LECTURE 36
The Planetary Boundary Layer (continued)

Ekman layer (continued)

We've described Ekman flow in a coordinate system where ***x* is aligned with geostrophic wind** $\vec{V}_g$ ($u_g = |\vec{V}_g|$, and $v_g = 0$), so ***y* points toward low p.** We assumed $\vec{V}_g$ is independent of *x* and *y*. The wind components are:

$$\bar{u} = \bar{u}_g\left(1 - e^{-\gamma z}\cos\gamma z\right), \qquad \bar{v} = \bar{u}_g\, e^{-\gamma z}\sin\gamma z\,, \qquad \text{where } \gamma \equiv \sqrt{\frac{f}{2K}}\,.$$

The **hodograph is a spiral** (but there's no curvature of any streamline or trajectory!)

The cross-isobar volume flux Q (volume flux in direction of low pressure) is **also known as Ekman transport.** It's the vertical integral of $\bar{v}$:

$$Q \equiv \int_0^\infty \bar{v}\,dz = \int_0^\infty \boxed{\bar{u}_g}\, e^{-\gamma z}\sin\gamma z\,dz \;= \bar{u}_g\, I\,, \qquad \text{where } I \equiv \int_0^\infty e^{-\gamma z}\sin\gamma z\,dz$$

↓ independent of *z*, so pull it out of integral

In prob set 7 we find that $I = \sqrt{\dfrac{K}{2f}}$. $\quad\therefore\quad$ $\boxed{Q = \bar{u}_g\sqrt{\dfrac{K}{2f}}}$

So, the stronger the mixing (larger K), the greater the cross-isobar volume flux.

The volume flux deficit *D* in direction of geostrophic wind (volume flux of $\bar{u}$ minus volume flux of u_g) is:

$$D \equiv \int_0^\infty \bar{u}\,dz - \int_0^\infty \bar{u}_g\,dz = \int_0^\infty \left(\bar{u} - \bar{u}_g\right)dz \qquad \text{plug in formula for } \bar{u}$$

$$= \int_0^\infty \left[\bar{u}_g \left(1 - e^{-\gamma z} \cos\gamma z \right) - \bar{u}_g \right] dz \qquad \text{get some cancellation}$$

$$= -\bar{u}_g J \qquad \text{where } J \equiv \int_0^\infty e^{-\gamma z} \cos\gamma z \, dz$$

In prob set 7 we find that $J = I = \sqrt{\frac{K}{2f}}$. $\therefore$ $\boxed{D = -\bar{u}_g \sqrt{\frac{K}{2f}}}$ Negative implies **deficit**.

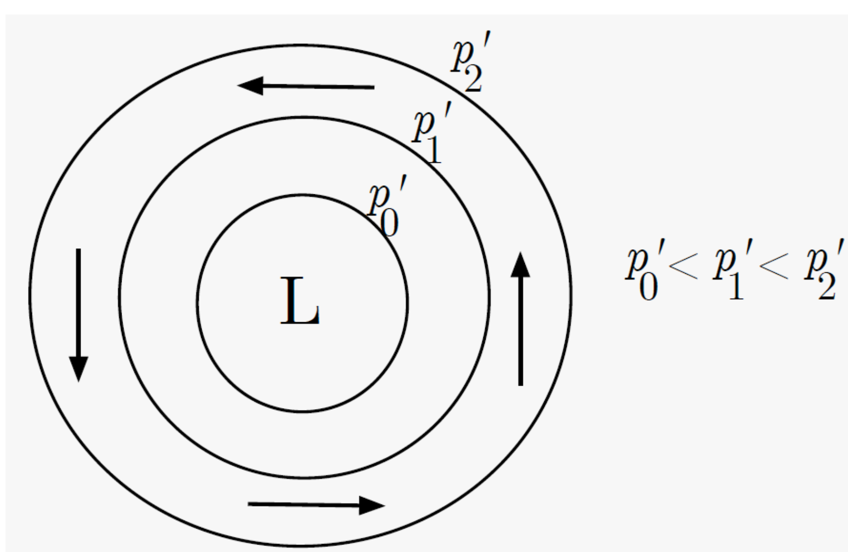
Stronger mixing ($K \uparrow$) implies **larger volume flux deficit** in direction of geostrophic wind ($\bar{u}$ **is weaker**). This isn't surprising: the larger the K, the deeper the boundary layer ($H \uparrow$), and thus a larger thickness of the atmosphere where $\bar{u}$ is weak.

