METR 5113, Advanced Atmospheric Dynamics I
Alan Shapiro, Instructor
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1 handout: fig. 9.3 of Dutton

Can use scale analysis to determine relative importance of acceleration to Coriolis force.

Let U be typical magnitude of horiz velocity.

Let L by typical horiz length scale (scale over which u and v change by an amount approx equal to U).

Then the typical ratio of acceleration to Coriolis force is:

$$\left| \frac{(\vec{u} \cdot \nabla)\vec{u}}{2\vec{\Omega} \times \vec{u}} \right| \approx \frac{U^2/L}{fU} = \frac{U}{fL}$$

Rossby number $Ro \equiv \frac{U}{fL}$

If Ro >> 1, acceleration is much larger than Coriolis force.

If Ro << 1, acceleration is much smaller than Coriolis force.

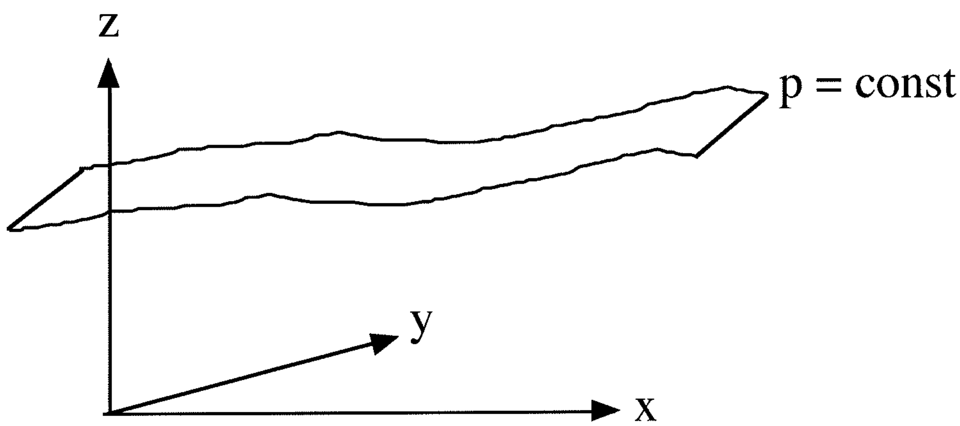
For large-scale flows in mid-latitudes, $U \sim 10$ m/s, $L \sim 1000$ km (10^6 m), $f \sim 10^{-4}\ s^{-1}$ and therefore $Ro \sim 0.1 << 1$
$\therefore$ acceleration << Coriolis force for large-scale flows.

In tornados, $U \sim 100$ m/s, $L \sim 100$ - 1000 m, $f \sim 10^{-4}\ s^{-1}$. Ro can be 1000 to 10,000 or more (>>1). $\therefore$ acceleration >> Coriolis force. Coriolis force is negligible.

Isobaric (pressure) coordinates

As long as flow is hydrostatic (e.g., synoptic-scale flows and many meso-scale flows), it is convenient to work with isobaric (pressure) coordinates rather than Cartesian coordinates.

Consider a surface of constant pressure p = const. This sfc will be nearly horizontal:



As long as we're on an isobaric sfc, $\delta p = 0$. Can move arbitrary increments δx, δy in x, y directions but then we must also move a definite amount δz in z direction to stay on sfc.

As long as we're on this p-surface:

$$\delta p = 0 = \left.\frac{\partial p}{\partial x}\right|_{y,z} \delta x + \left.\frac{\partial p}{\partial y}\right|_{x,z} \delta y + \left.\frac{\partial p}{\partial z}\right|_{x,y} \delta z + \text{h.o.t.}$$

where δx, δy are arbitrary but δz is not.

h.o.t. --> 0 for δx, δy, δz --> 0 (which is how we consider them)

By moving on this sfc in x dir^n but not y dir^n ($\delta y = 0$) we find

$$\left.\frac{\partial p}{\partial x}\right|_{y,z} \delta x + \left.\frac{\partial p}{\partial z}\right|_{x,y} \delta z = 0 \qquad (\delta x, \delta z \dashrightarrow 0)$$

$$\therefore\ \delta z = -\frac{\frac{\partial p}{\partial x}}{\boxed{\frac{\partial p}{\partial z}}}\,\delta x = \frac{1}{\rho g}\frac{\partial p}{\partial x}\,\delta x \qquad (\delta x, \delta z \dashrightarrow 0)$$

use hydrostatic eqn

Above eqn tells us how far to move in z-dir^n to stay on sfc.

Divide it by δx,

$$g\,\frac{\delta z}{\delta x} = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (\delta x, \delta z \dashrightarrow 0)$$

In the limit $\delta x \dashrightarrow 0$, get

$$g\left.\frac{\partial z}{\partial x}\right|_{p} = \frac{1}{\rho}\left.\frac{\partial p}{\partial x}\right|_{z} \qquad \text{y is also held const}$$

or, after introducing geopotential $\Phi = gz$:

$$\left.\frac{\partial \Phi}{\partial x}\right|_{p} = \frac{1}{\rho}\left.\frac{\partial p}{\partial x}\right|_{z}$$

Similarly, by moving in y-dir^n (holding x const), we get

$$\left.\frac{\partial \Phi}{\partial y}\right|_{p} = \frac{1}{\rho}\left.\frac{\partial p}{\partial y}\right|_{z}$$

Can combine these two (and mult by -1) to get:

$$\boxed{-\frac{1}{\rho}\nabla_z p = -\nabla_p \Phi}$$

It's an expression for horiz p.g.f. in isobaric coordinates.

