METR 5113, Advanced Atmospheric Dynamics I
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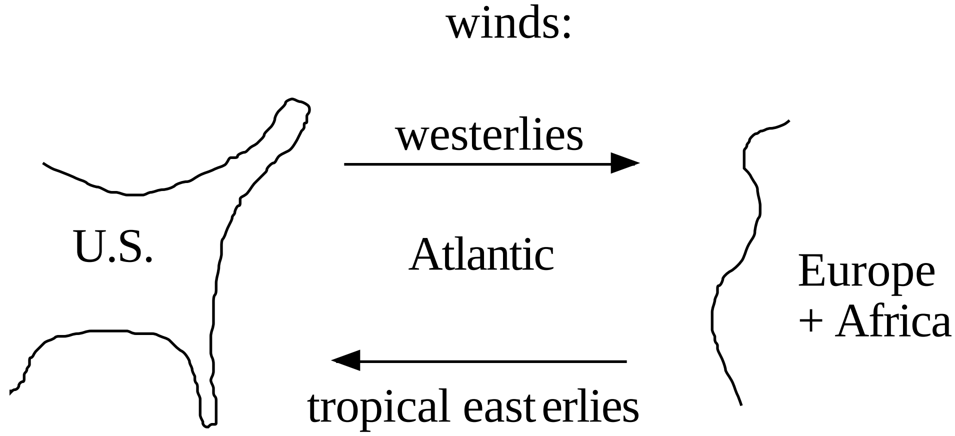
Friday, 9 November 2018 (lecture 34)

Reading: Kundu's chapter on Geophysical Fluid Dynamics (up through Ekman layers).

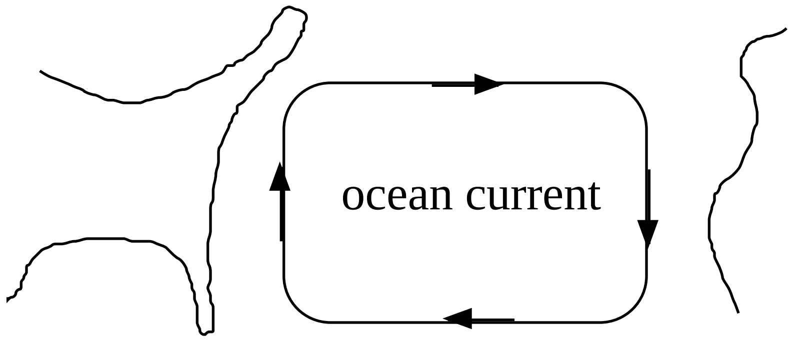
For flows in which accelerations are large compared to Coriolis acceleration, distinction btw rot and non-rot ref frames is not important (can ignore rot for small- and some meso-scale flows). However, for synoptic- and larger-scale flows the distinction is very important.

Let's look at some examples.

An application of a circ thm: Westward intensification of western ocean currents (e.g. Gulf stream).

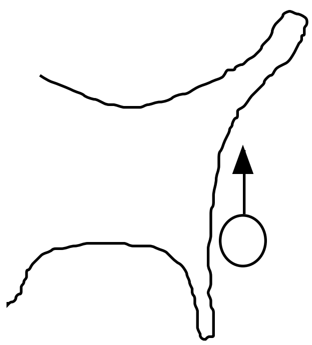


Prevailing winds are westerly up north and easterly in south. Wind stress on ocean induces oceanic currents. Currents are deflected by land. Net effect is a clockwise oceanic circulation:

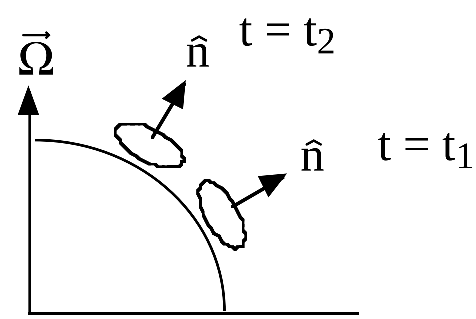


"Impermeable" east coast of US forces easterly current to deflect northward. Consider a horiz parcel of water near the coastline:

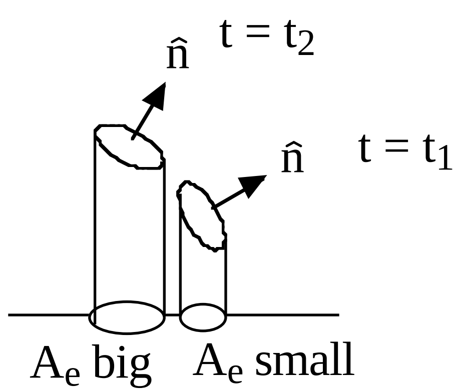
in plan view:



x-section through earth's rotation axis:



