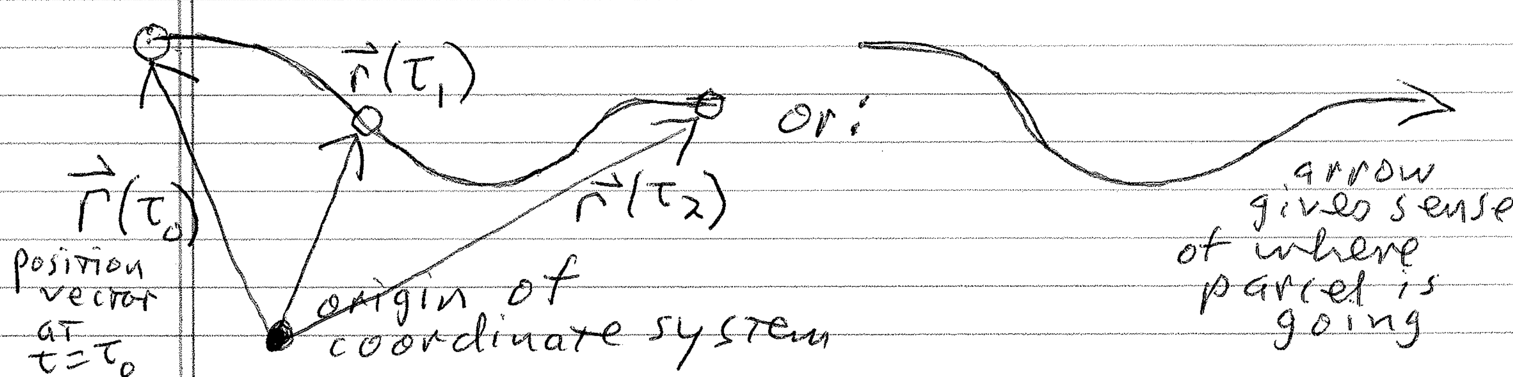
Lecture 9

- 1 handout on streamlines + trajectories

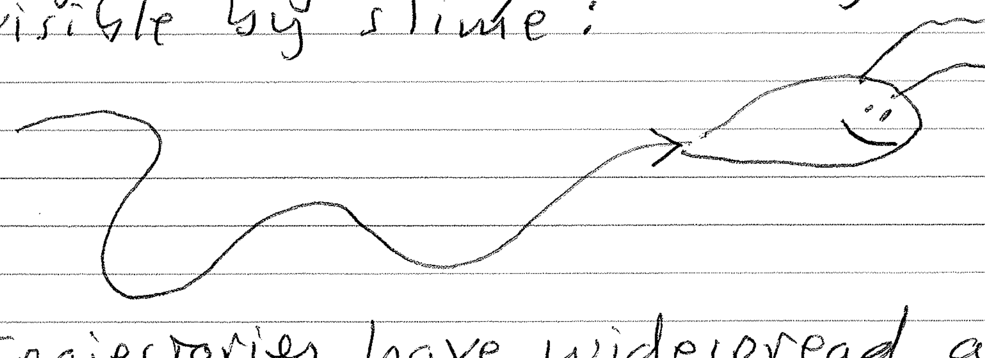
Trajectories

A trajectory is the curve traced out by the tip of the position vector of an air parcel (or other mass) as it moves around.



Gives history of motion of a parcel — a Lagrangian concept.

e.g. Trajectory of a slug is made visible by slime:



Trajectories have widespread applications in the environmental sciences:

- can be used to study the origin of low-level rotation in mesocyclones and tornadoes (where do the parcels in tornados come from)?

- can be used to study the source region of atmospheric pollutants at receptor sites.
- Can be used to track the spread of isotopes from nuclear reactor breaches and oil from offshore oil spills.
- Can be used to track floating mines and fish larvae, and can be used in search + rescue + recovery operations

If we know the velocity field as a function of time, we can calculate the trajectory from:

$$\boxed{\frac{D\vec{r}}{D\tau} = \vec{U}(\tau)} \leftarrow \text{Vector differential eq}^{n} \text{ for trajectories}$$

where $\vec{r} = x(\tau)\hat{i} + y(\tau)\hat{j} + z(\tau)\hat{k}$

$\vec{U} = u(\tau)\hat{i} + v(\tau)\hat{j} + w(\tau)\hat{k}$

Trajectory eq^{ns} (in component form):

$$\boxed{\frac{Dx}{D\tau} = u \qquad \frac{Dy}{D\tau} = v \qquad \frac{Dz}{D\tau} = w}$$

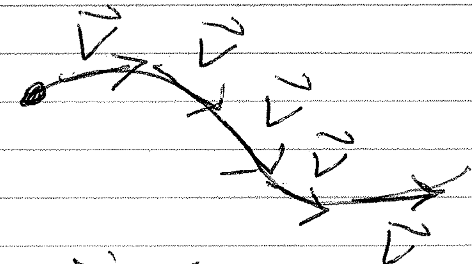
But the complication is that you need to know u, v and w at the parcel location in order to solve these equations.

