METR 5113, Advanced Atmospheric Dynamics I
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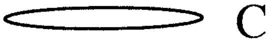
Wednesday, 29 August 2018 (lecture 5)

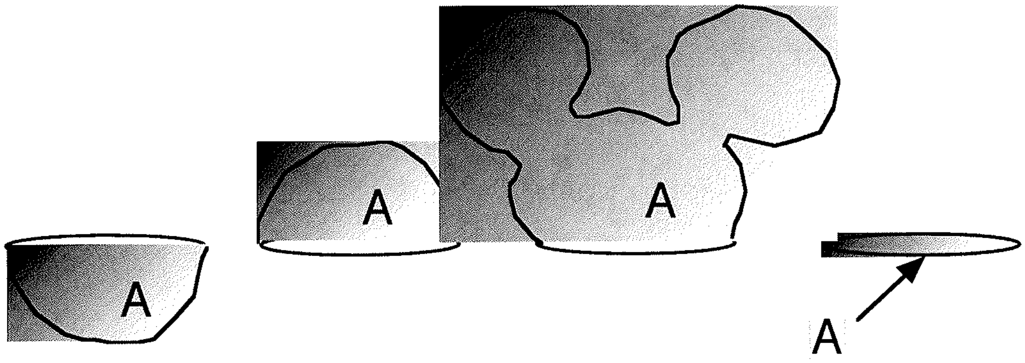
Stokes Th^{m}

Consider an arbitrary closed curve C and an open sfc A in 3-D space bounded by C. (open sfc means a sfc not enclosing any volume, so a cup is okay but NOT a balloon).

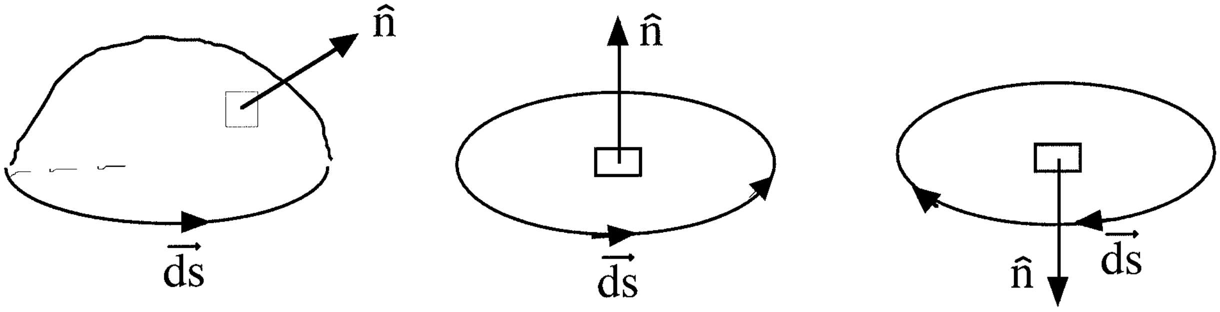
e.g. curve C



e.g. possible open surfaces A attached to this same curve C:



Let one side of sfc be "outside". $\hat{n}$ is unit outward normal vector. $\vec{ds}$ is element of C. Orientation of $\vec{ds}$: if you walk along C in direction of $\vec{ds}$ with head in direction of $\hat{n}$, the sfc is to your left.

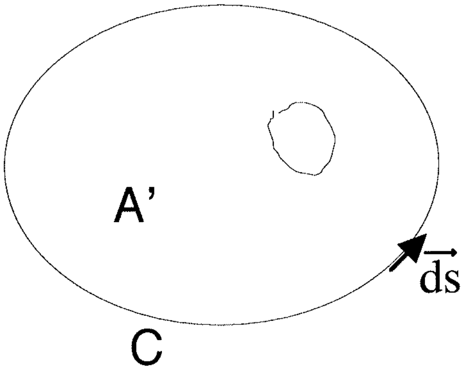


can have up as outside or down as outside
[either way is fine, but stick to your choice]

