Lecture 20 (1)

Horizontal vorticity equations

x-comp vorticity is $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$ (call it ω_x)

y-comp vorticity is $\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$ (call it ω_y)

To get the x-comp vorticity eq^n (eq^n for $\partial \omega_x / \partial t$, take $\partial/\partial y$ (z eq^n of motion) and subtract from it $\partial/\partial z$ (y eq^n of motion).

To get the y-comp vorticity eq^n (eq^n for $\partial \omega_y / \partial t$, take $\partial/\partial z$ (x eq^n of motion) and subtract from it $\partial/\partial x$ (z eq^n of motion).

Get:

$$\frac{\partial}{\partial t}\omega_x = \text{advection, stretching, tilting} + \hat{i}\cdot\left(\frac{1}{\rho^2}\nabla\rho \times \nabla p\right)$$

it's large where $\underline{\partial\rho/\partial y}$ is large

x-comp baroclinic vorticity generation.

$$\frac{\partial}{\partial t}\omega_y = \text{advection, stretching, tilting} + \hat{j}\cdot\left(\frac{1}{\rho^2}\nabla\rho \times \nabla p\right)$$

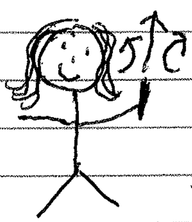
it's large where $\underline{\partial\rho/\partial x}$ is large

y-comp baroclinic vorticity generation

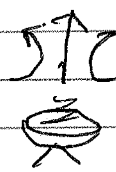
Flows where the $\boxed{\text{horizontal}}$ (x or y) components of <u>baroclinic</u> vorticity <u>generation</u> are important are found across almost all scales in the atmosphere, from microscale to synoptic-scale, for example:

- convective clouds, squall lines, supercells, etc.
- leading edge of fronts and t-storm outflows
- gravity waves
- katabatic flows (flows down cooled slopes)
- anabatic flows (" up heated ").
- sea-breezes
- man-made convective flows arising from:

cigarettes:

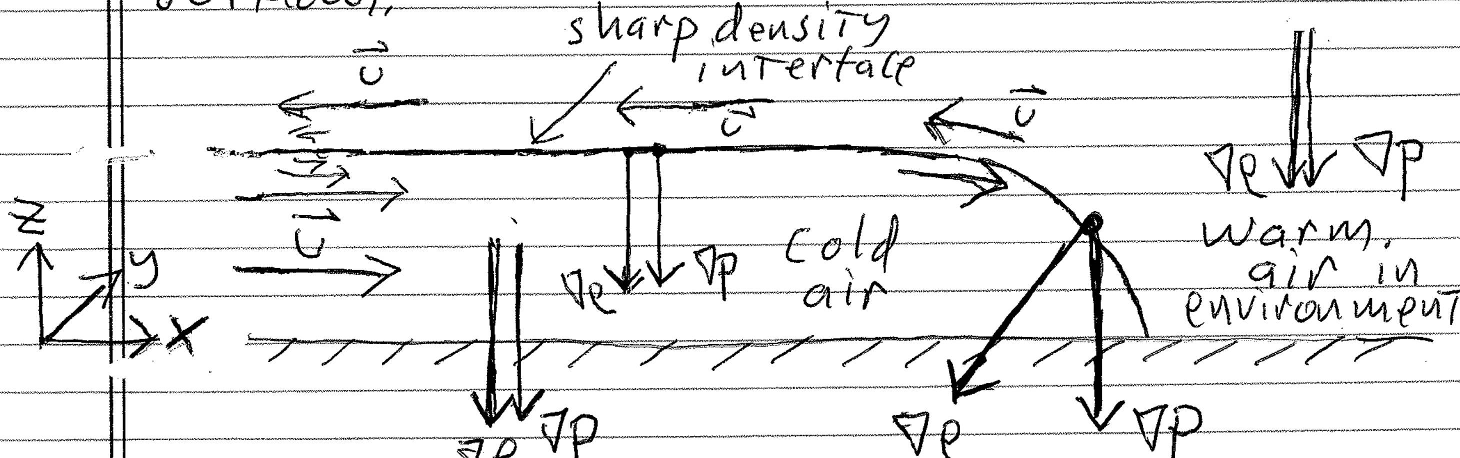


bar-b-cues:



atom bomb blasts: (mushroom cloud)

→ Let's focus on one example: a thunderstorm outflow moving in the $+\hat{i}$ direction (westerly outflow).

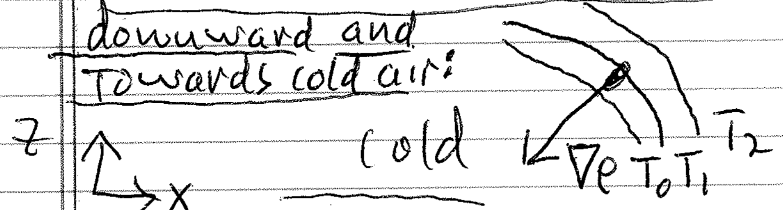


What's going on here?

