Lecture 7

Barotropic condition

If there's no vertical shear of the geostrophic wind, $-\frac{\partial \vec{V}_g}{\partial p} = 0$, then the thermal wind eqⁿ

$$-\frac{\partial \vec{V}_g}{\partial p} = \frac{R}{fp} \hat{k} \times \nabla_p T$$ yields $\nabla_p T = 0$, so

T is constant on an isobaric surface (though can differ from one isobaric surface to another).

So, in this case, $T = T(p)$

$\therefore$ $\rho = \frac{P}{RT} = \frac{P}{RT(p)}$ so density is

also const on an isobaric surface, so: $\rho = \rho(p)$

Also: $\theta = T\left(\frac{P_0}{p}\right)^{\kappa} = T(p)\left(\frac{P_0}{p}\right)^{\kappa}$

so potential temp θ is also const on an isobaric surface, so $\theta = \theta(p)$.

$$\boxed{\rho = \rho(p)}$$ Barotropic condition

This condⁿ implies all of the above statements (no shear of geos wind, etc.)

A barotropic atmosphere is pretty boring, thermodynamically.

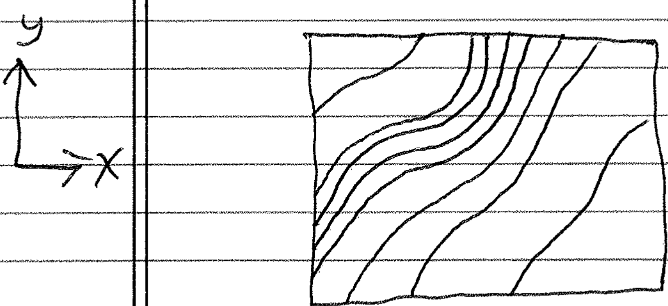
Early NWP models used the barotropic condⁿ because it greatly simplified the vorticity eqⁿ that was used in the numerical prediction.

In general, the atmosphere is baroclinic:

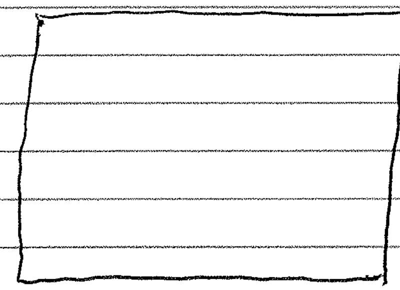
$\rho = \rho(p, T)$ without reducing to $\rho = \rho(p)$

An example of a "thermal field" (can be ρ or T or θ) on an isobaric sfc:

for a baroclinic case



for a barotropic case



blank. No isolines.

In a baroclinic flow,

$$\nabla_p T = \nabla_p\left(\frac{p}{\rho R}\right) = \frac{p}{R}\nabla_p \frac{1}{\rho}$$

$$= -\frac{p}{R\rho^2}\nabla_p \rho \neq 0$$

since ρ can vary on an isobaric surface in a baroclinic flow.

$\therefore \nabla_p T \neq 0$

$\therefore \frac{\partial \mathbf{V}_g}{\partial p} \neq 0$

So there is vertical shear of the geostrophic wind in a baroclinic atmosphere.

Techniques to estimate vertical motion

- Observations (e.g. from a vertically-pointing Doppler radar or lidar).
- Output from an N.W.P. analysis package (e.g. a Reanalysis package such as the NCEP/NCAR Reanalysis data set or the ECMWF Re-analysis).
- Infer it from the quasi-geostrophic ω eqn (obtained from a vorticity eqn and thermodynamic energy eqn).
- Infer it from the Adiabatic Method (thermodynamic energy eqn with the diabatic term neglect connects ω to u, v and T)
- Infer it from a <u>mass conservation</u> eqn

We'll study this one. But first, examine relation btw w and ω:

(<u>True</u>) vertical velocity: $w \equiv \frac{dz}{dt}$

(<u>Pressure</u>) vertical velocity: $\omega \equiv \frac{dp}{dt}$

For a <u>rising</u> air parcel $z\uparrow$ so $w > 0$
but $p\downarrow$ so $\omega < 0$

For a <u>sinking</u> air parcel $z\downarrow$ so $w < 0$
but $p\uparrow$ so $\omega > 0$

Connection between ω and w:

$$\omega = \frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}$$

If flow is approximately hydrostatic then

$$\omega = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} - \rho g w$$

For synoptic-scale flows, can use scale analysis to show that $\frac{\partial p}{\partial t}$, $u\frac{\partial p}{\partial x}$ and $v\frac{\partial p}{\partial y}$ are much smaller in magnitude than $-\rho g w$ [except right at ground where $w = 0$ (if ground is flat)].

So, for synoptic scale flows $\omega \simeq -\rho g w$ (except at ground)

Some common approximate forms of mass conservation:

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.}$$ Incompressibility condition

Also known as the Boussinesq form of mass conservation.

$$\boxed{\frac{\partial}{\partial x}(\bar{\rho} u) + \frac{\partial}{\partial y}(\bar{\rho} v) + \frac{\partial}{\partial z}(\bar{\rho} w) = 0. \quad \text{where } \bar{\rho} = \bar{\rho}(z).}$$ Anelastic mass conservation eq^n

omega

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0}$$ Isobaric-coordinate form of mass conservation. Assumes $\partial p/\partial z = -\rho g$

Can get w (or ω) from these expressions by <u>integrating</u> with respect to z (or p)

e.g. suppose we know u and v and $\bar{\rho}$ all the way down to the ground, and the ground is flat (so $w=0$ there!), then using the <u>anelastic mass cons eq^n</u> we get:

$$\frac{\partial}{\partial x}(\bar{\rho}u) + \frac{\partial}{\partial y}(\bar{\rho}v) + \frac{\partial}{\partial z}(\bar{\rho}w) = 0$$

$$\therefore \quad \frac{\partial}{\partial z}(\bar{\rho}w) = -\frac{\partial}{\partial x}(\bar{\rho}u) - \frac{\partial}{\partial y}(\bar{\rho}v)$$

integrate with respect to z from the <u>ground</u> (call it $z=0$) up to arbitrary height z

$$\bar{\rho}(z)w(z) - \bar{\rho}(0)\underbrace{w(0)}_{0 \text{ for flat ground}} = -\int_0^z \left(\frac{\partial}{\partial x}(\bar{\rho}u) + \frac{\partial}{\partial y}(\bar{\rho}v)\right)dz'$$

now $\div$ by $\bar{\rho}(z)$

$$\therefore \quad \boxed{w(z) = -\frac{1}{\bar{\rho}(z)}\int_0^z \left(\frac{\partial}{\partial x}(\bar{\rho}u) + \frac{\partial}{\partial y}(\bar{\rho}v)\right)dz'}$$

If ground is not flat but is mountainous with terrain height h that varies with x and y ($h=h(x,y)$) then lower limit of integration is $z=h(x,y)$ instead of $z=0$, and w at sfc is: $w(h) = u\,\partial h/\partial x + v\,\partial h/\partial y$