Get larger K (more intense mixing) from larger geostrophic winds, larger surface heating and/or a rougher surface, e.g., forest versus grassland, or land versus sea.

Ekman pumping in the atmosphere

Consider the (usual) case where the geostrophic wind does vary spatially, e.g., around a synoptic-scale low pressure system. Suppose the upper level (well above the PBL) perturbation pressure field and **geostrophic wind vectors** $\vec{V}_g$ look like this:

These are $\vec{V}_g$ vectors:

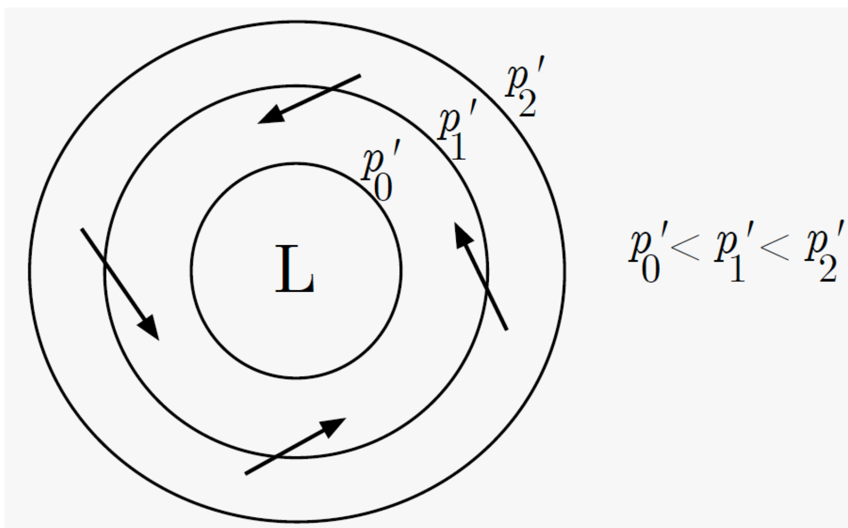


The actual winds are nearly equal to these $\vec{V}_g$ vectors (not exactly equal because trajectories are curved; there's a gradient wind balance, not a geostrophic balance).

Rather than abandon the Ekman theory (which was derived under the now-violated assumption that $\vec{V}_g$ doesn't vary in *x* or *y*), we can apply it **locally**. This is a valid approach as long as $\vec{V}_g$ doesn't vary too rapidly (synoptic scale variations are okay). So, at any *x*, *y* location plunk down the appropriate Ekman spiral, i.e., the $u(z)$ and $v(z)$ Ekman solution corresponding to $\vec{V}_g$ at that location.

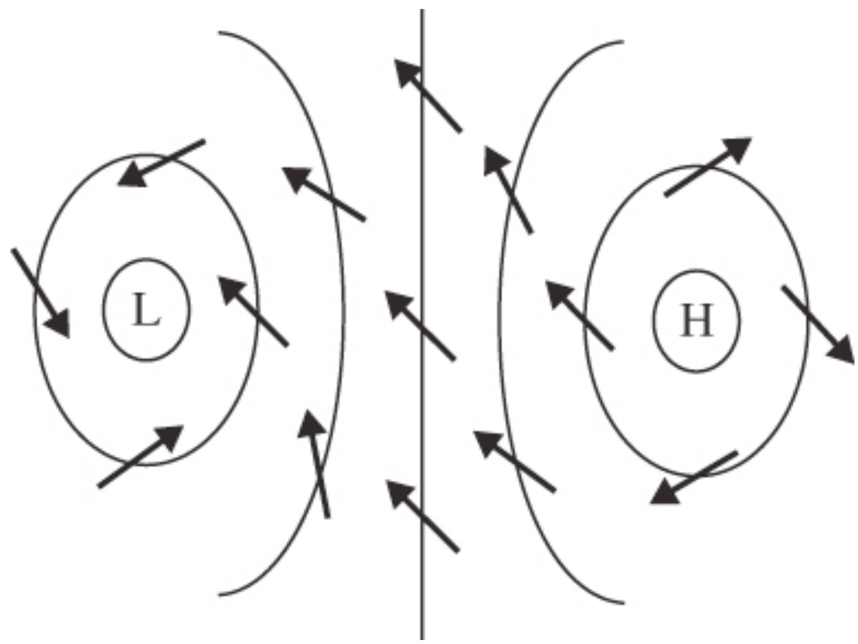
Assume horizontal pgf aloft is still largely impressed on the boundary layer. So at low altitudes (i.e., in the PBL) the perturbation pressure field and wind field from Ekman solution look like:

