METR 5113, Advanced Atmospheric Dynamics I
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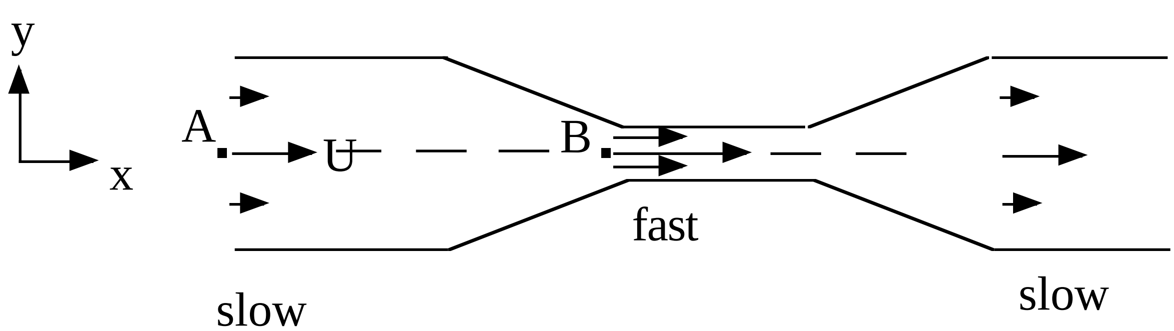
Friday, 12 Rocktober 2018 (lecture 22)

1 handout: answers to problem set 3.

Applications of Bernoulli's eq^{ns}

Example 1: Constriction in a channel (flow through a canyon)



Assume steady state and ρ = const. Use steady state version of B eqn (where vort may or may not be zero):

$$\frac{q^2}{2} + \frac{p}{\rho} + gz = C \quad \text{(along a streamline)}$$

Consider flow along central streamline (through A and B). Far upstream of constriction (at A) $\vec{u} = U\hat{i}$, so $q_A = U$.

$$\therefore \quad \frac{U^2}{2} + \frac{p_A}{\rho} + gz_A = C$$

At the constriction (B):

$$\frac{q_B^2}{2} + \frac{p_B}{\rho} + gz_B = C$$

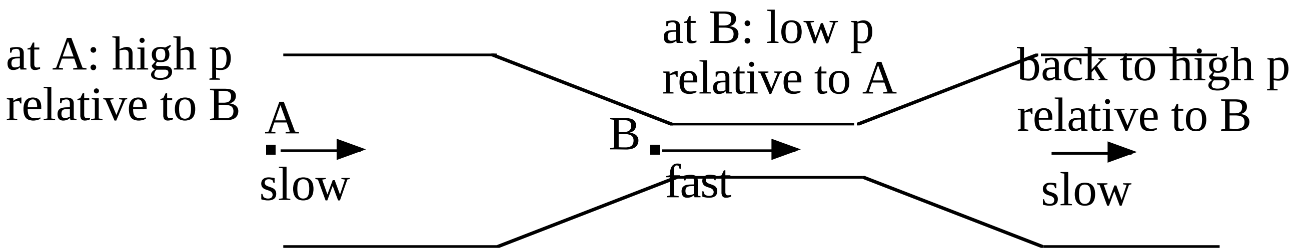
Same C in both eq^{ns} because we're on same streamline.

$$\therefore \quad \frac{U^2}{2} + \frac{p_A}{\rho} + \boxed{gz_A} = \frac{q_B^2}{2} + \frac{p_B}{\rho} + \boxed{gz_B}$$

cancellation since $z_A = z_B$

$$\therefore \quad p_A - p_B = \frac{\rho}{2}(q_B^2 - U^2) > 0 \quad \text{since } \underset{\text{fast}}{q_B} > \underset{\text{slow}}{U} \quad \text{(from mass cons)}$$

