METR 5113, Advanced Atmospheric Dynamics I
Alan Shapiro, Instructor
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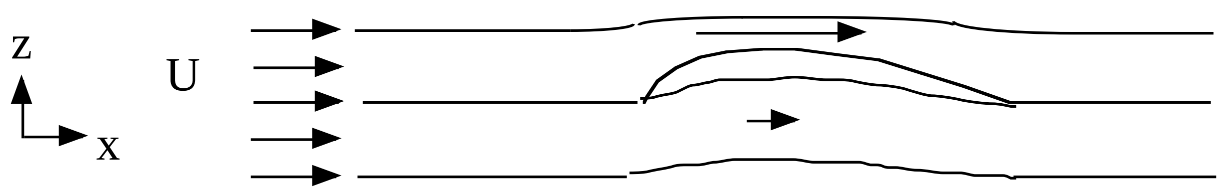
More Applications of Bernoulli's equation

Example 5: Flow over airplane wing

Consider airplane moving down runway at a constant speed U. Take frame of ref moving w/ plane. In this ref frame flow is steady and looks like:



Now analysis procedes as in Walmart example.

$$\frac{p'}{\rho} + \frac{q^2}{2} = C \quad \text{(same const everywhere).}$$

Look upstream, where q = U and flow is purely hydrostatic (so p' = 0). Get: $C = U^2/2$. So at any location:

$$\frac{p'}{\rho} + \frac{q^2}{2} = \frac{U^2}{2}$$

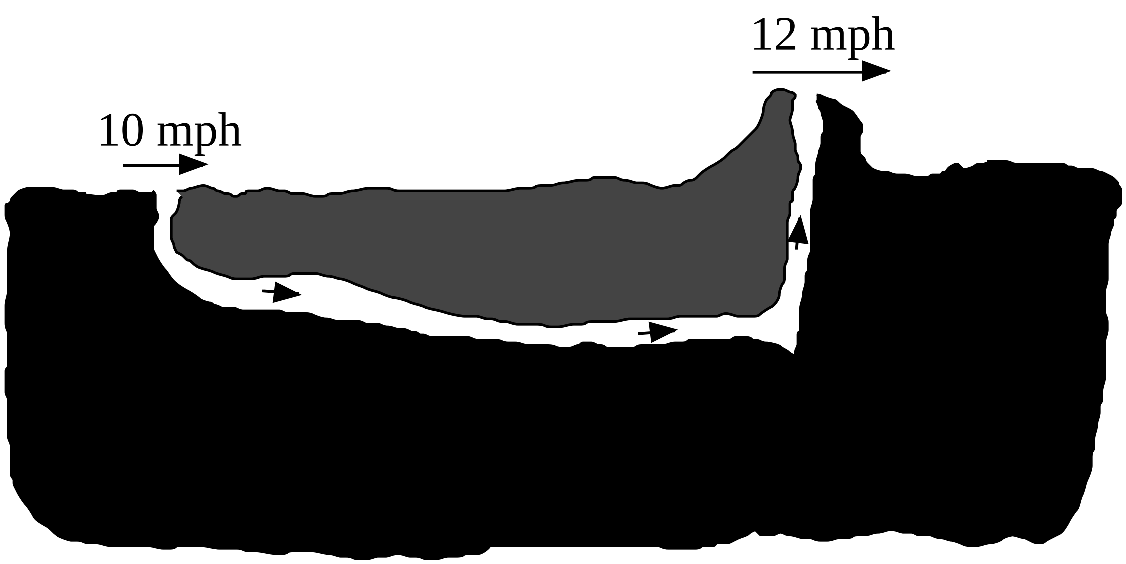
$$\therefore \; p' = \frac{\rho}{2}(U^2 - q^2)$$

Bow-shape of top of wing induces fast flow over top of wing.

So, $q > U$ so $U^2 - q^2 < 0$ so p' is negative there. Bow shape of bottom of wing induces slow flow beneath wing. So $q < U$ so $U^2 - q^2 > 0$ so p' is positive there. So get net upward pert pressure force on wing -- lift. Plane lifts off ground when integrated pert pressure force exceeds weight of plane.

B eqn also explains why planes can't take off when its very hot. (e.g., Phoenix at 120F). Since ρ is small (high temp), $|p'|$ on wing is less than what it would be at lower temps. So lift is reduced.