These are low-level wind vectors (from Ekman solution)



At each *x*, *y* location the wind vectors cut across isobars toward low *p*. Get convergence into the low. Integrating mass conservation equation (incompressibility condition or anelastic mass conservation equation) with respect to *z*, and assuming ground is flat (so $w = 0$ there), yields a formula for $w(z)$ that **reveals rising motion**. The associated adiabatic expansion and cooling can promote cloudiness and rain.

In a low-pressure system, friction (divergence of Reynolds stress) in the PBL induces rising motion at the top of PBL. This is called **Ekman pumping**. Similarly, in a high pressure system, get boundary-layer-induced subsidence at the top of the PBL. This is called **Ekman suction**. Here are low-level winds with both occurring:



Ekman layer in the ocean

Turbulent Ekman layers develop in the ocean near the surface (~ 50 - 100 m deep). They **arise in response to a wind stress** rather than to a perturbation pgf (but if sea surface is sloping, pgf can be important). Again adopt eddy viscosity closure,

$$\overline{u'w'} = -K\frac{d\overline{u}}{dz}, \tag{33}$$

$$\overline{v'w'} = -K\frac{d\overline{v}}{dz}. \tag{34}$$

where *K* is the eddy viscosity. If the perturbation pgf is negligible ($u_g = v_g = 0$), the equations of motion are a simplified version of (35) and (36):

$$0 = f\,\overline{v} + K\frac{d^2\overline{u}}{dz^2},$$

$$0 = -f\,\overline{u} + K\frac{d^2\overline{v}}{dz^2}.$$

The solution procedure parallels what we did in the atmospheric case. Only the boundary conditions are different: here $\bar{u}$ and $\bar{v} \to 0$ as $z \to -\infty$, and at the surface we replace the no-slip condition with wind stresses:

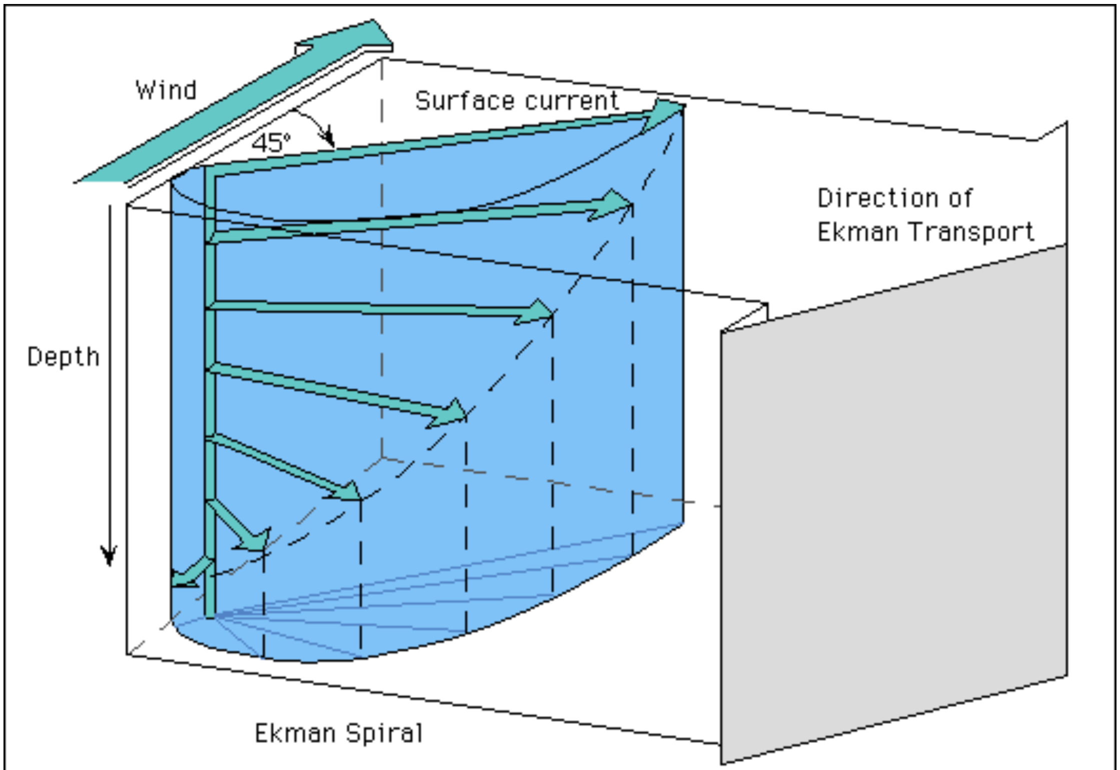
$$K \left. \frac{d\bar{u}}{dz} \right|_{z=0} = \text{specified } x\text{-component wind stress},$$
$$K \left. \frac{d\bar{v}}{dz} \right|_{z=0} = \text{specified } y\text{-component wind stress}.$$

