Lecture 18

Baroclinic Term

$$\frac{\partial \zeta}{\partial t} = \ldots \hat{k} \cdot \left(\frac{1}{\rho^2} \nabla \rho \times \nabla p \right)$$

vert vorticity due to non-alignment (skewness) of $\nabla \rho$ and ∇p vectors in horizontal plane. [verify that $\hat{k} \cdot (\nabla \rho \times \nabla p) = \hat{k} \cdot (\nabla_H \rho \times \nabla_H p)$]

→ If flow is <u>barotropic</u>, $\rho = \rho(p)$ then

$$\frac{\partial \rho}{\partial x} = \frac{d\rho}{dp} \frac{\partial p}{\partial x} \quad \text{[a form of chain rule]}$$

Get similar expressions for $\partial\rho/\partial y$ and $\partial\rho/\partial z$.
So if flow is <u>barotropic</u> then

$$\nabla \rho = \hat{\imath} \frac{\partial \rho}{\partial x} + \hat{\jmath} \frac{\partial \rho}{\partial y} + \hat{k} \frac{\partial \rho}{\partial z}$$

$$= \hat{\imath} \frac{d\rho}{dp} \frac{\partial p}{\partial x} + \hat{\jmath} \frac{d\rho}{dp} \frac{\partial p}{\partial y} + \hat{k} \frac{d\rho}{dp} \frac{\partial p}{\partial z}$$

$$= \frac{d\rho}{dp} \underbrace{\left(\hat{\imath} \frac{\partial p}{\partial x} + \hat{\jmath} \frac{\partial p}{\partial y} + \hat{k} \frac{\partial p}{\partial z} \right)}_{\nabla p}$$

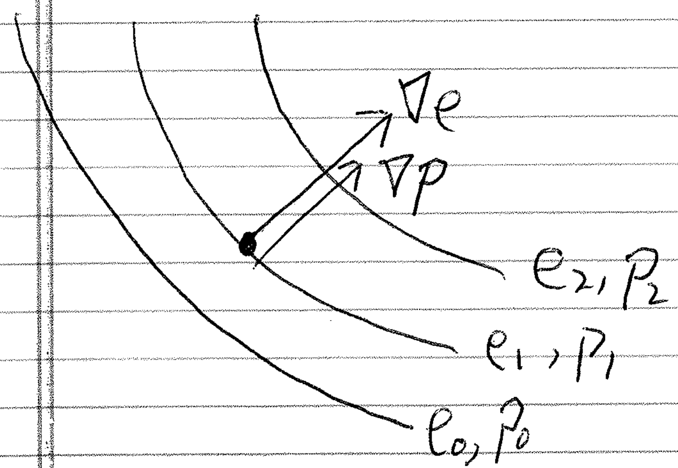
$$\therefore \nabla \rho = \frac{d\rho}{dp} \nabla p \quad \text{[vector form of chain rule]}$$

So, in the barotropic case, the baroclinic term is:

$$\hat{k} \cdot \left(\frac{1}{\rho^2} \nabla \rho \times \nabla p \right) = \frac{1}{\rho^2} \hat{k} \cdot \left(\frac{d\rho}{dp} \underbrace{\nabla p \times \nabla p}_{0} \right) = 0$$

vector × itself = 0

Barotropic case:



lines of constant p are also lines of constant ρ.

So $\nabla\rho$ is aligned with ∇p.

