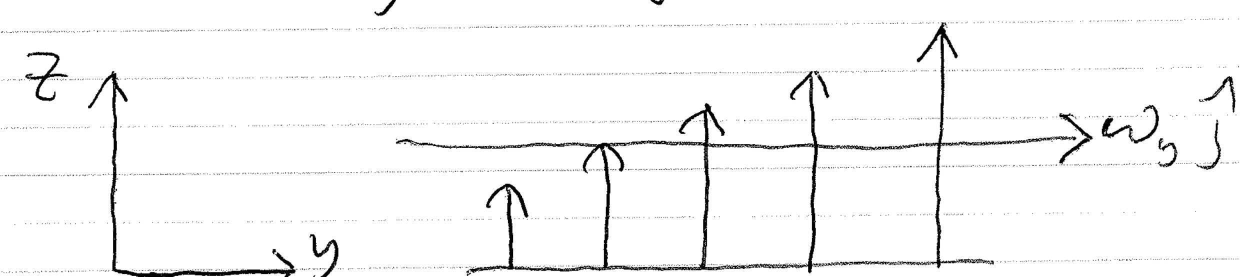


vorticity is all into-the-page
(+ĵ vort in this case)

So vortex line associated with $\frac{\partial u}{\partial z}$ looks like:

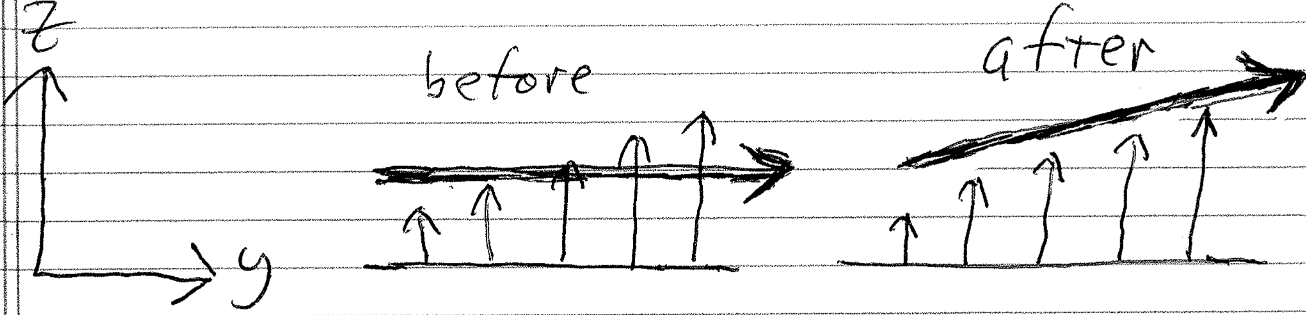


w increasing with y:



Vertical motion rotates this vortex line into the vertical direction!

The vortex line is "passive" and moves with the flow.



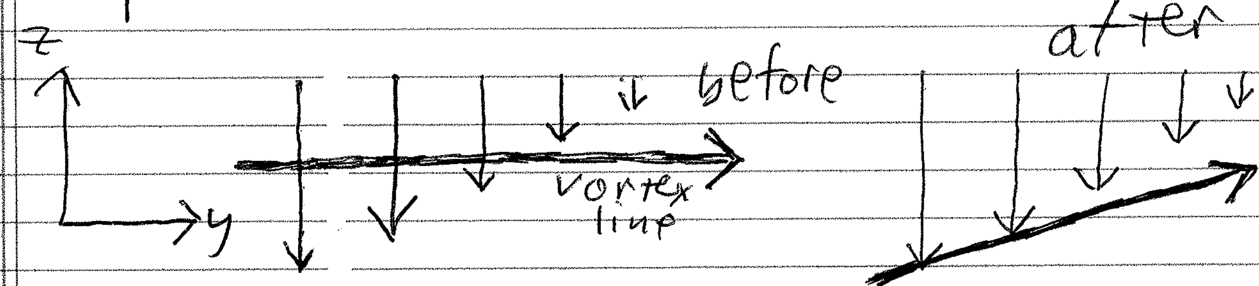
So we end up with a <u>positive vertical vorticity component</u>. [Note positive tilt to vortex line]

verify: $\frac{\partial w}{\partial y}\frac{\partial u}{\partial z} = (+)(+) > 0$

$\therefore$ acts to make $\frac{\partial \zeta}{\partial t} > 0$.