The solution (not shown) describes:

- a spiral hodograph

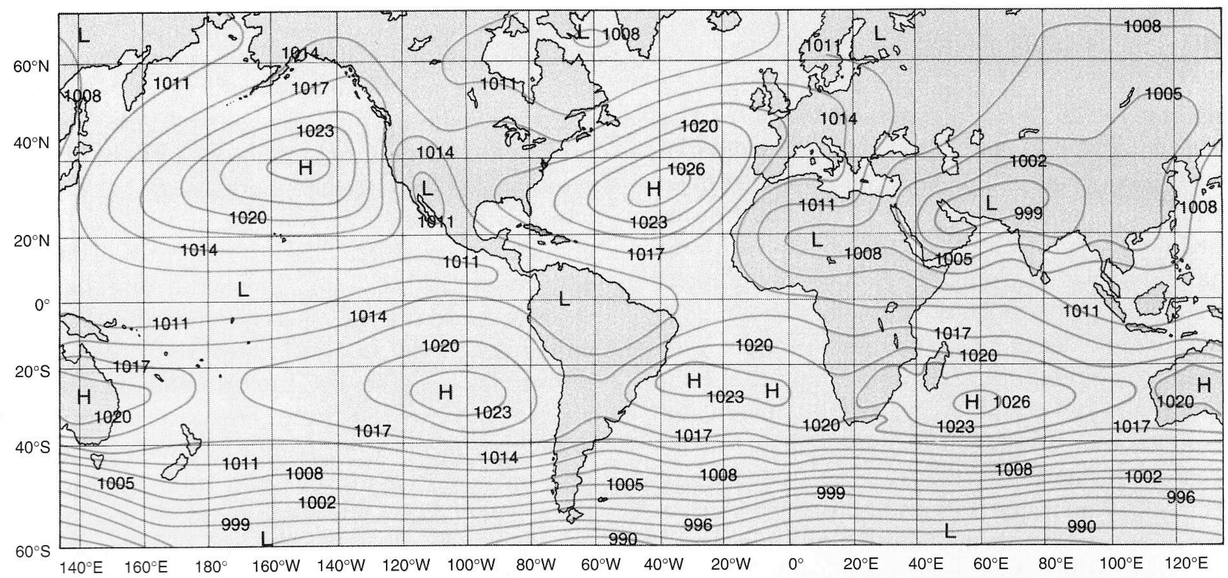
- **a surface current flowing at 45º to right of wind** (and wind stress) in N. hemisphere.

- **mass transport (Ekman transport) to right of wind** (and wind stress) in N. hemisphere.

