Now leave katabatic flows and look at another b.l. phenomenon:

The nocturnal low-level jet also known as the <u>southerly low-level jet</u>. We'll use the acronym LLJ.

- The LLJ is a warm-season b.l. phenomenon common to the Great Plains of the U.S., and other places worldwide, typically in regions east of mountain ranges or in vicinity of strong land-sea temperature contrasts.

  - LLJ typically develops around sunset under dry cloudless conditions conducive to strong radiational cooling.

  - At a fixed location in the jet, the wind vector turns <u>anti-cyclonically</u> with time (<u>clockwise</u> in N. hemisphere).

  - LLJ often associated with <u>southerly</u> geostrophic winds.

  - Reaches peak intensity in early morning hours, and dissipates shortly after dawn with onset of daytime convective mixing.

  - Peak jet winds can be <u>strongly</u> <u>supergeostrophic</u>. The peak wind component is often <u>southerly</u> (so same sense as the geostrophic wind).

  - Peak jet winds typically occur at heights less than 1 km and often at heights less than 500 m above ground level.

  - Jets can be several 100s of km wide and 1000s of km long.

LLJ exert significant impact on weather and regional climate:

  - LLJ support convective storms over the Great Plains by transporting gulf moisture northward and also enhancing convergence and lift at leading edge.

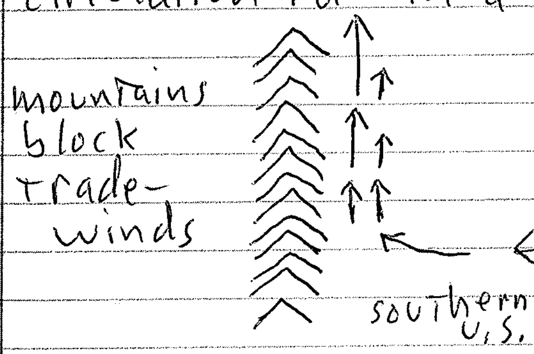
  - LLJ transport ozone and other pollutants 100s of miles over the course of a night.

– LLJs are an aviation hazard (large low-level shear)

– LLJs are an important source of wind for the wind-energy industry but large shear (and associated turbulence) can damage rotor blades.

Several theories have been advanced ~~to~~ to explain various aspects of the LLJ.

Wexler (1961) described the Great Plains LLJ as a northward-flowing inertial boundary layer associated with blocking of the easterly trade winds by the Rocky Mountains. His theory is the atmospheric analogue of Stommel's theory for westward intensification of oceanic boundary currents along eastern seabords (which we covered in Atmos. Dyn. I, → discussion of circulation th^{m} in a rotating reference frame.)



$$\frac{D}{Dt}(\zeta + f) = 0$$

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ vert. vort.

$f = 2\Omega \sin\phi(y)$

As parcels are forced to move north, their f increases so their ζ must decrease. Get strong anticyclonic shear.

→ If the Wexler mechanism or related blocking mechanism were operating, it could explain the existence of strong southerly mean winds during the summer. Such winds are likely an important precursor for the development of the jet. But the Wexler blocking mechanism does not explain the jet-like vertical structure or the temporal behavior (nocturnal oscillation).

→ Recently, however, Parish and Oolman (2010) argue that the strong southerly mean winds over the Great Plains in summer can be explained by heating of the sloping plains, with blocking being largely ineffectual.

Inertial-oscillation Theory of the LLJ

Blackadar (1957) envisioned the LLJ as an inertial oscillation (I.O.) arising in the boundary layer in response to the sudden release of the frictional constraint (shutdown of dry-convective thermals) near sunset.

Blackadar's Theory (paraphrased):

Consider a statistically steady daytime (late afternoon) flow induced by a constant (in x, y, z and t) horizontal pressure gradient force (p.g.f.). In the free atmosphere this p.g.f. is balanced by the Coriolis force (so geostrophic balance). But in the boundary layer, convective mixing by thermals is associated with turbulent stresses ("frictional" stresses) that acts on the flow (in addition to Coriolis force and p.g.f.).

Assuming ~~say~~ the ensemble mean flow is steady and 1-dimensional and density is treated as constant then

$$u = u(z), \quad v = v(z), \quad w = 0, \quad \rho = \text{const}$$

~~then~~ and mass conservation is automatically satisfied.

Also, in this case, the 1-D steady state assumption means all acceleration terms are 0:

$$\frac{D\vec{v}}{Dt} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\substack{\downarrow \\ 0 \\ \text{steady-state}}} + \boxed{(\vec{v}\cdot\nabla)\vec{v}} = 0$$

$$(\vec{v}\cdot\nabla)\vec{v} \to \underbrace{u\frac{\partial \vec{v}}{\partial x}}_{\substack{\downarrow \\ 0}} + \underbrace{v\frac{\partial \vec{v}}{\partial y}}_{\substack{\downarrow \\ 0}} + \underbrace{w\frac{\partial \vec{v}}{\partial z}}_{\substack{\downarrow \\ 0}}$$

So the eq^{ns} of motion during the daytime are:

(1) x-comp: $$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + \frac{d}{dz}\left(\nu \frac{du}{dz}\right)$$

(2) y-comp: $$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + \frac{d}{dz}\left(\nu \frac{dv}{dz}\right)$$

where ν is the eddy viscosity (in general, ν is a f^n of z).
Consider flow to be driven by the p.g.f.. In b.l. the pgf opposed by cor + friction.
Above the boundary layer the flow is considered to become geostrophic (with geostrophic winds being constant i.e. no shear of geostrophic winds, which is consistent with our taking ρ = const since that implies no thermal wind).