So $\nabla\rho \times \nabla p = 0$

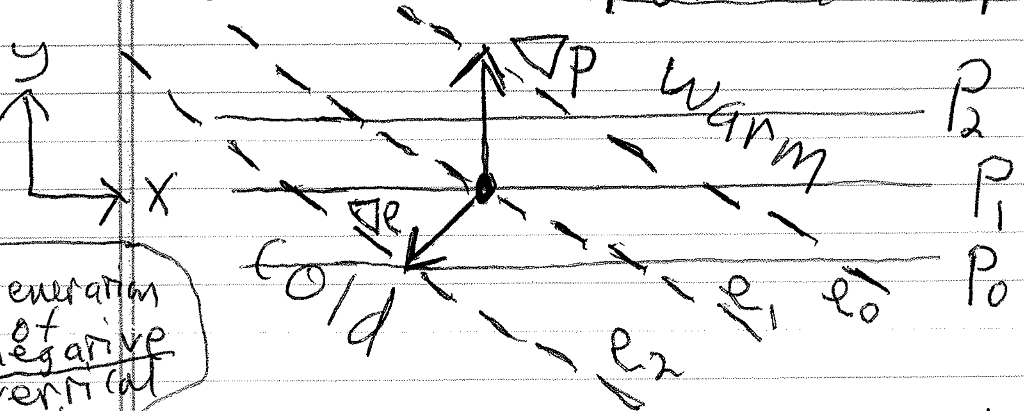
In contrast, if flow is baroclinic, $\rho = \rho(p, T)$ Then

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial x} \quad \text{[from chain rule]}$$

with similar expressions for $\partial\rho/\partial y$ and $\partial\rho/\partial z$. In this case we find that:

$$\nabla\rho = \frac{\partial \rho}{\partial p}\nabla p + \frac{\partial \rho}{\partial T}\nabla T$$

So $\nabla\rho$ and ∇p are generally not aligned – they can be skewed with respect to each other. For example:



$\boxed{P_2 > P_1 > P_0}$

$\boxed{\rho_2 > \rho_1 > \rho_0}$

generation of negative vertical vorticity.

In this case $\nabla\rho \times \nabla p$ points into-the-page ($-\hat{k}$ direction.) $\therefore$ $\hat{k}\cdot(\nabla\rho\times\nabla p) = \hat{k}\cdot(-\hat{k}\,\text{stuff}) < 0$ $\therefore$ $\partial\zeta/\partial t < 0$.

Stretching Term

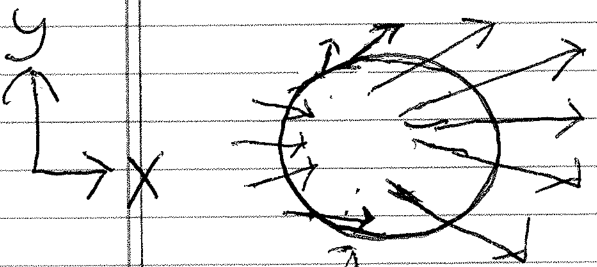
$$\frac{\partial \zeta}{\partial t} = \ldots \boxed{-\delta(\zeta + f)}$$

where $\delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is <u>horizontal</u> <u>divergence</u>.

δ is related to <u>rate of change</u> of horizontal <u>area</u> of an infinitesimal air parcel.

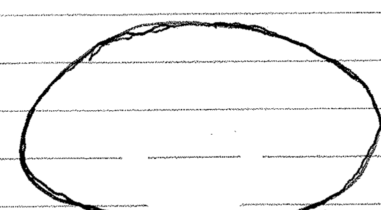
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{A}\frac{DA}{Dt}$$ (A is horiz area of infinitesimal parcel).

Consider a flow in which $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} > 0$ (divergent flow):



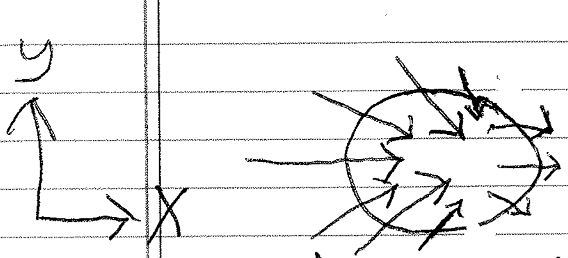
wind vectors reveal <u>divergence</u>.

initial horiz area A of a parcel.
In this flow field $\frac{1}{A}\frac{DA}{Dt} > 0$.