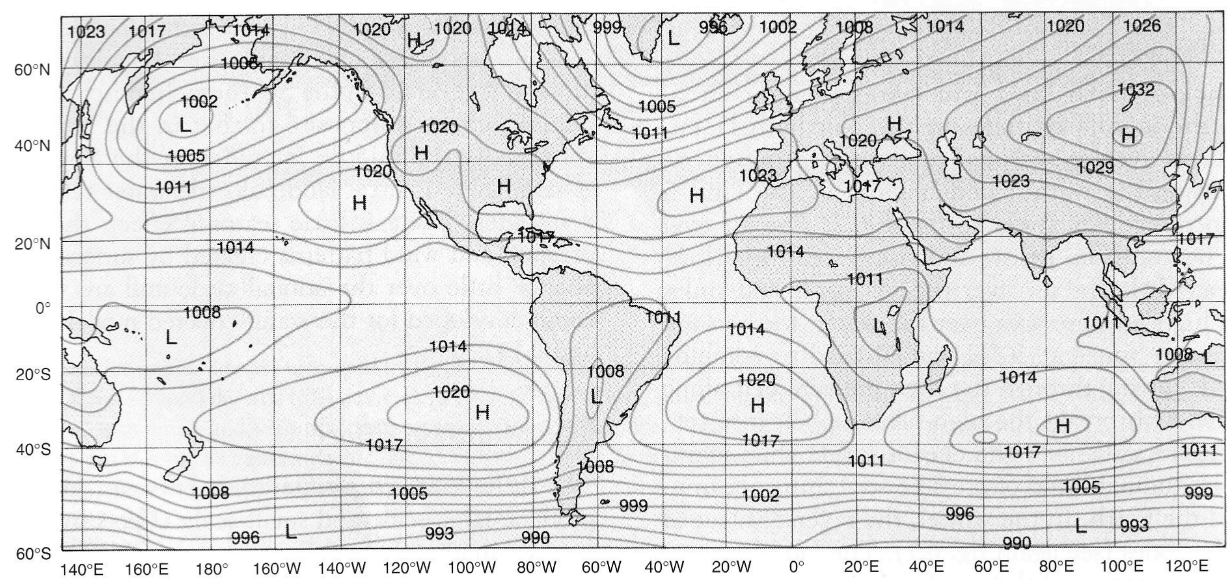


Adapted from Thurman, Harold V. **Essentials of Oceanography, 5th ed.** Prentice-Hall, Inc., 1996.

Sea level atmospheric pressure for July and January



(a) July



(b) January

Figure 7.14

Average sea-level atmospheric pressures expressed in millibars for (a) July and (b) January. Large landmasses at mid-latitudes in the Northern Hemisphere cause high and low atmospheric pressure cells to change their positions with the seasons. In the Southern Hemisphere, large landmasses do not exist at mid-latitudes, and average air pressure distribution changes little with the seasons.

