# Lecture 8



Note: The <u>anelastic mass conservation eq$^n$</u>,

$$(\star) \quad \boxed{\frac{\partial}{\partial x}(\bar{\rho} u) + \frac{\partial}{\partial y}(\bar{\rho} v) + \frac{\partial}{\partial z}(\bar{\rho} w) = 0},$$

where $\bar{\rho} = \bar{\rho}(z)$ is sometimes written in an alternative form:

since $\bar{\rho}$ is NOT a function of $x$ or $y$ pull it out of $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

$$\bar{\rho}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \bar{\rho}\frac{\partial w}{\partial z} + w\frac{d\bar{\rho}}{dz} = 0$$

$\div$ by $\bar{\rho}$, get:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dz} w = 0$$

Define $\kappa \equiv \frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dz} = \frac{d}{dz}\ln\bar{\rho}$

So $(\star)$ is equivalent to:

$$(\star\star) \quad \boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \kappa w = 0}$$

We've seen that if we know $u$, $v$, and $\bar{\rho}$ then we can integrate $(\star)$ to get $w$. We can also get $w$ from $(\star\star)$ but it requires a lot more work. Think of $(\star\star)$ as a <u>first order</u> <u>linear</u> <u>inhomogeneous</u> differential equation for $w$. Can solve it using method of <u>integrating factors</u>...

... But solving $(\star)$ for $w$ is the easier way to go (and the solutions are the same).

You can also integrate the isobaric-coordinate form of mass conservation,

$$(\star\star\star) \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

To get $\omega$ but you need a boundary condition on $\omega$ (e.g. $\omega$ on the ground) to evaluate the solution. Often you won't have that information.

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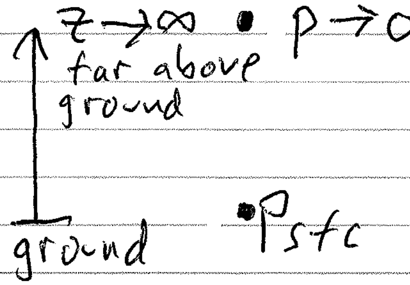
## Surface pressure tendency eq$^n$

Start with $(\star\star\star)$. Isolate the $\omega$ term:

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\therefore \quad d\omega = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp$$




Integrate from ground (where $p = p_{sfc}$) up far above ground ($z \to \infty$) where $p \to 0$ and where $\omega \equiv Dp/Dt \to 0$.

$$\int_{\omega(p_{sfc})}^{0} d\omega' = -\int_{p_{sfc}}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp'$$

$$\therefore \quad 0 - \omega(p_{sfc}) = -\int_{p_{sfc}}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp'$$

$$\omega(p_{sfc}) = \int_{p_{sfc}}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp'$$

or: $$\omega(p_{sfc}) = -\int_{0}^{p_{sfc}} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp'$$

Focus on lefthand side:

$$\omega(p_{sfc}) = \left.\frac{Dp}{Dt}\right|_{sfc} = \left.\frac{\partial p}{\partial t}\right|_{sfc} + u\left.\frac{\partial p}{\partial x}\right|_{sfc} + v\left.\frac{\partial p}{\partial y}\right|_{sfc} + w\left.\frac{\partial p}{\partial z}\right|_{sfc}$$

- On the ground $w$ is 0 (if ground is flat) or very small (on most slopes).
- Obs show that near the ground $\left|u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right| << \left|\frac{\partial p}{\partial t}\right|$

plug in here

so $$\omega(p_{sfc}) \approx \left.\frac{\partial p}{\partial t}\right|_{sfc}$$

$\therefore$ we get sfc pressure tendency eq$^n$:

$$\boxed{\left.\frac{\partial p}{\partial t}\right|_{sfc} = -\int_{0}^{p_{sfc}} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dp'}$$

or $$\left.\frac{\partial p}{\partial t}\right|_{sfc} = -\int_{0}^{p_{sfc}} \nabla_p \cdot \underset{\substack{\uparrow \\ \text{horiz velocity}}}{\vec{v}_h}\, dp'$$

$\nabla_p \cdot \vec{v}_h$ is called the divergence of the wind field. (really it's the 2-dimensional divergence).