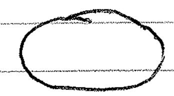


a little later, horizontal area A of this parcel has <u>increased</u>.

Consider a flow in which $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} < 0$ (<u>convergent flow</u>):



wind vectors reveal <u>convergence</u>.
so $\frac{1}{A}\frac{DA}{Dt} < 0$



A little later, horiz area of parcel has <u>shrunk</u>.

Suppose we have positive absolute vertical vorticity $\zeta + f > 0$ and positive divergence ($\delta > 0$ so $DA/Dt > 0$) Then:

$$-\delta(\zeta + f) = -(+)(+) < 0$$

$\therefore \frac{\partial \zeta}{\partial t} < 0$ so $\zeta \downarrow$ with time.

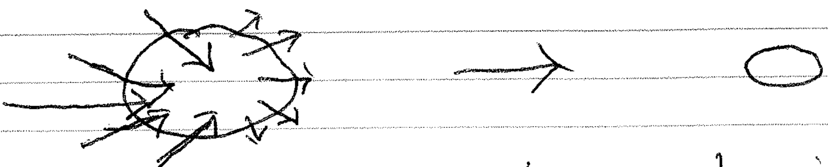
Conversely, if we have positive absolute vertical vorticity $\zeta + f > 0$ and negative divergence (so convergence) ($\delta < 0$ so $DA/Dt < 0$) Then:

$$-\delta(\zeta + f) = -(-)(+) > 0$$

$\therefore \frac{\partial \zeta}{\partial t} > 0$ so $\zeta \uparrow$ with time.

What's happening?

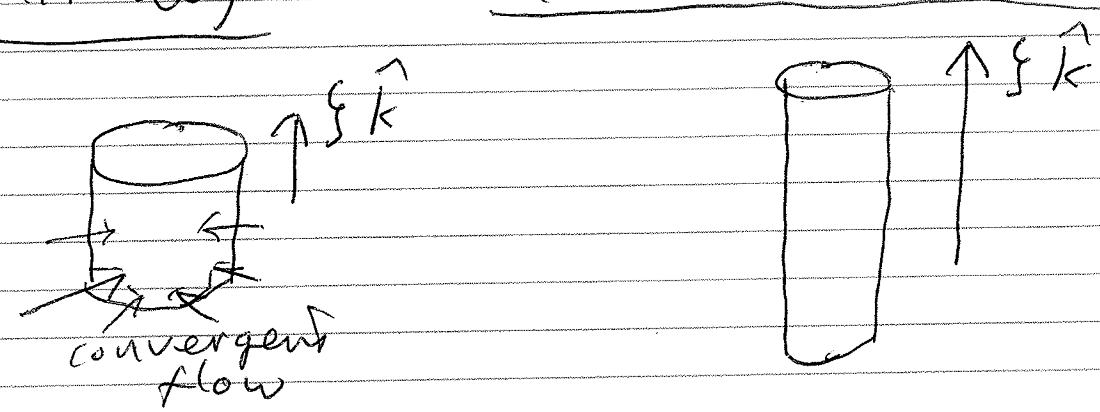
In case of horizontal convergence,



parcel mass is brought in closer to axis of rotation (central axis). Vorticity is amplified. Effect is analogous to a spinning figure skater. Bringing arms in brings mass closer to axis of rotation $\therefore$ faster spin.

This is called a "stretching effect" because a decrease in a parcel's horiz area is associated with vertical elongation – stretching of parcel in vertical (necessitated by mass conservation).

initially: a little time later:



For stretching term to operate, need both absolute vertical vorticity and a stretching mechanism.

Vorticity amplification through stretching (or reduction through compression) is an important effect in synoptic-scale flows, mesoscale flows (thunderstorms, especially supercells and mesoscale convective systems), small-scale flows (e.g. tornadoes) and turbulence.