Effect of wind stress on surface currents and upwelling/downwelling

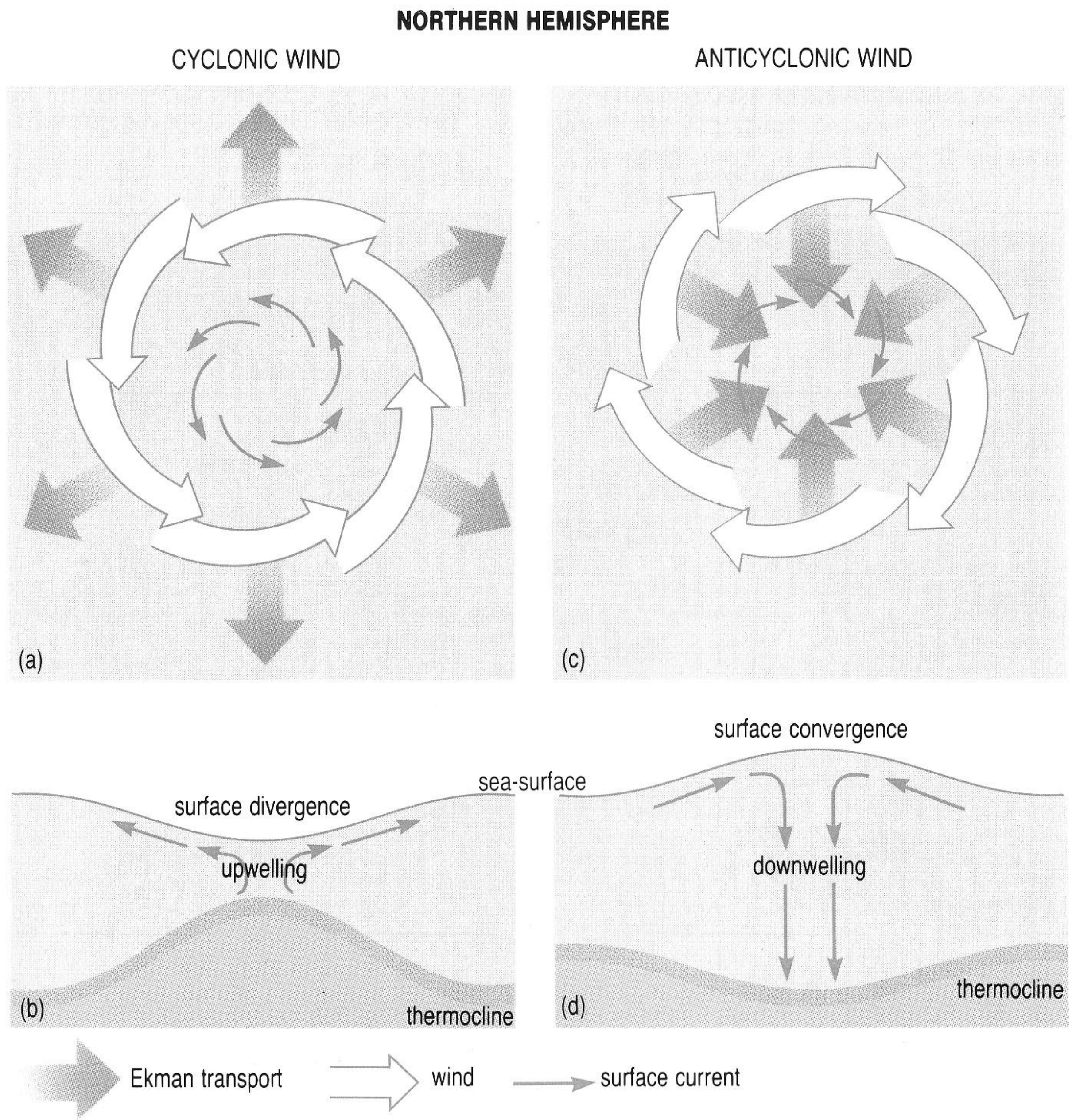


Figure 3.24 The effect of a cyclonic wind in the Northern Hemisphere (a) on surface waters, (b) on the shape of the sea-surface and thermocline. Diagrams (c) and (d) show the effects of an anticyclonic wind in the Northern Hemisphere. (Remember that in the Southern Hemisphere, cyclonic = clockwise and anticyclonic = anticlockwise.)

from "Ocean Circulation" A. Colling (Editor)