If parcel moves northward with its area unchanged (and tangent to earth's sfc) then comp of $\hat{n}$ in dir^n of $\vec{\Omega}$ increases $\therefore A_e \uparrow$



Since $\Gamma_R + 2\Omega A_e = C$, and $A_e \uparrow$, must have $\Gamma_R \downarrow$. Then, from Stoke's th^m, <u>area-integrated vorticity comp normal to area</u>

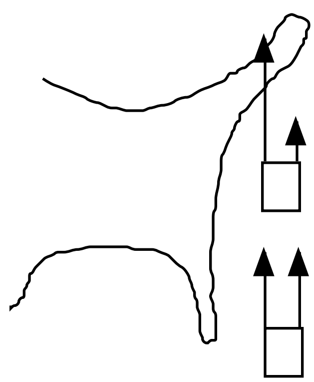
(which is vertical comp ζ) is decreasing. Since A is const, $\zeta \downarrow$.

$$\therefore \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \downarrow$$

But $u \approx 0$ all along coast (so all along y dir^n) due to impermeability cond^n, so $\partial u / \partial y \approx 0$ all along coast so

$$\therefore \quad \frac{\partial v}{\partial x} \downarrow$$

Suppose parcel starts out (in south) with v indep of x, so initially $\partial v/\partial x = 0$. As parcel moves north, $\partial v/\partial x \downarrow$ so $\partial v/\partial x$ becomes negative and increases its magnitude the further north it goes. A negative value of $\partial v/\partial x$ means v decreases in x direction (toward east) so v increases toward west. So v is max at coast.



Get westward intensification!

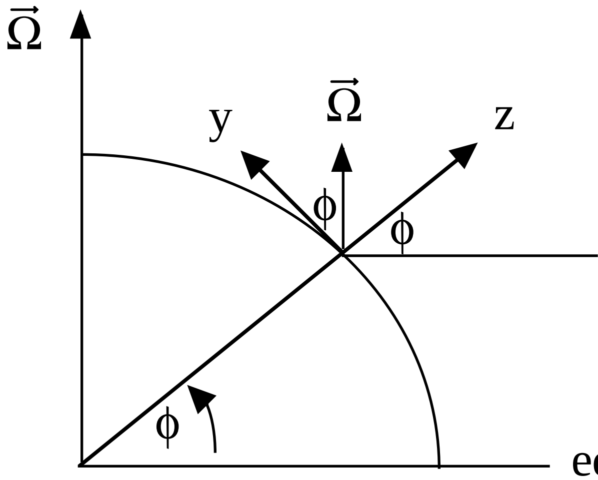
Conversely, ocean currents on eastern edge of ocean basins are relatively weak (flow weakens as coast is approached).

Geostrophic flow

Consider N-S eqns in rot ref frame:

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \nabla\right)\vec{u} = -\frac{1}{\rho}\nabla p - 2\vec{\Omega}\times\vec{u} + \vec{g} + \nu\nabla^2\vec{u}$$

For large-scale flows, it's convenient to work in spherical coordinates. However, if the regions of interest are much smaller than the earth's radius, it's even more convenient to use a local Cartesian system (x points east, y points north, z points up):



[To get x, y, z comp^s of Coriolis force, use above diagram to get x, y, z comp^s of $\vec{\Omega}$, then evaluate cross-product $-2\vec{\Omega}\times\vec{u}$. Get:
$-2\vec{\Omega}\times\vec{u} = \hat{i}\,(2\Omega\sin\phi\ v - 2\Omega\cos\phi\ w) - \hat{j}\,2\Omega\sin\phi\ u + \hat{k}\,2\Omega\cos\phi\ u$.
Then take dot product of this eqn with $\hat{i}$, $\hat{j}$, and $\hat{k}$, in turn,]

Now consider the special case where:

u = const (for all space/time)
v = const "
w = 0 "

so flow is not accelerating (neglecting curvature of streamlines around curved earth), and friction terms are 0.

$$\therefore\ \ 0 = -\frac{1}{\rho}\nabla p - 2\vec{\Omega}\times\vec{u} + \vec{g}$$

in component form, get:

x-comp: $$0 = fv - \frac{1}{\rho}\frac{\partial p}{\partial x}$$ where $f = 2\Omega\sin\phi$, ϕ is latitude

y-comp: $$0 = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

z - comp: $$\frac{\partial p}{\partial z} = -\rho g + \boxed{\rho 2\Omega\cos\phi\, u}$$ (negligible!)