If ζ is initially 0, ζ <u>becomes positive</u>.

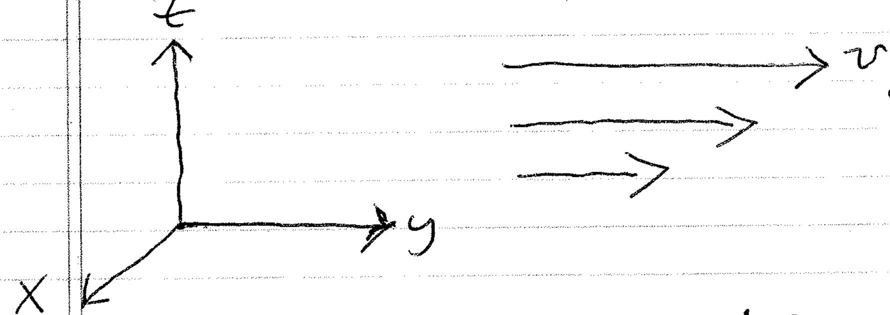
<u>Important</u>! Get same result (positive vertical vorticity generation) for a w profile that looks like:



w is clearly different in these lower pictures compared to the top pictures but $\frac{\partial w}{\partial y}$ is the <u>same</u>! The <u>change in w as y increases</u> is the same, so get same <u>tilting</u> effect on vortex line.

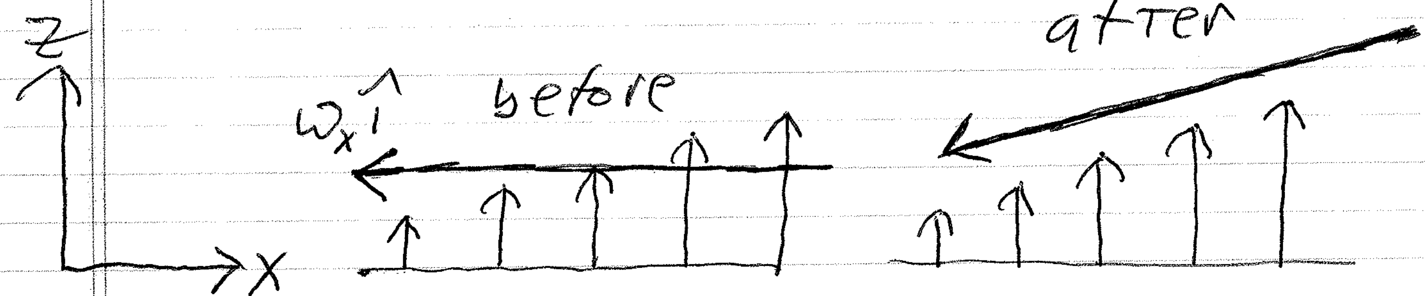
Now look at $\boxed{-\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}}$

e.g. suppose <u>v increases with z</u> and <u>w increases with x</u>:



vorticity is into-the-page, which is the $-\hat{i}$ direction!

So vortex line associated with $\frac{\partial v}{\partial z}$ points in $-\hat{i}$ direction. It gets rotated as follows:



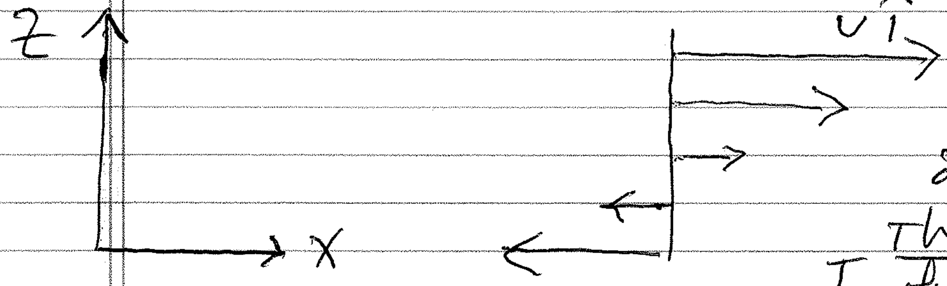
So, a <u>negative vertical vorticity component</u> is generated.

verify: $-\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} = -(+)(+) < 0$

So it contributes to $\frac{\partial \zeta}{\partial t} < 0$

If ζ is initially 0, it becomes negative.