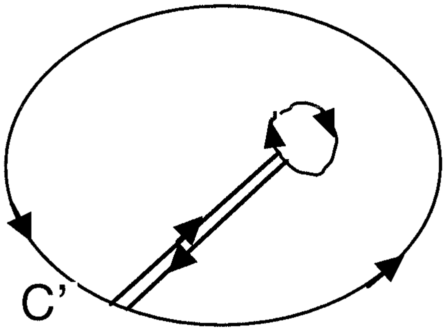
And consider any vector field $\vec{u}$ with continuous derivatives in A (isolated singularities are allowed). Then <u>Stokes Th^{m} says</u>:

$$\boxed{\underbrace{\int_A \left(\nabla \times \vec{u}\right) \cdot \hat{n}\, dA}_{\text{area integral}} = \underbrace{\oint_C \vec{u} \cdot \vec{ds}}_{\text{line integral}}}$$

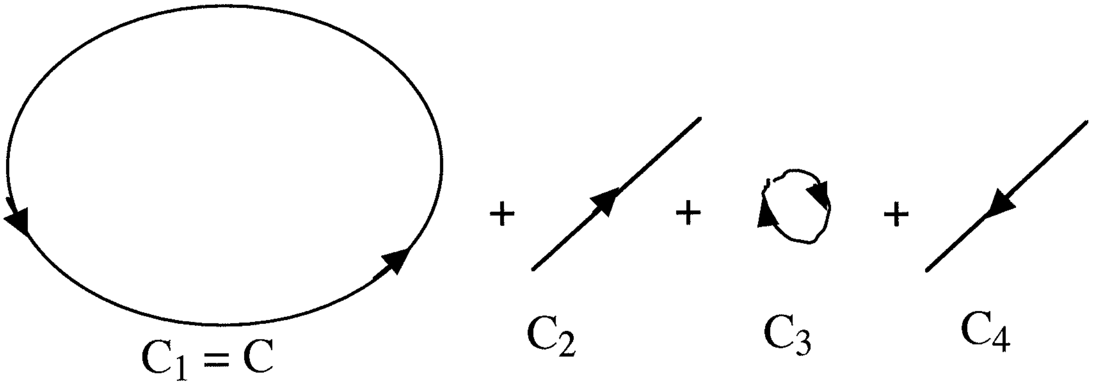
Can modify Stokes Th^{m} to exclude holes:



A' (shaded area) excludes the hole. The following closed curve C' encloses A' so we can apply Stokes th^{m} to C', A':



C' is comprised of 4 curves:



So we can apply Stokes Th^{m} to area A' bounded by C' :

$$\int_{A'} \left(\nabla \times \vec{u}\right) \cdot \hat{n}\, dA' = \oint_{C'} \vec{u} \cdot \overrightarrow{ds}$$

$$= \oint_{C_1} \vec{u} \cdot \overrightarrow{ds} + \int_{C_2} \vec{u} \cdot \overrightarrow{ds} + \oint_{C_3} \vec{u} \cdot \overrightarrow{ds} + \boxed{\int_{C_4} \vec{u} \cdot \overrightarrow{ds}}$$

counter- clockwise

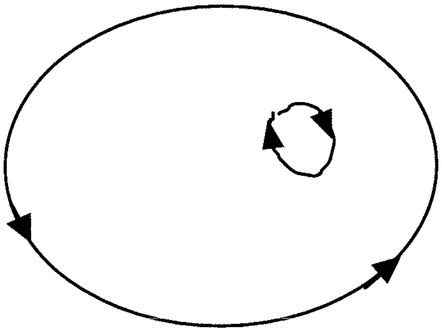
clockwise

= <u>minus</u> C_2 integral since $\overrightarrow{ds}_{C_4} = -\overrightarrow{ds}_{C_2}$

$$= \oint_{C_1} \vec{u} \cdot \overrightarrow{ds} + \oint_{C_3} \vec{u} \cdot \overrightarrow{ds}$$