$$\therefore \quad p_A > p_B$$

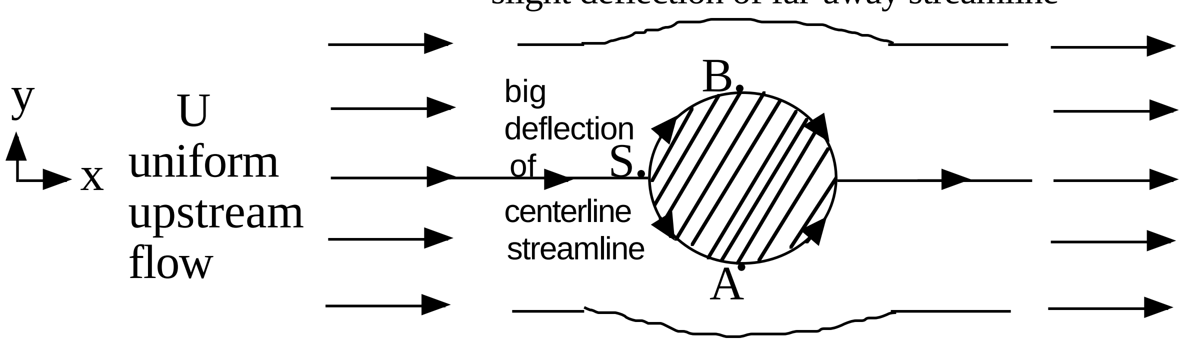


Flow is faster at B than at A (from mass cons eq^n). So a parcel accelerates from A to B. Bernoulli's eq^n says this acceleration is associated with a pressure drop.

<u>Get qualitatively same result if you assume uniform flow upstream and use irrotational version of Bernoulli's eqn.</u>

Theoretically, very fast speeds can be associated w/ <u>negative</u> pressures (tensions rather than compressions). In liquids get <u>cavitation</u> (boiling) when vapor pressure is reached. Collapsing vapor bubbles can damage dam spillways, submarine propellers and pumps/pipes.

<u>Example 2</u> - Steady, 2D, deflection of a uniform flow around a cylinder (e.g. telephone pole). (t-storm updrafts sometimes behave as if they're solid cylinders)



S is a <u>stagnation point</u> -- all velocity comps are 0 at S. Normal comp at S is 0 due to impermeability cond^n. Tangential comp at S is 0 from symmetry (<u>not</u> from no-slip cond^n). $\therefore$ $q_s = 0$.

From mass cons and streamline deflection (confluence) north and south of cylinder, expect fast flow north and south of cylinder.

Use Bernoulli eq^n for irrotational flow but w/ unsteady term = 0 since this is a steady state. Assume ρ = const.

$$\frac{q^2}{2} + \frac{p}{\rho} + gz = C \quad \text{(same const everywhere)}$$

Far upstream (anywhere upstream):

$$\frac{U^2}{2} + \frac{p_{up}}{\rho} + gz = C$$

At stagnation point at S:

$$0 + \frac{p_s}{\rho} + gz = C \qquad \text{(same const as above)}$$

$$\therefore \quad \frac{U^2}{2} + \frac{p_{up}}{\rho} + \boxed{gz} = \frac{p_s}{\rho} + \boxed{gz} \quad \text{cancellation}$$

$$\therefore \quad p_s = p_{up} + \rho \frac{U^2}{2}$$

$$\therefore \quad p_s > p_{up}$$ $\therefore$ <u>high pressure at stagnation point.</u>

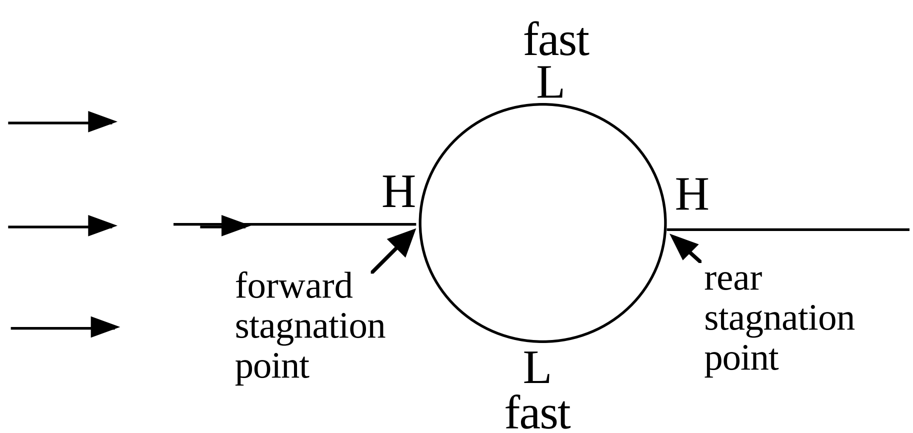
Now look on north point of cylinder:

$$\frac{q_B^2}{2} + \frac{p_B}{\rho} + gz = C$$

$$\therefore \quad \frac{q_B^2}{2} + \frac{p_B}{\rho} + gz = \frac{U^2}{2} + \frac{p_{up}}{\rho} + gz$$

