LECTURE 24: Asymmetry of the spin-up process for mid-latitude synoptic-scale flows

Observations show that for mid-latitude synoptic-scale flows in the N. hemisphere ($f > 0$), systems with positive values of ζ tend to have larger magnitudes of ζ than systems with negative values of ζ. To see why this is so, consider the approximate form of the absolute vertical vorticity equation (derived in class):

$$\frac{D_h \eta}{Dt} = -\eta\,\delta, \tag{1}$$

where $\eta \equiv \zeta + f$ is the absolute vertical vorticity, and $\delta \equiv \partial u/\partial x + \partial v/\partial y$ is the horizontal divergence. To solve (1), separate variables and integrate it (allowing for initial condition). Get:

$$\eta(t) = \eta_0 \exp[-\int_0^t \delta(t')dt'], \tag{2}$$

where η_0 is the parcel's initial value of η. Since $\eta \equiv \zeta + f$, we can rewrite (2) in terms of ζ as:

$$\zeta(t) = -f(t) + \left(\zeta_0 + f_0\right) \exp[-\int_0^t \delta(t')dt']. \tag{3}$$

To most clearly show the response of a parcel's ζ to stretching vs compression, assume that:

(i) initially the air is not moving (relative to earth) so $\zeta_0 = 0$ for all parcels.

(ii) each parcel experiences a δ of one sign only (pos or neg) for all time (but with $\delta = 0$ initially). So $\int_0^t \delta(t')\,dt'$ has same sign as δ, and increases in magnitude with time.

(iii) parcel doesn't change its latitude (so parcel's f at any time is just its initial value f_0).

Given these assumptions, (3) becomes:

$$\zeta(t) = \left\{ \exp[-\int_0^t \delta(t')dt'] - 1 \right\} f_0. \tag{4}$$

Reality check that (4) is consistent with the initial condition $\zeta_0 = 0$: for $t = 0$ the integral $\int_0^t \delta(t')dt'$ is zero, so $\exp[-\int_0^t \delta(t')dt']$ is exp(0) which is 1, so then the right hand side of (4) reduces to $\{1 - 1\} f_0$, which is 0. So (4) does yield $\zeta = 0$ at $t = 0$.

Now let's examine the implications of (4) for cases of stretching and compression:

Stretching of air parcels in the northern hemisphere

Stretching is associated with horizontal convergence (negative divergence, $\delta < 0$) so $\int_0^t \delta(t')dt'$

< 0. So minus this integral is positive: $-\int_0^t \delta(t')dt' > 0$ (and increases with time). So $\exp[-\int_0^t \delta(t')dt']$, which is 1 initially, grows exponentially with time. So (4) says that ζ is positive and can get huuuge.

Compression of air parcels in the northern hemisphere

Compression is associated with horizontal divergence (positive divergence, $\delta > 0$) so $\int_0^t \delta(t')dt'$ > 0. So minus this integral is negative: $-\int_0^t \delta(t')dt' < 0$. So $\exp[-\int_0^t \delta(t')dt']$ dies exponentially with time, and (4) says that ζ is negative and levels off at $-f_0$.

These results are easier to understand if you visualize the problem in an absolute sense, i.e., as an observer in an inertial (non-rotating) reference frame would see it:

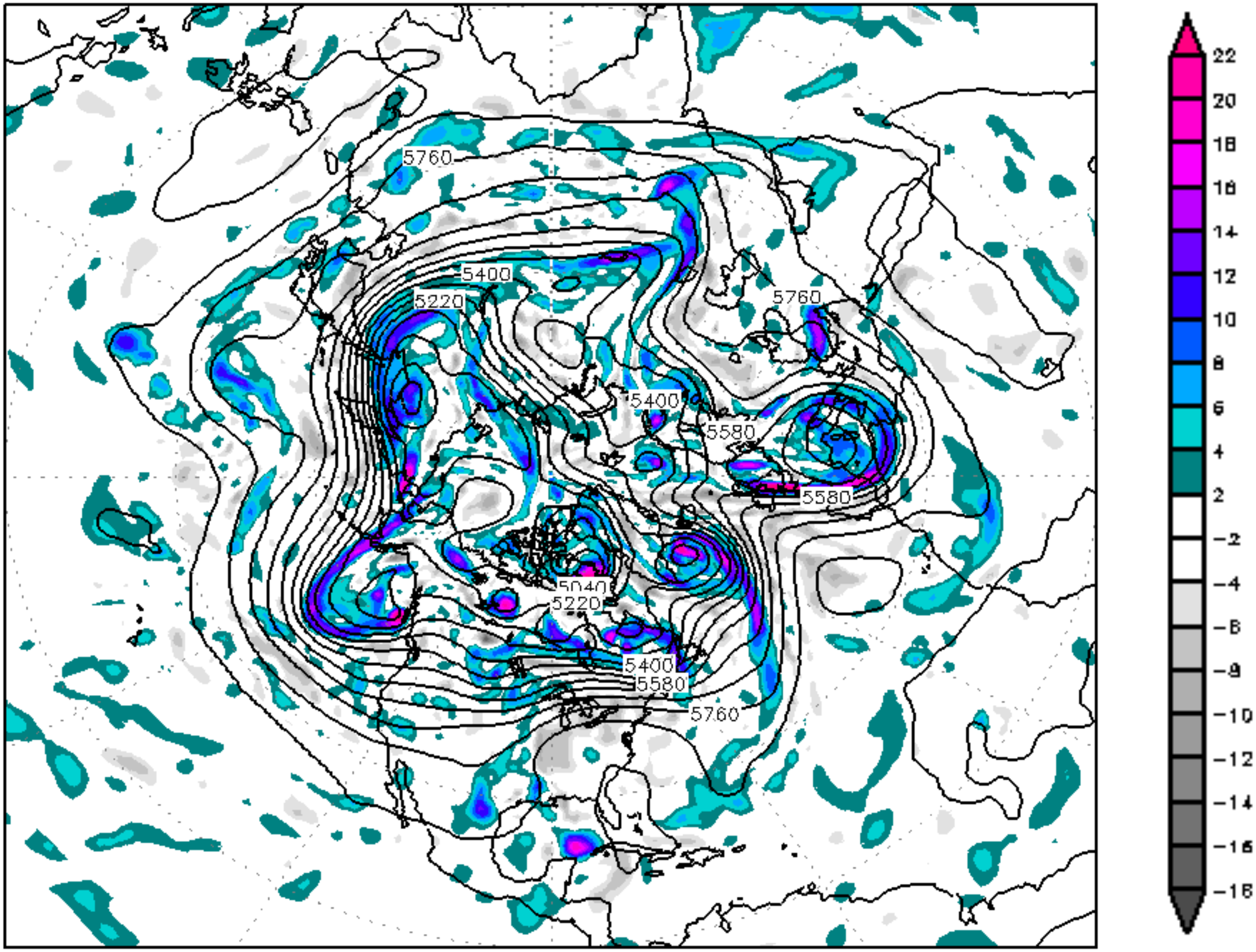
Recall that we're considering an initial state of no motion relative to the earth. That means that in an absolute sense, the atmosphere is in solid body rotation. So the atmosphere is chock full of actual vorticity (and of the cyclonic sense)! And since the parcels we're considering are part of this atmosphere, they all start out with positive absolute vorticity.

Now along comes a disturbance. In the case of stretching, a parcel's positive initial absolute vorticity gets amplified. The more the stretching, the more amplification. There's no limit to how large this absolute vorticity can get. To an observer in the rotating frame, the relative vorticity appears huge (and it's positive).

In the case of compression, a parcel's positive initial absolute vorticity gets reduced. For prolonged compression, the absolute vorticity approaches 0. An observer in the rotating reference frame would see relative vorticity with the value of $-f$ (because relative vorticity plus f add up to zero absolute vorticity).

These results suggest that for a population of divergent and convergent systems of equal frequency and intensity in an atmosphere that is "roughly in solid body rotation", the largest magnitudes of ζ will be found for the systems with positive ζ.

500 mb heights (m) and relative vorticity ζ ($10^{-5}s^{-1}$), 00Z 9 Nov 2009



This figure is from Brian Fiedler's notes, "Forces and Motion in the Atmosphere". Brian credits the figure from: http://www.nrlmry.navy.mil/metoc/nogaps/NOGAPS_nh_net.html