METR 5113, Advanced Atmospheric Dynamics I
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Ekman flow (continued)

From the Ekman solution we see that friction induces a flow component directed toward low pressure.

Ekman layer depth δ_E is measure of thickness of frictional boundary layer. (at $z = \delta_E$, the wind is about 80% geostrophic)

$\delta_E \equiv \sqrt{\frac{2\nu}{f}}$. [As $\nu \uparrow$, $\delta_E \uparrow$. As $f \uparrow$, $\delta_E \downarrow$]

Observed Ekman depths in atmosphere ~1000 m. Theory yields:

$$\delta_E = \sqrt{\frac{2\nu}{f}} = \sqrt{\frac{2 \times 1.4 \times 10^{-5} \mathrm{m}^2 \mathrm{s}^{-1}}{10^{-4} \mathrm{s}^{-1}}} \sim 0.5 \mathrm{~m}$$

Whoa! Much smaller than observations! Why? Real atmosphere is turbulent (so $\vec{u} = \vec{u}(x,y,z,t)$, not $\vec{u} = \vec{u}(z)$). However, if we take an "appropriate" average of N-S eq^{ns}, the averaged eq^{ns} look like the un-averaged eq^{ns} but with molecular viscosity ν replaced with a much larger eddy viscosity ν_E.

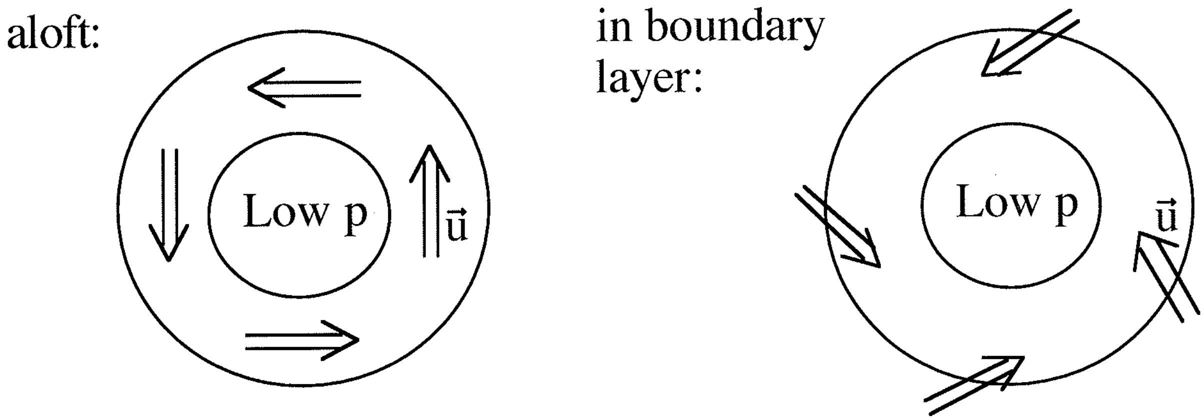
Calculate eddy viscosity based on observed Ekman depth:

$$\sqrt{\frac{2\nu_E}{f}} = 1000 \mathrm{~m} \quad \therefore \quad \nu_E = \frac{f}{2}\left(1000 \mathrm{~m}\right)^2 \sim 50 \mathrm{~m}^2/\mathrm{s}$$

Pure Ekman spirals don't exist in nature but modified (flatter) spirals are observed, as well as main theoretical result that low level flow cuts across isobars towards low pressure.

Ekman pumping in low pressure systems

If streamlines are curved, Ekman theory is not strictly valid (u,v, vary in x,y as well as z), but it's approximately valid. Apply Ekman concepts locally -- assume velocity profile in vicinity of any point of interest behaves like an Ekman velocity profile (the Ekman spiral profiles can differ from point to point).



Horiz p gradient aloft is largely present at low levels. At low levels, friction induces a flow component toward low pressure. Get horiz convergence into low pressure zone, get rising motion (from mass conservation), condensation, rain, etc.

An approximate vorticity equation for mid-latitude synoptic-scale flows above boundary-layer.

Such flows are observed to have the following characteristic scales of motion:

horiz length scale L ~ 1000 km = 10^6 m
vert length scale H ~ 10 km = 10^4 m

horiz velocity scale U ~ 10 m/s

vert velocity scale $W \sim 0.01$ m/s

scale for typical change in pressure across a horiz distance L:
$\Delta p \sim 10$ mb $= 10^3$ Pa

density scale $\rho \sim 1$ kg/m^3

scale for typical change in density across a horiz distance L:
$\Delta\rho \sim 0.01$ kg/m^3 (about 1% of the typical value for density)

time scale T (advective time scale -- time it takes a parcel moving horizontally at speed U to traverse distance L):
$T = L/U = 10^5$ s

Using these values, typical sizes of terms in the equations of motion can be estimated, e.g., $u \dfrac{\partial u}{\partial x} \sim \dfrac{U^2}{L} = 10^{-4}$ m/s^2, etc.

Performing a synoptic-scale analysis of the horizontal eqns of motion (see Holton, section 2.4) we find that we can safely neglect friction and vertical advection of wind, obtaining:

$$(1) \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$(2) \quad \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

[Here x and y are local Cartesian coords, x--> east, y --> north]

Taking $\partial/\partial x$ (2) - $\partial/\partial y$ (1) yields:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\zeta = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)f - v\frac{df}{dy} + \frac{1}{\rho^2}\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{1}{\rho^2}\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}$$

Can also use scale analysis to show that baroclinic terms are <u>much smaller</u> than other terms in vert vort eqn so neglect 'em.

Get approx vert vort eqn for synoptic-scale flows:

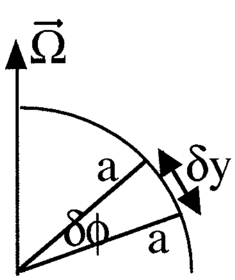
$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)(\zeta + f) - v\frac{df}{dy}$$

stretching of absolute vorticity — advection of earth vort

Since df/dy > 0, southerly winds ($v > 0$) yield $- v\, df/dy < 0$ so relative vorticity ζ decreases. <u>Southerly winds bring in lower values of earth vort</u>. Conversely, <u>northerly winds bring in higher values of earth vort</u> so ζ increases.

Can relate df/dy to Ω, latitude, and earth's radius a (~ 6300 km)

$$\frac{df}{dy} = \frac{d}{dy}(2\Omega \sin\phi) = 2\Omega\frac{d}{dy}\sin\phi = 2\Omega\cos\phi\frac{d\phi}{dy}$$



$$\delta y = a\,\delta\phi \qquad \therefore \quad \frac{d\phi}{dy} = \lim_{\delta y \to 0}\frac{\delta\phi}{\delta y} = \frac{1}{a}$$

$$\therefore \quad \frac{df}{dy} = \frac{2\Omega}{a}\cos\phi$$

Consider a Taylor expansion of f about a reference latitude:

$$f(y) = f_0 + \beta(y - y_0) + \text{h.o.t.}, \quad \text{where } \beta \equiv \left.\frac{df}{dy}\right|_{y=y_0}.$$

Take deriv w.r.t y:

$$\frac{df}{dy}(y) = \beta + \text{h.o.t.}$$

<u>Beta-plane approximation</u>: keep only first term on rhs of the above expansions for f and df/dy. So $f(y) \approx f_0$, and $\frac{df}{dy}(y) \approx \beta$, i.e., f and df/dy are both treated as constants in the places where they each appear in the vorticity equation.

So, synoptic-scale vort eqn on beta-plane is:

$$(3) \quad \frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)(\zeta + f_0) - \beta v$$