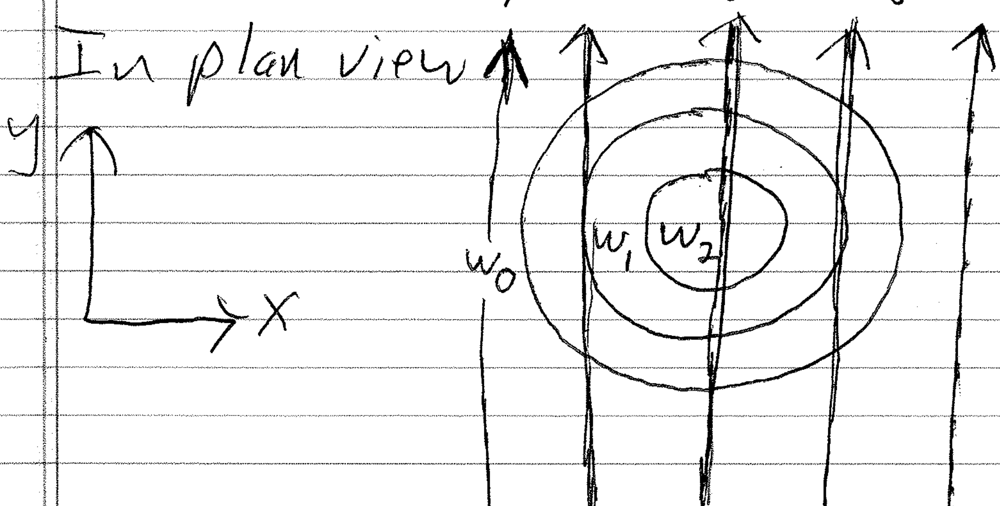
e.g. Consider a thunderstorm growing in an environment in which v increases with z and wind is easterly at surface:



vorticity associated with $\partial u/\partial z$ points into the page everywhere [which is $+\hat{j}$ direction]

Now consider an updraft growing in that environment.

In plan view

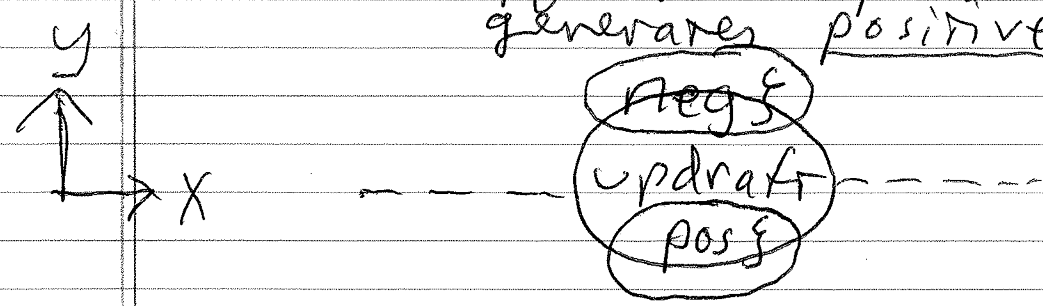


environmental vortex lines

circles are w-isolines with $w_2 > w_1 > w_0$

result: To N. of updraft center, tilting generates negative ζ.

To S. of updraft center, tilting generates positive ζ.



ζ is 0 on east/west line running through peak updraft [why?]

→ Look at handouts on T-storm in a sheared environment.

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Can rewrite Tilting Terms compactly in terms of horizontal vorticity vector $\vec{\omega}_H$

$$\vec{\omega} = \nabla \times \vec{V} = \underbrace{\hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \hat{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}_{\text{This is } \vec{\omega}_H = \omega_x \hat{i} + \omega_y \hat{j}} + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Tilting Terms:

$$\frac{\partial w}{\partial y}\boxed{\frac{\partial u}{\partial z}} - \frac{\partial w}{\partial x}\boxed{\frac{\partial v}{\partial z}}$$

$$= \frac{\partial w}{\partial y}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} + \frac{\partial w}{\partial x}\right) - \frac{\partial w}{\partial x}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} + \frac{\partial w}{\partial y}\right)$$

$$= \frac{\partial w}{\partial y}\,\omega_y \boxed{+\frac{\partial w}{\partial y}\frac{\partial w}{\partial x}} - \frac{\partial w}{\partial x}\left(-\omega_x\right)\boxed{-\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}}$$

cancel

$$= \omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y}$$

$$= \vec{\omega}_H \cdot \nabla w$$