Ekman transport and upwelling near a coastline

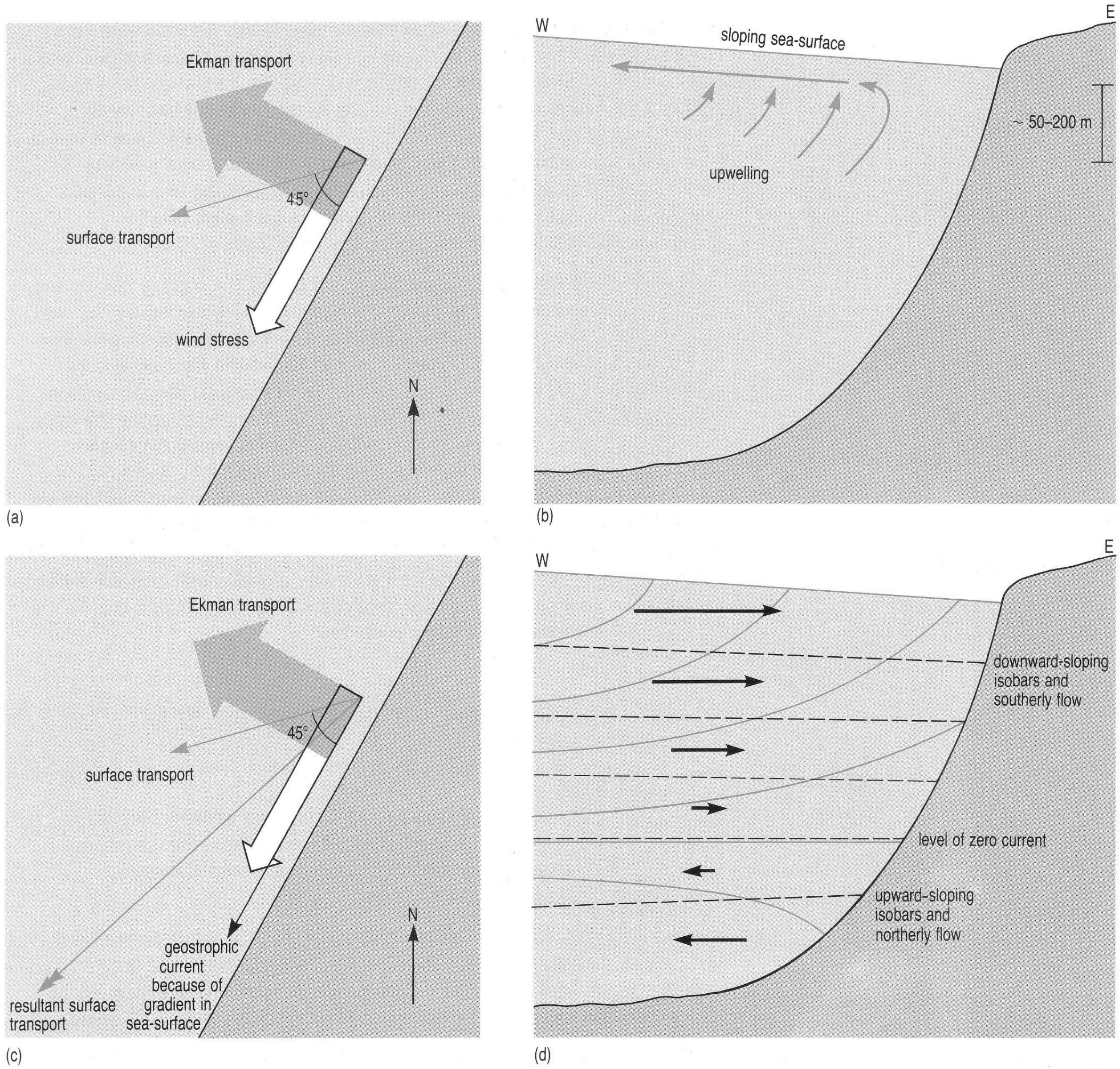


Figure 4.36 Diagrams (*not to scale*) to illustrate the essentials of coastal upwelling (here shown for the Northern Hemisphere).
(a) Initial stage: wind stress along the shore causes surface transport 45° to the right of the wind, and Ekman transport (average motion in the wind-driven layer) 90° to the right of the wind (cf. Figure 3.6(b)). (*Note:* this shows the idealized situation which is never observed in reality.)
(b) Cross-section to illustrate the effect of conditions in (a): the divergence of surface waters away from the land leads to their replacement by upwelled subsurface water, and to a lowering of sea-level towards the coast.
(c) As a result of the sloping sea-surface, there is a horizontal pressure gradient directed towards the land (black arrows in (d)) and a geostrophic current develops 90° to the right of this pressure gradient. This 'slope' current flows along the coast and towards the Equator. The resultant surface transport, i.e. the transport caused by the combination of the surface transport at 45° to the wind stress and the slope current, still has an offshore component so upwelling continues.
(d) Cross-section to illustrate the variation with depth of density (the blue lines are isopycnals) and pressure (the dashed black lines are isobars and the horizontal arrows represent the direction and relative strength of the horizontal pressure gradient force). Isopycnals slope up towards the shore as cooler, denser water wells up to replace warmer, less dense surface waters. The shoreward slope of the isobars decreases progressively with depth until they become horizontal; at this depth the horizontal pressure gradient force is zero, and so the velocity of the geostrophic current is also zero. At greater depths, isobars slope up towards the coast indicating the existence of a *northerly* flow; a deep counter-current is a common feature of upwelling systems.