METR 5113, Advanced Atmospheric Dynamics I
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Wednesday, 12 September 2018 (lecture 10)

- 1 handout: frontogenesis/frontolysis in shear flow

[Finish up transparencies on linear algebra]

Rate of strain tensor e is 2^{nd} order, real, symmetric. In principal axes coordinates it appears in diagonal form:

$$e' = \begin{pmatrix} e'_{11} & 0 & 0 \\ 0 & e'_{22} & 0 \\ 0 & 0 & e'_{33} \end{pmatrix}$$

So, if you consider a fluid box oriented s.t. faces are $\perp$ to the principal axes, only <u>normal</u> strains exist. "One box's shear strains are another box's normal strains"

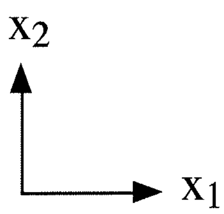
As expected, sum of diag elements of e (vol strain rate) is invariant under a coord rotation:

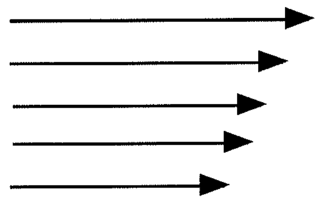
$$e'_{ij} = C_{ki} C_{lj} e_{kl}$$

$$\therefore \quad e'_{ii} = C_{ki} C_{li} e_{kl} = C_{ki} C^T_{il} e_{kl} = \left(C \cdot C^T\right)_{kl} e_{kl}$$

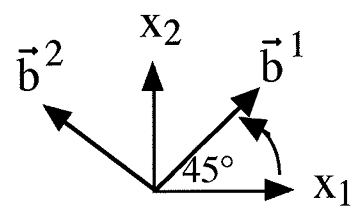
$$= \delta_{kl} e_{kl} = e_{kk} \,(\text{or } e_{ll}) = e_{ii}$$

Back to our shear flow example (see calculation in Kundu)



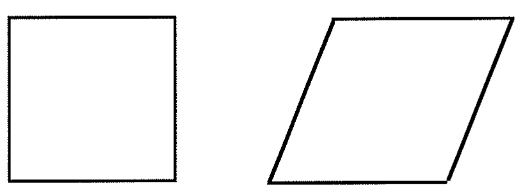


principal axes:



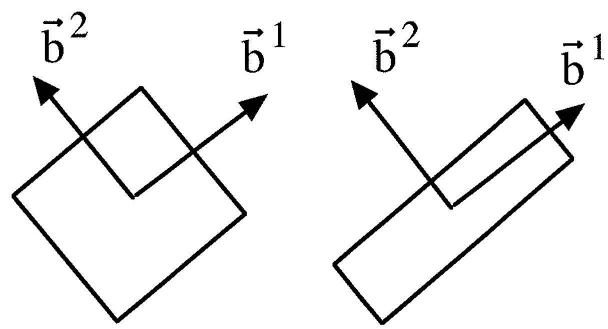
stretching along $\vec{b}^1$ axis (axis of dilatation)
compression along $\vec{b}^2$ axis (axis of contraction)

consider a box aligned with original coord axes:



$t = t_0$ $\quad$ $t = t_0 + \delta t$

consider a box aligned with principal axes:



$t = t_0$ $\quad$ $t = t_0 + \delta t$

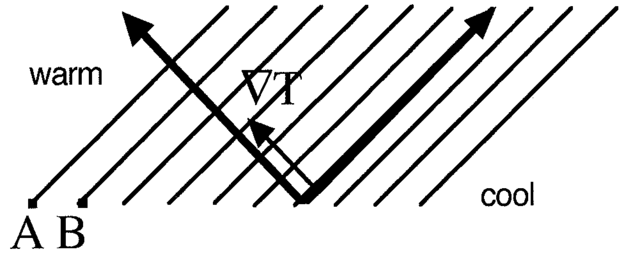
How does horiz deformation affect frontogenesis or frontolysis?

Effect of horizontal deformation alone is to promote fronto-genesis when axis of dilatation lies within 45° of the isotherms, and to promote frontolysis when axis of dilatation lies between 45° and 90° of the isotherms (Vol 2, Bluestein's Synoptic Met.)

