METR 5113, Advanced Atmospheric Dynamics I
Alan Shapiro, Instructor
First day of class (Mon, 20 Aug 2018)

4 handouts: (i) syllabus (ii) useful references (iii) class exam time (please fill out and return by next Monday!), and (iv) Greek alphabet.

Class introductions.

Go through the 4 handouts.

Goal of course: develop quantitative skills in dynamics so that you can read journal articles in dynamics (e.g. J. Atmos. Sci., Mon. Wea. Rev., Quart. J. Roy. Meteor. Soc.) as well as perform research. This course is an <u>in-depth study of basic concepts in dynamics</u> -- <u>it is not a survey course</u>.

Advice:

1. Read the notes carefully. Be sure you understand the derivations thoroughly and can reproduce them.

2. Take a first pass at the material yourself then discuss it in a study group (you're encouraged to join a study group).

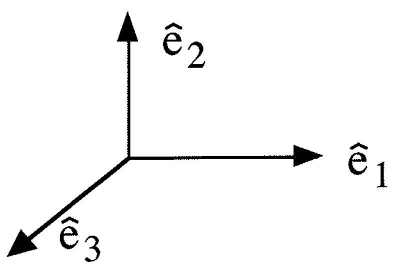
3. Keep a running list of questions to ask me (and members of your study group).

4. Buy or borrow books to help amplify material you find particularly interesting (or to help shore up material you are having difficulty with).

Reading: Kundu: Author's notes, Ch1 (skip sfc tension and dimensional analysis) and Ch2

<u>Vector and Cartesian Tensor Analysis</u> (Ch 2 of Kundu).

Consider a Cartesian coord system w/ <u>unit</u> vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ in the x_1, x_2 and x_3 directions (a right-handed triple).



Equivalent notation:

$\hat{e}_1, \hat{e}_2, \hat{e}_3$

$\hat{i}, \hat{j}, \hat{k}$

[Discuss what a right-handed triple is. Draw examples]

$$\hat{e}_1 \cdot \hat{e}_1 = \overset{1}{|\hat{e}_1|}\overset{1}{|\hat{e}_1|}\cos\overset{0}{(\hat{e}_1, \hat{e}_1)} = \cos 0 = 1 \,,$$

$$\hat{e}_2 \cdot \hat{e}_2 = 1 \,,$$

$$\hat{e}_3 \cdot \hat{e}_3 = 1 \,,$$

$$\hat{e}_1 \cdot \hat{e}_2 = \overset{1}{|\hat{e}_1|}\overset{1}{|\hat{e}_2|}\cos\overset{90^\circ}{(\hat{e}_1, \hat{e}_2)} = \cos 90^\circ = 0 \,,$$

$$\hat{e}_1 \cdot \hat{e}_3 = 0 \,,$$

$$\hat{e}_2 \cdot \hat{e}_1 = 0 \,,$$

$$\hat{e}_2 \cdot \hat{e}_3 = 0 \text{, etc.}$$

The position vector $\vec{x}$ of a fluid element can be decomposed into the $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ directions as,

$$\vec{x} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3 = \sum_{k=1}^{3} x_k\hat{e}_k$$ (k not just vertical!)

Note that $\vec{x} \cdot \hat{e}_1 = x_1 \underbrace{\hat{e}_1 \cdot \hat{e}_1}_{1} + x_2 \underbrace{\hat{e}_2 \cdot \hat{e}_1}_{0} + x_3 \underbrace{\hat{e}_3 \cdot \hat{e}_1}_{0} = x_1$

So $\quad x_1 = \vec{x} \cdot \hat{e}_1$

Similarly, $\quad x_2 = \vec{x} \cdot \hat{e}_2$

$x_3 = \vec{x} \cdot \hat{e}_3$

In view of the above, we can rewrite $\vec{x}$ as:

$$\vec{x} = (\vec{x} \cdot \hat{e}_1)\hat{e}_1 + (\vec{x} \cdot \hat{e}_2)\hat{e}_2 + (\vec{x} \cdot \hat{e}_3)\hat{e}_3$$

or: $\vec{x} = \sum_{k=1}^{3} (\vec{x} \cdot \hat{e}_k)\hat{e}_k$

or: $\vec{x} = \sum_{i=1}^{3} (\vec{x} \cdot \hat{e}_i)\hat{e}_i$, etc

In general, for <u>any</u> vector $\vec{F}$ (velocity, vorticity, acceleration, temperature gradient, etc):

$$\boxed{\vec{F} = (\vec{F} \cdot \hat{e}_1)\hat{e}_1 + (\vec{F} \cdot \hat{e}_2)\hat{e}_2 + (\vec{F} \cdot \hat{e}_3)\hat{e}_3}$$

or: $\boxed{\vec{F} = \sum_{k=1}^{3} (\vec{F} \cdot \hat{e}_k)\hat{e}_k}$

or: $\boxed{\vec{F} = \sum_{i=1}^{3} (\vec{F} \cdot \hat{e}_i)\hat{e}_i}$, etc.

