Lecture 5

Thermal Wind

give out handout on jet stream, pg. 88 of Dutton's textbook.

<u>Geostrophic wind</u>: $\vec{V}_g \equiv \frac{1}{\rho f} \hat{k} \times \nabla p$

<u>Thermal wind</u>: $\vec{V}_T \equiv \vec{V}_{g\,\text{upper level}} - \vec{V}_{g\,\text{lower level}}$

The thermal wind is the <u>vector difference</u> btw the geos wind at an upper and lower level. ["Thermal wind" is a misnomer – it's NOT a wind or even a potential or hypothetical wind].

$$\vec{V}_T \equiv \vec{V}_{g\,upper} - \vec{V}_{g\,lower}$$

$$= \frac{1}{f\rho_{upper}} \hat{k} \times \nabla p_{upper} - \frac{1}{f\rho_{lower}} \hat{k} \times \nabla p_{lower} \quad \text{[Messy!]}$$

This simplifies in isobaric coords. Use $\frac{1}{\rho}\nabla_z p = \nabla_p \Phi$:

$$\vec{V}_T = \frac{1}{f} \hat{k} \times \nabla_p (\Phi_{upper} - \Phi_{lower}) \quad \text{[Simpler!]}$$

We can relate $\vec{V}_T$ to the <u>horizontal</u> gradient of temperature within this layer. Begin by considering "vertical shear" of geostrophic wind in isobaric coordinates.

Since $p\downarrow$ as $z\uparrow$, $-\frac{\partial}{\partial p}()$ behaves like $+\frac{\partial}{\partial z}()$

$$\vec{V}_g = \frac{1}{f}\hat{k} \times \nabla_p \Phi \quad \text{Take } -\frac{\partial}{\partial p}$$

$$\therefore -\frac{\partial \vec{V}_g}{\partial p} = -\frac{1}{f}\hat{k} \times \nabla_p \frac{\partial \Phi}{\partial p}$$

scratch paper. what is $\frac{\partial \Phi}{\partial p}$?

$$\frac{\partial p}{\partial z} = -\rho g$$

Use $\left.\frac{\partial p}{\partial z}\right|_{x,y} = \frac{1}{\left.\frac{\partial z}{\partial p}\right|_{x,y}}$

[valid since same stuff (x,y) is held constant]

$$\therefore \frac{1}{\left.\frac{\partial z}{\partial p}\right|_{x,y}} = -\rho g$$

$$\therefore \frac{\partial z}{\partial p} = -\frac{1}{\rho g}$$ use ideal gas law

$$\therefore \frac{\partial z}{\partial p} = -\frac{RT}{pg}$$

or: $$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

$$\therefore -\frac{\partial \vec{V}_g}{\partial p} = -\frac{1}{f}\hat{k} \times \nabla_p\left(-\frac{RT}{p}\right)$$ pull $\frac{1}{p}$ out in front of ∇_p

Get the "Thermal Wind Equation":

$$\boxed{-\frac{\partial \vec{V}_g}{\partial p} = \frac{R}{fp}\,\hat{k} \times \nabla_p T}$$

Really it's an eq^n for vertical shear of the geostrophic wind (since $-\frac{\partial \vec{V}_g}{\partial p}$ behaves like $+\frac{\partial \vec{V}_g}{\partial z}$).

Examine January jet stream diagram, pg. 88 of Dutton's "Dynamics of Atmospheric Motion"

- Look at temp profile beneath the jet max (latitude of jet max ~ 40°N)

- On any constant pressure level beneath the jet max, the temp increases towards the south. So $\nabla_p T$ points south (assuming the east/west component of $\nabla_p T$ is much smaller than the north/south comp, which is reasonable in January).

- So $\hat{k} \times \nabla_p T$ points into the page, which is towards the east (so $\hat{k} \times \nabla_p T$ is westerly).

- Since p is always positive and gas constant R is always positive and we're looking at the Northern hemisphere (where f is positive) then $\frac{R}{fp} > 0$. So $\frac{R}{fp}\hat{k} \times \nabla_p T$ points toward the east.

– So $-\frac{\partial \vec{V_g}}{\partial p}$ (which behaves as $+\frac{\partial \vec{V_g}}{\partial z}$) points toward the east.

– Since $\frac{\partial \vec{V_g}}{\partial z}$ points toward east, the change in $\vec{V_g}$ as you move upward is toward the east.

– Since the surface geostrophic wind points toward east (i.e., is westerly), the geostrophic wind intensifies with height.

– As long as temp gradient points south, the westerly geostrophic wind continues to increase with height.

– The jet maximum occurs at about 200 mb. By definition, this maximum is characterized by $\frac{\partial V_g}{\partial p} = 0$.

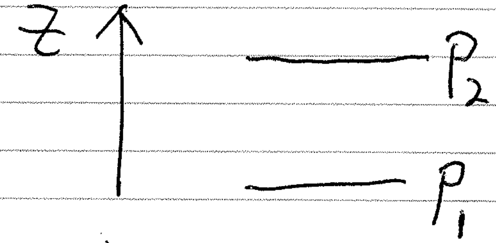
From thermal wind equation this maximum should be associated with $\nabla_p T = 0$ i.e. no horiz change in T. Notice how isotherms on diagram tend to flatten out near the jet max.

——————————— end of handout discussion.

Can relate thermal wind $\vec{V}_T \equiv \vec{V}_{g\,upper} - \vec{V}_{g\,lower}$ to horiz gradient of T by integrating the thermal wind equation:

$$-\frac{\partial \vec{V}_g}{\partial p} = \frac{R}{fp}\,\hat{k} \times \nabla_p T$$

Integrate it from a lower level to an upper level.



$$p_1 > p_2$$

$$\int_{\vec{V}_g(p_1)}^{\vec{V}_g(p_2)} d\vec{V}_g = -\frac{R}{f}\,\hat{k} \times \int_{p_1}^{p_2} \nabla_p T \,\frac{dp}{p}$$

$$\underbrace{\vec{V}_g(p_2)}_{\text{it's } \vec{V}_{g\,upper}} - \underbrace{\vec{V}_g(p_1)}_{\text{it's } \vec{V}_{g\,lower}} = -\frac{R}{f}\,\hat{k} \times \nabla_p \int_{\ln p_1}^{\ln p_2} T\, d\ln p'$$

$$\therefore \quad \vec{V}_T = -\frac{R}{f}\,\hat{k} \times \nabla_p \boxed{\int_{\ln p_1}^{\ln p_2} T\, d\ln p'}$$

we already simplified it (Lecture 4, pg 2)

$$\therefore \quad \vec{V}_T = -\frac{R}{f}\,\hat{k} \times \nabla_p \left[\underbrace{(\ln p_2 - \ln p_1)}_{\text{pull in front of } \nabla_p} \overline{T} \right]$$

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Use $-(\ln p_2 - \ln p_1) = -\ln\left(\frac{p_2}{p_1}\right) = \ln\left(\frac{p_1}{p_2}\right)$

$$\therefore \boxed{\vec{V}_T = \frac{R}{f} \ln\left(\frac{p_1}{p_2}\right) \hat{k} \times \nabla_p \overline{T}}$$