If you know u, v, w at regular fixed grid points (e.g. from a NWP) model, you can interpolate their values to the parcel location.

Streamlines

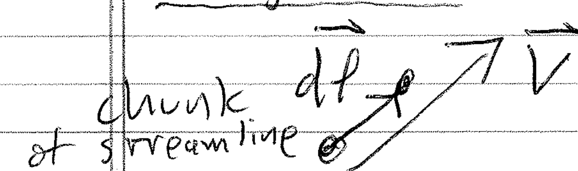
A streamline is a curve that's everywhere tangent to the local velocity vectors at a given time. [It cuts through many parcels at a given time].

streamline through a point of interest at some given time:



Derive the differential equations for streamlines:

Let $\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ be a small chunk of a streamline. It's tangential to $\vec{V}$:



So the cross product btw $\vec{dl}$ and $\vec{V}$ must be 0.

$$\vec{dl} \times \vec{V} = 0$$ write as a determinant

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$ expand it out

$$\hat{i}(w\,dy - v\,dz) + \hat{j}(u\,dz - w\,dx) + \hat{k}(v\,dx - u\,dy) = 0$$

Take dot product of prev eq^n with $\hat{i}$ (and use $\hat{i}\cdot\hat{i}=1$, $\hat{i}\cdot\hat{j}=0$, $\hat{i}\cdot\hat{k}=0$)

$$w\,dy - v\,dz = 0 \rightarrow \frac{dy}{dz} = \frac{v}{w}$$

Take dot product with $\hat{j}$, get:

$$u\,dz - w\,dx = 0 \rightarrow \frac{dx}{dz} = \frac{u}{w}$$

Take dot product with $\hat{k}$, get:

$$v\,dx - u\,dy = 0 \rightarrow \frac{dy}{dx} = \frac{v}{u}$$

These are the differential eq^{ns} for streamlines. Can use them in these or rewritten forms such as

$$\frac{dz}{dx} = \frac{w}{u}, \quad \frac{dz}{dy} = \frac{w}{v} \quad \text{etc.}$$

Streamlines and trajectories usually do NOT coincide! But they do coincide in steady-state flows.

[Steady state flow: No change in u, v or w with time at any location, that is, $\frac{\partial u}{\partial t}=0$, $\frac{\partial v}{\partial t}=0$, $\frac{\partial w}{\partial t}=0$]

→ Examine handout on streamlines + traj.

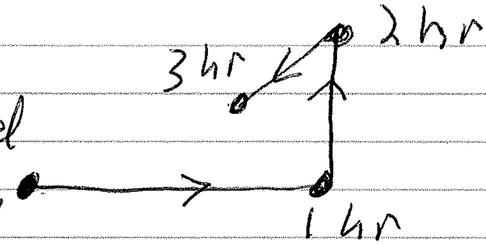
e.g. Describe trajectories for a flow in which wind is <u>westerly</u> at 3 m/s everywhere from $\tau = 0$ to $\tau = 1$ hr

Then wind is <u>southerly</u> at 2 $\frac{m}{s}$ everywhere from $\tau = 1$ hr to $\tau = 2$ hr

Then wind is north-easterly at 1 $\frac{m}{s}$ everywhere from $\tau = 2$ hr to $\tau = 3$ hr.

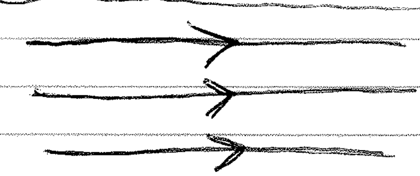
<u>Trajectory</u> :

consider a parcel that's here at $\tau = 0$



<u>streamlines at 3 dif times</u> :

$\boxed{\tau = 0.5\text{ hr}}$
<u>(or any time btw 0 and 1 hr)</u>



$\boxed{\tau = 1.5\text{ hr}}$

$\boxed{\tau = 2.5\text{ hr}}$