Can write $\vec{x}$ as column vector: $$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Transpose of $\vec{x}$ is row vector: $\vec{x}^T = (x_1, x_2, x_3)$

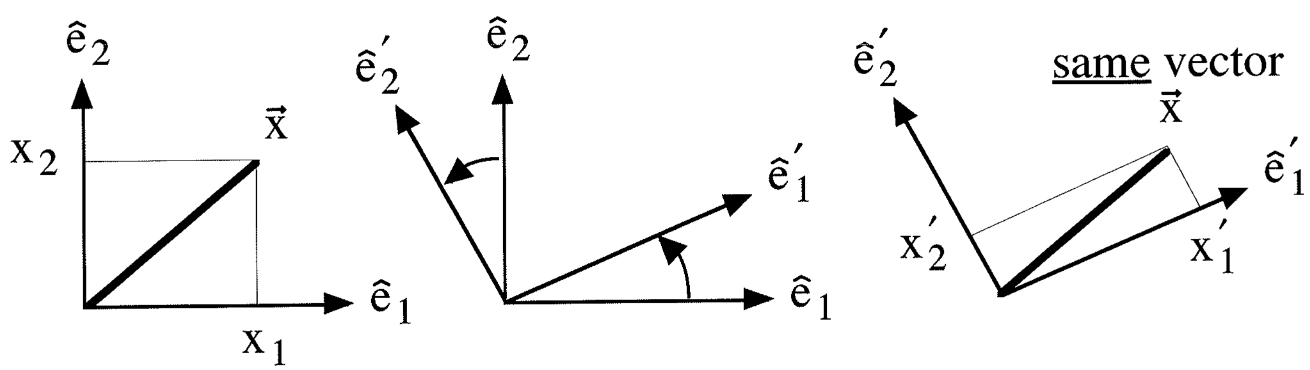
Now introduce a new Cartesian coord system with the <u>same origin</u> as original, obtained from original by a <u>rotation</u> of axes. <u>Rotated</u> not rotat<u>ing</u>. (rotate coord system then leave it alone).

Unit vectors in the new (rotated) system are: $\hat{e}'_1, \hat{e}'_2, \hat{e}'_3$

<u>Same</u> pos^n vector $\vec{x}$ can be rewritten in new coord system as,

$$\vec{x} = x'_1 \hat{e}'_1 + x'_2 \hat{e}'_2 + x'_3 \hat{e}'_3 = \sum_{k=1}^{3} x'_k \hat{e}'_k$$

$\vec{x}$ itself doesn't care about coord system but its <u>components</u> do care. $x'_1 \neq x_1$, $x'_2 \neq x_2$, etc. To see this, consider:



Graphically, we see above that $x'_1 > x_1$ while $x'_2 < x_2$

How are components of $\vec{x}$ in new system related to components of $\vec{x}$ in the old system? Look at x'_2 component:

$$x_2' = \vec{x} \cdot \hat{e}_2' = \sum_{k=1}^{3} x_k \, \hat{e}_k \cdot \hat{e}_2'$$

$$= \sum_{k=1}^{3} x_k \underbrace{|\hat{e}_k|}_{1} \underbrace{|\hat{e}_2'|}_{1} \cos\left(\hat{e}_k, \hat{e}_2'\right) = \sum_{k=1}^{3} x_k \cos(\hat{e}_k, \hat{e}_2')$$

Define $C_{k2} \equiv \cos(\hat{e}_k, \hat{e}_2')$ 1st index of C -- old system
2nd index of C -- new system

So $x_2' = \sum_{k=1}^{3} C_{k2} x_k$

Get similar result for x_1' and x_3'. In general,

$$x_j' = \sum_{k=1}^{3} C_{kj} x_k \,, \qquad \text{where } C_{kj} \equiv \cos(\hat{e}_k, \hat{e}_j')$$

Equivalently, define $C_{kj} \equiv \hat{e}_k \cdot \hat{e}_j'$. C is direction-cosine matrix.
1st index of C goes with old system, 2nd index goes with new.