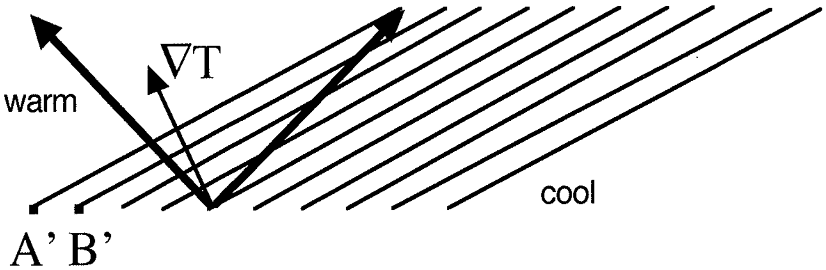
Consider same shear flow as above, with 2 different distributions of isotherms (following drawings are in the x_1 x_2 plane):

e.g. Frontogenesis: increase in magnitude of horiz temp gradient

isotherms
at $t = t_0$:



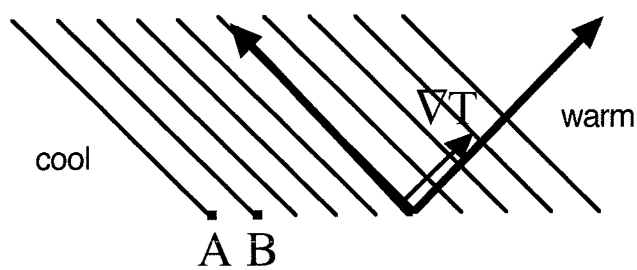
isotherms
at $t = t_1$:



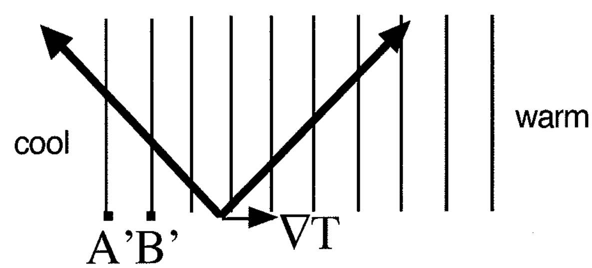
distance AB and distance A'B' are the same, but isotherms are now packed closer together -- $|\nabla T|$ has increased.

e.g. Frontolysis: decrease in magnitude of horiz temp gradient

isotherms
at $t = t_0$:

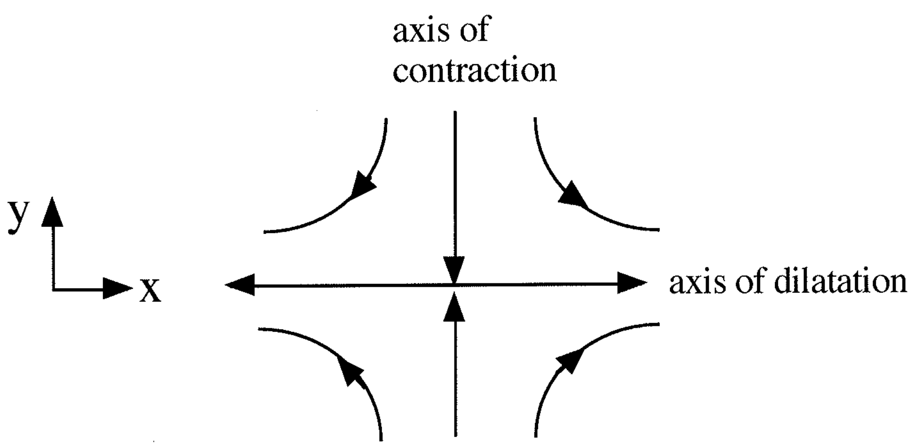


isotherms
at $t = t_1$:

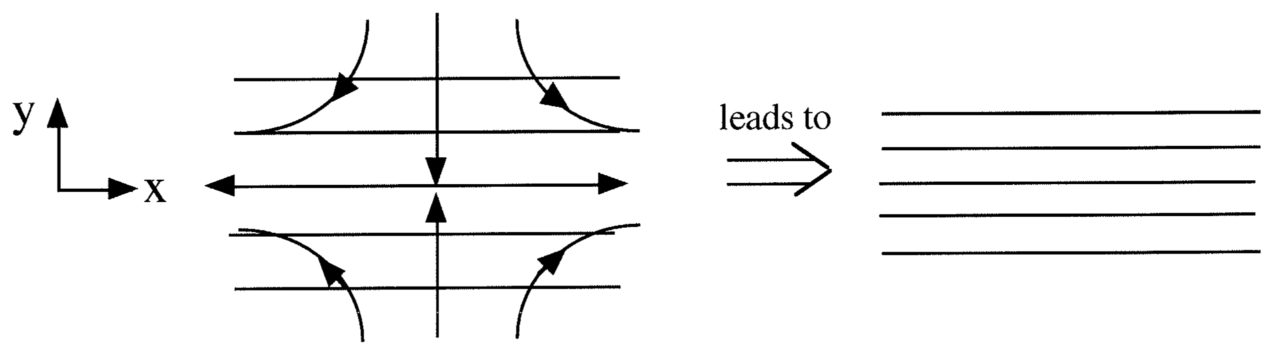


distance AB and distance A'B' are the same, but isotherms are now further apart than before so $|\nabla T|$ has decreased.

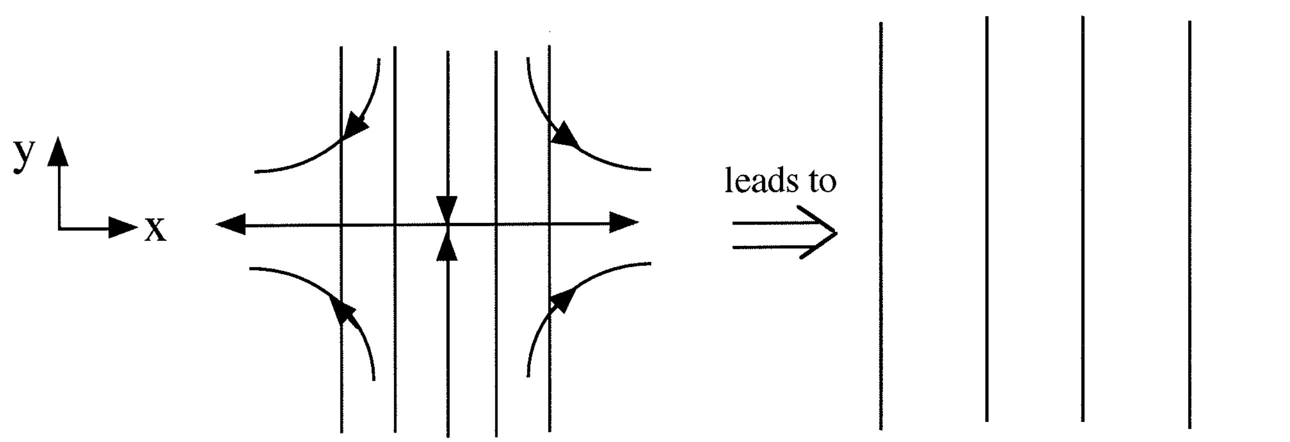
Another example of horiz deformation affecting frontogenesis/ frontolysis. Consider $u = Ax$, $v = - Ay$ (let $A > 0$). Solve odes for streamlines, get $xy = \text{const}$. [see next page] Flow looks like:



If isotherms are parallel to axis of dilatation (0° angle w.r.t. axis of dilatation) get frontogenesis:



If isotherms are perpendicular to axis of dilatation (90° angle w.r.t. the axis of dilatation) get <u>frontolysis</u>:



Calculation of streamlines for a flow in which $u = Ax$, $v = -Ay$ (with $A > 0$):

$$\frac{dx}{ds} = u, \qquad \frac{dy}{ds} = v$$

so $$\frac{dx}{ds} = Ax, \qquad \frac{dy}{ds} = -Ay$$ now separate variables:

$$\frac{dx}{x} = A\,ds, \qquad \frac{dy}{y} = -A\,ds$$ integrate, get

(1) $\ln x = A\,s + \text{const}$, (2) $\ln y = -A\,s + \text{dif const}$

From (2): $As = -\ln y + \text{dif const}$. Plug this into (1),

$\ln x = -\ln y + \text{const} + \text{dif const}$

$\ln x + \ln y = C$ where C is some const

$\ln(xy) = C$ exp both sides [exp(C) is a const, call it D]

$xy = D$

To describe particular streamlines, choose dif values for D. For example, corresponding to the choice $D = 0$ we find a streamline along $x = 0$, $y =$ anything, and $x =$ anything, $y = 0$, i.e., running along the coordinate axes, as in diagram on prev page.