Example 6: Bernoulli Ventilation of Prairie dog burrows



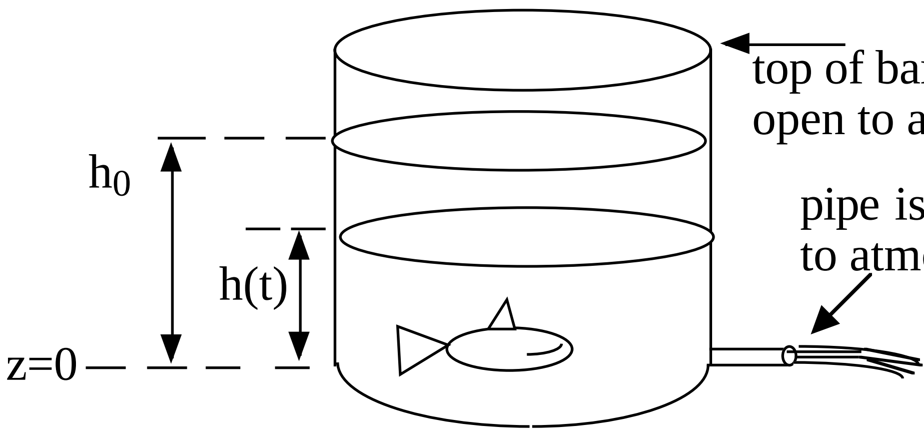
Prairie dog burrows have at least two openings, one of which is elevated. Near-surface wind increases as it passes over mound (just as flow is faster over top of cylinder or over aircraft wing). Work w/ irrot B eqn. Pert pressure is lower at mound opening than at lower openings. Pert pressure drives slow flow within burrow from high p' toward low p' (friction is probably important within burrow so don't apply B th^m down there).

Example 7: Bernoulli Ventilation of underground cities

-- Hittites of mountainous central Turkey built underground cities in soft rock that held up to 20,000 people and were up to 20 stories deep. Ventilation shafts sunk into mountain tops were connected to lateral shafts that surfaced at lower elevations.

-- Byzantines and Romans also built underground cities with ventilation systems..

Example 8: Flow of water out of a barrel



Initial height of free surface (air/water interface) is h_0.
Height of free sfc at time t is h(t).
Cross-sectional area of barrel is A
Cross-sectional area of pipe is a   (a << A)

Speed of descending free sfc is V(t) [so velocity of free sfc is $-V(t)\hat{n}_{sfc}$ where $\hat{n}_{sfc}$ is unit outward normal to free sfc, i.e., in $\hat{k}$ direction]. Speed of outward jet is v(t) [so velocity is $v(t)\,\hat{n}_{pipe}$ where $\hat{n}_{pipe}$ is unit outward normal to pipe face].

From mass cons, VA = va. But let's prove it!:

___________scratch paper:

Mass conservation eq^n for liquid is incompressibility cond^n, $\nabla \cdot \vec{u} = 0$. Integrate it over all water in barrel-pipe system: $\int \nabla \cdot \vec{u}\, dV = 0$. Div thm says $\int \nabla \cdot \vec{u}\, dV = \int \vec{u} \cdot \hat{n} dA$. So $\int \vec{u} \cdot \hat{n} dA = 0$. Integral is over whole area bounding barrel-pipe system. $\vec{u} \cdot \hat{n} = 0$ on all solid bdries (impermeability cond^n), but not on free surface or at pipe face.

$$\therefore \quad \underbrace{\int \vec{u} \cdot \hat{n}\, dA}_{\text{over free sfc}} + \underbrace{\int \vec{u} \cdot \hat{n}\, dA}_{\text{over pipe face}} = 0$$

$$\therefore \quad -V \underbrace{\boxed{\hat{n}_{sfc} \cdot \hat{n}_{sfc}}}_{1} A + v \underbrace{\boxed{\hat{n}_{pipe} \cdot \hat{n}_{pipe}}}_{1} a = 0$$

$$\therefore \quad -VA + va = 0$$

$$\therefore \quad \boxed{VA = va}$$

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So $\frac{V}{v} = \frac{a}{A} << 1$

So $V << v$

Use Bernoulli's eqn to find the relation between the speed of water out of barrel and the height of the free surface.

Assume flow starts from rest ∴ initial vorticity is 0. Can show vort remains 0 (in absence of baroclinic and frictional processes). ∴ Use <u>irrot</u> form of B eq^n. Free surface moves down very very slowly so flow behaves as if it's "nearly" in a steady state. So can neglect unsteady term (a good approx), and get:

$$\frac{q^2}{2} + \frac{p}{\rho} + gz = C \quad \text{(same const everywhere)}$$

At free sfc (on liquid side of free surface):

$z = h(t)$

$q = V$, slow descent of fluid surface.

$p = p_{at}$, liquid at free sfc has atmos pressure (pressure is continuous across air/liquid interface)

ρ is the density of the water.

$$\therefore \quad \frac{V^2}{2} + \frac{p_{at}}{\rho} + gh = C \qquad (*)$$

In jet:

$z = 0$

$q = v$ (fast!)

$p = p_{at}$, (assume same atmospheric pressure as above)

ρ is the density of the water.

$$\therefore \quad \frac{v^2}{2} + \frac{p_{at}}{\rho} + 0 = C \qquad (**)$$

Since rhs's of (*) and (**) are equal to each other, the lhs's must also be equal to each other,

$$\therefore \quad \frac{V^2}{2} + \boxed{\frac{p_{at}}{\rho}} + gh = \frac{v^2}{2} + \boxed{\frac{p_{at}}{\rho}} + 0 \qquad \text{(cancellation)}$$

$$\therefore \quad \frac{V^2}{2} + gh = \frac{v^2}{2} \qquad (***)$$

Quickie answer: neglect V compared to v (since V << v), get

$$\therefore \quad \frac{v^2}{2} = gh$$

$$\therefore \quad v = \sqrt{2gh}$$

Torricelli's Theorem (1643) [this was actually known before Bernoulli eq^{ns}]