Flow is in <u>geostrophic balance</u>: balance btw Cor force and pgf

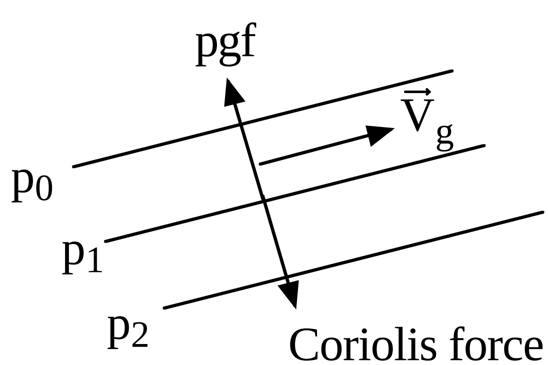
Define the geostrophic wind components to be:

$$u_g \equiv -\frac{1}{\rho f}\frac{\partial p}{\partial y}, \qquad v_g \equiv \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

or in vector form: $\vec{V}_g \equiv \hat{k} \times \frac{1}{\rho f}\nabla p$, where $\vec{V}_g = u_g\,\hat{i} + v_g\,\hat{j}$

[Show on board how to get this vector geos wind from the components of the geos wind].

From these definitions, geostrophic wind is really a normalized pressure gradient (well, $\hat{k}\times$ normalized pressure gradient).



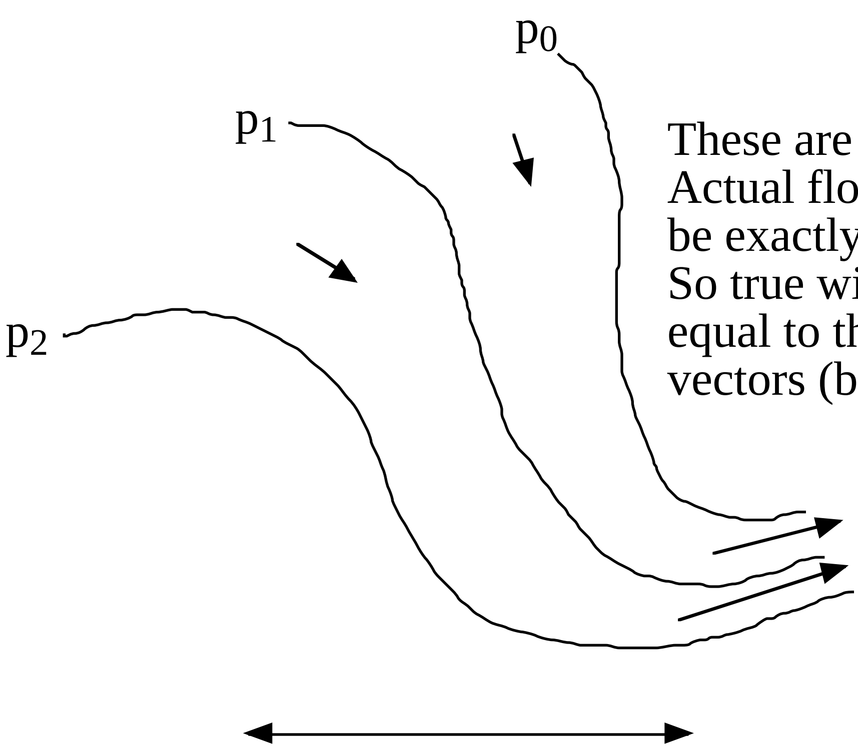
Geostrophic wind blows parallel to the isobars.

The stronger the pgf, the stronger the geostrophic wind.

$\vec{V}_g$ blows to the left of ∇p (to right of pgf) in N. Hemisphere.

Buys Ballot's Law: With the (geostrophic) wind at your back, low pressure is on your left in the northern hemisphere.

Geostrophic balance is a good approx to real flows in the atmosphere and oceans for <u>synoptic-scale flows</u> above boundary layer. Even though real streamlines can be curved and flow can be unsteady, the magnitude of the acceleration associated with these effects is usually only 10% of the mag of the Coriolis force.



These are geos wind vectors. Actual flow in this case can't be exactly geostrophic (why?) So true wind won't be exactly equal to these geos wind vectors (but can be close).

In general, can use scale analysis to determine the relative importance of acceleration to Coriolis force.

Let U be typical magnitude of the horiz velocity.

Let L by typical length scale (scale over which u and v change by an amount approx equal to U)

Then the typical ratio of acceleration to Coriolis force is:

$$\left| \frac{\left(\vec{u} \cdot \nabla\right) \vec{u}}{2\vec{\Omega} \times \vec{u}} \right| \approx \frac{U^2 / L}{fU} = \frac{U}{fL}$$

Rossby number $\mathrm{Ro} \equiv \frac{U}{fL}$