counter-clockwise

clockwise

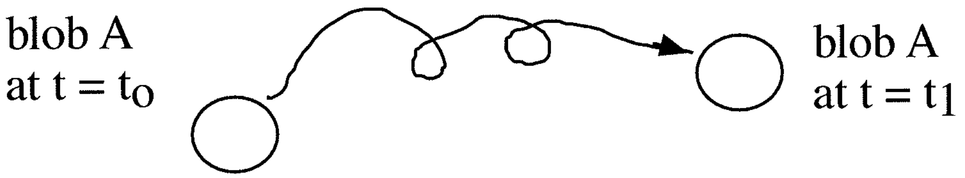


<u>Kinematics</u> (Ch. 3)

Kinematics: study of motion w/out regard for underlying forces.

<u>Lagrangian Description</u>

Describe motion of individual fluid elements (parcels, blobs, etc.)

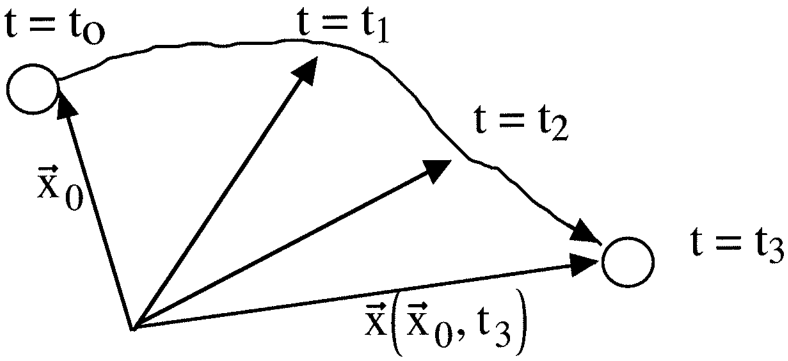


Rather than refer to blobs as A or B, refer to them by their position $\vec{x}_0$ at initial time t_o.

The blob that was at $\vec{x}_0$ at time t_o is now (time t) at the pos^n $\vec{x}(\vec{x}_0, t)$, with velocity $\vec{u}(\vec{x}_0, t)$, and temp $T(\vec{x}_0, t)$.

In Lag^n description, $\vec{x}$ is a dependent variable. So are velocity and temperature, etc. But t and $\vec{x}_0$ are independent variables.

Trajectory (pathline): curve traced out by tip of the pos^n vector of a fluid blob, $\vec{x}(\vec{x}_0, t)$ over a period of time.



[can redraw diagram with dif coord system (dif origin) to show that even though pos^n vectors are dif in dif coord systems, the curve traced out by tip of pos^n vectors doesn't change]

Differential equations for trajectories:

$$\frac{d\vec{x}}{dt} = \vec{u}(\vec{x}_0, t)$$

$$\text{or:} \quad \frac{d\vec{x}}{dt} = \vec{u}(t) \quad [\vec{x}_0 \text{ is implied}]$$

or: $$\frac{dx_i}{dt} = u_i(t)$$

or: $$\frac{dx}{dt} = u(t)\,, \quad \frac{dy}{dt} = v(t)\,, \quad \frac{dz}{dt} = w(t)\,.$$

If you know parcel's velocity as a f^n of time then can get traj by integrating these eq^{ns}. The consts of integration can be fixed in terms of the initial parcel location $\vec{x}_0$.

The rate of change of any variable F (e.g. temperature) for an infinitesimal blob is usually written as: $\frac{dF}{dt}$ or $\frac{DF}{Dt}$. This is the

total derivative of F.
material "
substantial "
individual "
particle "

Eulerian Description

Describe motion at fixed locations $\vec{x}$ rather than following blobs. $\vec{x}$ and t are treated as independent variables.

e.g., Oklahoma mesonet stations yield Eulerian data: u, v, T as functions of time at fixed locations, NOT following blobs.

[On board consider case of a cold front sliding through to the east without diabatic effects. Temp of parcels is conserved but temp decreases with time at fixed points.]