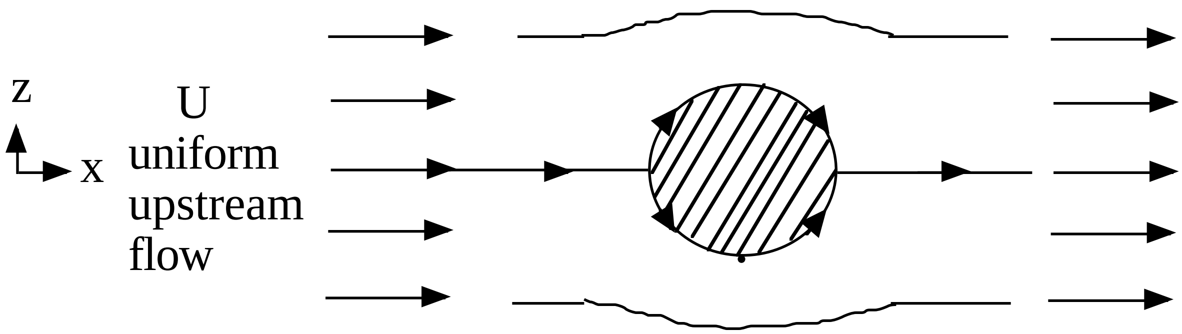
$$\therefore \quad p_B = p_{up} + \frac{\rho}{2}\boxed{(U^2 - q_B^2)} \rightarrow \text{negative since } U < q_B$$

$\therefore \quad p_B < p_{up} \quad \therefore$ <u>low pressure at north point of cylinder</u> (and at south point)



In real life p at rear stag point is not as big as p at forward stag point (due to friction, boundary layer separation in lee of cylinder, and possible turbulence). End up with a net pressure force on cylinder in direction of flow. However, analysis of forward part of cylinder and upstream still valid (a la Prandtl).

Example 3 - Same as example 2 but consider flow over horizontally-oriented cylinder (or tree branch).



$$\frac{q^2}{2} + \frac{p}{\rho} + gz = C \quad \text{(same const everywhere)}$$

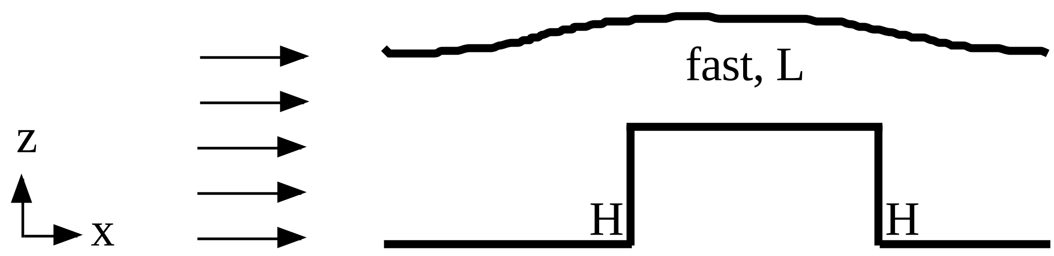
Rewrite using perturbation pressure p'. [$p = p' + \bar{p}(z)$
$= p' + (\text{const} - \rho g z) = p' - \rho g z + \text{const}$].

$$\therefore \quad \frac{p}{\rho} + gz = \frac{p'}{\rho} + \text{const}$$

$$\therefore \quad \frac{q^2}{2} + \frac{p'}{\rho} = C \quad \text{(this C differs from old C)}$$

Proceed as in example 2, but work w/ pert pressure p' instead of full p. Get low perturbation pressure p' at top and bottom of cylinder, and high perturbation pressure at stag point.

<u>Example 4</u>: Wind-storm over a Walmart.



Similar to previous example. Can use irrot version of B eqn w/unsteady term set to 0.

$$\frac{q^2}{2} + \frac{p}{\rho} + gz = C \text{ (same const everywhere)}$$

$$\therefore \frac{q^2}{2} + \frac{p'}{\rho} = C$$

Get high p' at 2 stag points. Get fast flow above roof. There q^2 is really big so p' is large negative (low p) but p inside building is nearly the same as before storm (it was hydrostatic so p'=0 there). $\therefore$ net pert pressure force on roof is large and upward -- roof lifts off. Then pressures equalize and roof falls into store.