If $\nabla_p \cdot \vec{v}_h > 0$ then we have <u>positive divergence</u>, which we refer to simply as "<u>divergence</u>."

e.g. $\leftarrow \; \rightarrow$ or $\rightarrow \; \longrightarrow$

If $\nabla_p \cdot \vec{v}_h < 0$ then we have <u>negative divergence</u>, which we refer to simply as "<u>convergence</u>."

e.g. $\rightarrow \leftarrow$ or $\longrightarrow \rightarrow$

So, if we have <u>net convergence</u> in a column above the surface, (net convergence of mass), into column

$$\int_0^{p_{sfc}} \nabla_p \cdot \vec{v}_h \, dp' < 0$$

$$\text{so } -\int_0^{p_{sfc}} \nabla_p \cdot \vec{v}_h \, dp' > 0$$

$$\therefore \left.\frac{\partial p}{\partial t}\right|_{sfc} > 0$$ <u>surface pressure rises!</u>

But if we have <u>net divergence</u> in a column above the surface, (net divergence of mass out of column),

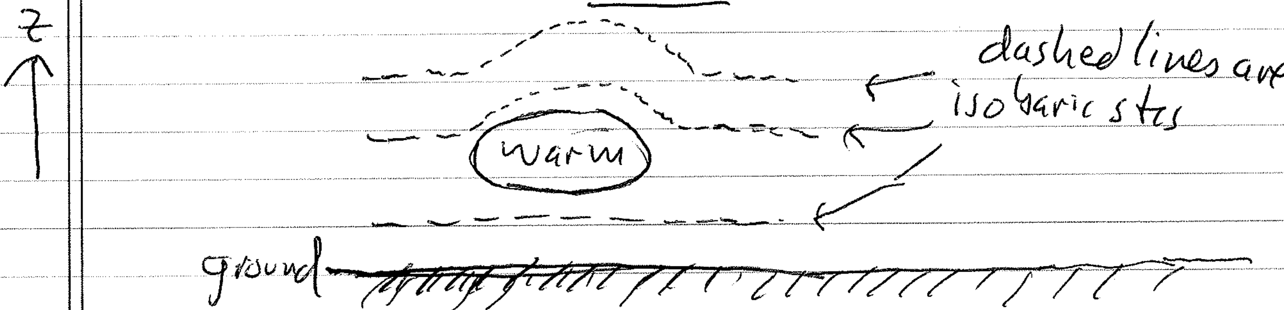
$$\int_0^{p_{sfc}} \nabla_p \cdot \vec{v}_h \, dp' > 0$$

$$\text{so } -\int_0^{p_{sfc}} \nabla_p \cdot \vec{v}_h \, dp' < 0$$

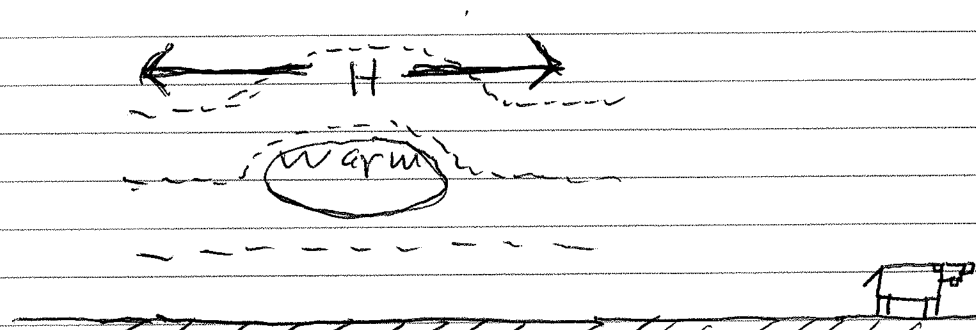
$$\therefore \left.\frac{\partial p}{\partial t}\right|_{sfc} < 0$$ <u>surface pressure falls!</u>

Example: <u>Development of a thermal cyclone.</u>

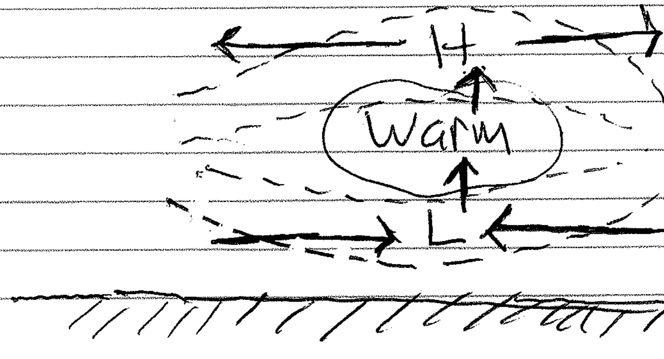
- Suppose a local heat source acts in the midtroposphere.
- From hypsometric eq$^n$, heights of pressure surfaces are <u>raised</u> above the warm zone.




- So now there's high pressure above the warm zone.
- Upper level p.g.f. drives a divergent flow, aloft.




- <u>Net divergence in column above surface causes surface pressure to fall</u> (according to surface pressure tendency equation).




Low level p.g.f. drives a convergent flow at low levels [and associated rising motion]