We idealize the outflow as a long blob of cold air displacing relatively warm air in environment. In environment, p and ρ both increase with decreasing z, so ∇p and $\nabla\rho$ both point downward.

Similarly, ∇p and $\nabla\rho$ both point downward inside cold pool (except at leading edge).

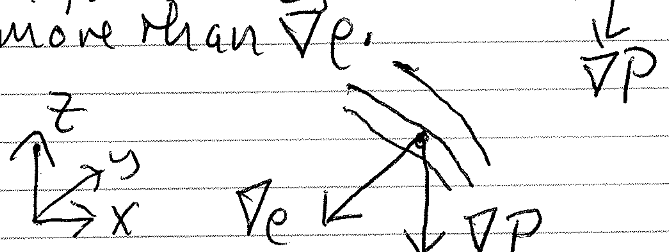
But something interesting is happening along the leading edge, where there are horizontal changes in ρ. Along the leading edge, ρ increases downward and also increases toward cold air, so $\nabla\rho$ points downward and towards cold air:



$T_2 > T_1 > T_0$

so there's a horizontal (x-comp) part to $\nabla\rho$.

In contrast, the pressure field is still largely hydrostatic, and thus tends to point downward more than $\nabla\rho$.



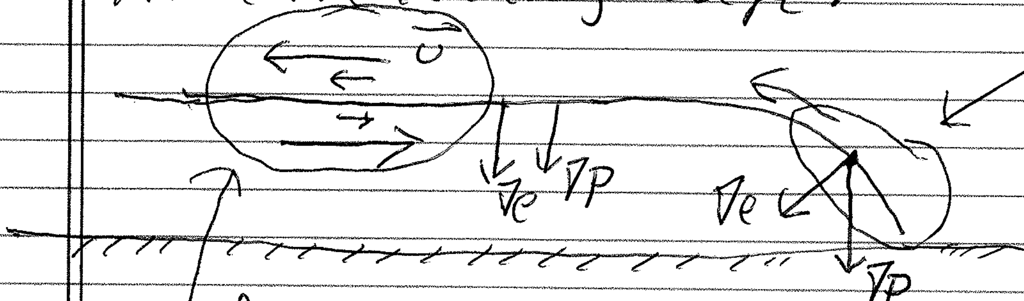
$$\nabla p \approx \frac{\partial p}{\partial z}\hat{k} \approx -\rho g \hat{k}$$

So, on leading edge of interface, $\nabla\rho \times \nabla p$ points out-of-page, which is in the minus $\hat{j}$ direction!

$$\therefore \hat{j}\cdot\left(\frac{1}{\rho^2}\nabla\rho\times\nabla p\right) = \hat{j}\cdot(-\hat{j}\sin\theta) < 0$$

$$\therefore \frac{\partial \omega_y}{\partial t} < 0 \quad \therefore \text{Generate negative (out of page) } y\text{-comp vorticity.}$$

Note that out-of-page vorticity is what we see along the interface further back from the leading edge:



Vorticity is produced here and is then advected rearward

the vorticity in here was produced baroclinically at the leading edge.
But there is no baroclinic generation going on here because $\nabla\rho$ and ∇p are both pointing nearly straight downward.

A theory for tornadogenesis (for some tornados)

(i) Horiz vorticity is generated baroclinically at leading edge of t-storm outflow. (baroclinic term in x or y comp vorticity equation).

(ii) This horiz vorticity gets tilted into the vertical by an updraft or a downdraft (tilting term in z-comp vorticity eqn). So now we have ζ.

(iii) ζ gets stretched (amplified) in a convergent updraft → get a tornado (from stretching term in z-comp vorticity eqn).

Scale analysis of vertical vorticity eq^n for mid-latitude synoptic-scale flows.

Characteristic scales of motion for synoptic-scale flows (based on typical observations):

Horizontal length scale $L \sim 1000\,km = 10^6\,m$

Vertical length scale $H \sim 10\,km = 10^4\,m$

Horizontal velocity scale $U \sim 10\,m/s$

Vertical velocity scale $W \sim 0.01\,m/s$
[1 cm/s]

Time scale based on horizontal advection

(time it takes a parcel moving horizontally at speed U to traverse the distance L), $T_{horiz} = \frac{L}{U} = 10^5\,s$ (about 1 day)

Time scale based on vertical advection
(time it takes a parcel moving vertically at speed W to traverse the vertical distance H), $T_{vert} = \frac{H}{W} = 10^6\,s$ (about 10 days)

Typical density
$\rho \sim 1\,kg/m^3$

Typical horizontal change in density (across horizontal distance L),

$\Delta\rho_{horiz} \sim 0.01\,kg/m^3$ [about 1% of ρ]

Typical horizontal change in pressure (across horiz distance L),
[1 mb = 100 Pa]

$\Delta P_{horiz} \sim 10\,mb = 1000\,Pa$