Pressure contours on constant height surface carry same information as geopotential contours on a const pressure surface.

Isobaric form is simpler -- it doesn't have density in it! Only 1 variable to worry about (Φ) rather than 2 variables p, ρ.

As a consequence, the expression for geostrophic wind is also simpler in isobaric coordinates:

$$\vec{V}_g = \hat{k} \times \underbrace{\frac{1}{\rho f}\nabla_z p}_{\rho \text{ and } p} = \hat{k} \times \underbrace{\frac{1}{f}\nabla_p \Phi}_{\text{just } \Phi}$$

Other expressions are also simpler in isobaric coordinates (e.g., mass conservation, vort eqn, thermal wind, etc.)

Thermal Wind

It's a misnomer. Not a wind. Not even a virtual (thought-experiment) wind. It's a vertical shear of the geos wind. As we'll see, it's related to the horiz temp gradient.

Consider geos wind in pressure coords:

$$\vec{V}_g = \hat{k} \times \frac{1}{f}\nabla_p \Phi$$

Take deriv w.r.t. p

$$\frac{\partial \vec{V}_g}{\partial p} = \hat{k} \times \frac{1}{f} \nabla_p \boxed{\frac{\partial \Phi}{\partial p}} \longrightarrow \text{what's meaning of this?}$$

---------------- scratch paper.

write hydrostatic eqn as:

$$\frac{\partial z}{\partial p} = -\frac{1}{\rho g} \quad \text{mult by g and use } \Phi \equiv g z$$

$$\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p} \quad \text{used ideal gas law } p = \rho RT$$

$$\frac{\partial \vec{V}_g}{\partial p} = -\hat{k} \times \frac{1}{f} \nabla_p \frac{RT}{p} \quad \text{mult by -1 and rearrange}$$

$$\boxed{-\frac{\partial \vec{V}_g}{\partial p} = \frac{R}{fp} \hat{k} \times \nabla_p T}$$

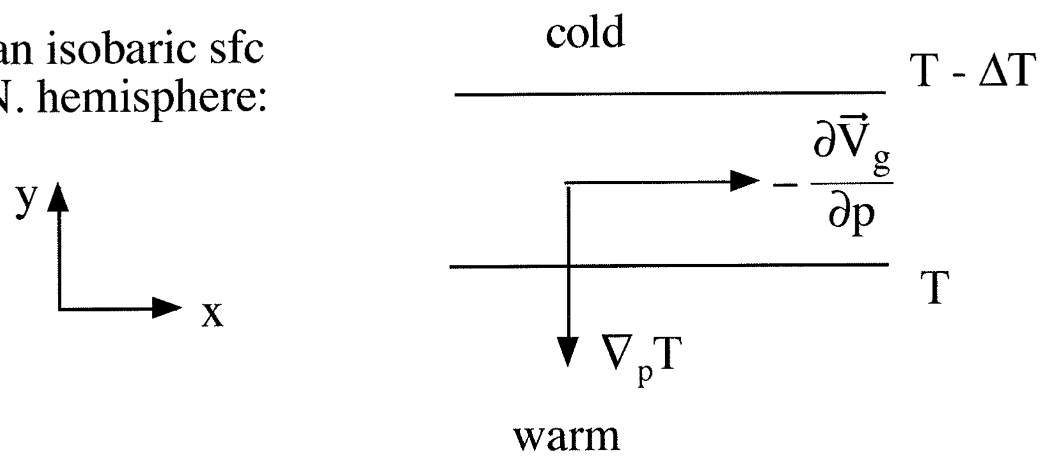
$-\frac{\partial}{\partial p}$ behaves like $\frac{\partial}{\partial z}$ so $-\frac{\partial \vec{V}_g}{\partial p}$ behaves like vertical shear of geos wind. Call it thermal wind shear vector or just thermal wind.

Thermal wind shear vector is parallel to isotherms on an isobaric surface.

With thermal wind shear vector at your back, cold air is to your left (in the N. hemisphere).

Example:

on an isobaric sfc
in N. hemisphere:



So in this case if geostrophic wind vector is westerly at surface, it intensifies with height. If geostrophic wind vector is easterly at the surface, it weakens with height, then reverses (i.e., becomes westerly), then intensifies with height.

[fig. 9.3 of Dutton, transparency]

Thermal wind shear eqn can also be used to explain why:

- cyclonic winds (counter-clockwise winds in N. hemisphere) around cold core lows increase with z

- anticyclonic winds (clockwise winds in N. hemisphere) around warm core highs increase with z.