So above b.l. $u \to u_G$, $v \to v_G$ and the eq^{ns} of motion reduce to:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv_G \qquad \text{(above b.l.)}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu_G \qquad \text{(above b.l.)}$$

Since the p.g.f. is considered to be const (indep of $\underline{\underline{z}}$, x, y, t), you can think of fv_G and $-fu_G$ as proxies for the constant p.g.f. [i.e. when you see fv_G and fu_G in the eq^{ns} of motion, just think: they are the horiz. pgf].

Now the sun is setting and the turbulent mixing is greatly reduced ($\nu \to 0$ or some small value). So the <u>last term</u> of eq^n (1) and of eq^n (2) <u>drops out and no additional force steps into take its place</u>!

So the surviving forces (pgf and Coriolis) are <u>unbalanced</u> and they force flow acceleration.

So the nighttime eq^{ns} of motion are:

(3) x-comp: $$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

(4) y-comp: $$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

Near the ground, shortly after sunset, u and v are still pretty small (they were slowed by the frictional stresses during the day). So, near the ground, shortly after sunset, the accelerations are largely forced just by the p.g.f. Get a large kick in the direction of the p.g.f.

Lets solve (3) and (4). First rewrite them using

$-\frac{1}{\rho}\frac{\partial p}{\partial x} = -fv_G$ and $-\frac{1}{\rho}\frac{\partial p}{\partial y} = fu_G$ (fv_G, fv_G are proxies for p.g.f)

$$\therefore \quad \frac{Du}{Dt} = f(v - v_G)$$

$$\frac{Dv}{Dt} = -f(u - u_G)$$

$$\therefore \quad \frac{D}{Dt}(u - u_G) = f(v - v_G) \quad \left(\text{since } \frac{Du_G}{Dt} = 0\right)$$

$$\frac{D}{Dt}(v - v_G) = -f(u - u_G) \quad \left(\text{since } \frac{Dv_G}{Dt} = 0\right)$$

Now define the ageostrophic wind components:

$$u_a = u - u_G$$

$$v_a = v - v_G$$

$\therefore$ nighttime eq^{ns} of motion become:

$$(5) \quad \frac{Du_a}{Dt} = f v_a$$

$$(6) \quad \frac{Dv_a}{Dt} = -f u_a$$

Take $u_a * (5) + v_a * (6)$, get:

$$u_a \frac{Du_a}{Dt} + v_a \frac{Dv_a}{Dt} = f u_a v_a - f v_a u_a = 0$$

$$\therefore \frac{D}{Dt}\left(\frac{u_a^2}{2}\right) + \frac{D}{Dt}\left(\frac{v_a^2}{2}\right) = 0 \quad \text{Integrate w.r.t. } t$$

$$\therefore \underline{\frac{u_a^2(t) + v_a^2(t)}{2}} = \text{const} \quad (\text{for a } \underline{\text{parcel}})$$

$\therefore \sqrt{u_a^2(t) + v_a^2(t)}$ is const.

$\therefore$ The kinetic energy and the <u>speed</u> associated with the <u>ageostrophic</u> wind is <u>independent of time</u>.

Different parcels can have dif ageostrophic kinetic energies, but these do not change in time.

Let's get the full sol^n of (5) and (6). Take D/Dt of one of them (okay, of (5), but it doesn't matter) and then use the other one:

$$\frac{D}{Dt}\left(\frac{Du_a}{Dt}\right) = f \frac{Dv_a}{Dt}$$

from (6) this is $-f u_a$

$$\therefore \boxed{\frac{D^2 u_a}{Dt^2} = -f^2 u_a}$$

2nd order linear homogeneous constant coefficient o.d.e.

So seek sol^{ns} of the form $u_a = \text{const}\, e^{mt}$. Plug this into the o.d.e., get $m^2 = -f^2 \therefore m = \pm i f$.

∴ General solⁿ is:

$$u_a(\tau) = A e^{if\tau} + B e^{-if\tau}$$

where A and B are unknown constants. Using Euler's formula we can rewrite this as:

$$u_a(\tau) = A(\cos f\tau + i\sin f\tau) + B(\cos f\tau - i\sin f\tau)$$

$$= \underbrace{(A+B)}_{\text{call it } C}\cos f\tau + \underbrace{i(A-B)}_{\text{call it } D}\sin f\tau$$

$$\therefore \boxed{u_a(\tau) = C\cos f\tau + D\sin f\tau}$$

To get an expression for v_a, use this u_a formula in (5):

$$v_a = \frac{1}{f}\frac{Du_a}{D\tau} = \frac{1}{f}\left(-Cf\sin f\tau + Df\cos f\tau\right)$$

$$\therefore \boxed{v_a = -C\sin f\tau + D\cos f\tau}$$

To pin down C and D in these expressions, use initial condⁿˢ.

Let $\tau = 0$ coincide with sunset, i.e. time at which vigorous mixing by thermals gets shut down.

$$u_a(0) = C\cos 0 + D\sin 0 = C \quad \therefore \boxed{C = u_a(0)}$$

$$v_a(0) = -C\sin 0 + D\cos 0 = D \quad \therefore \boxed{D = v_a(0)}$$

$$\therefore \boxed{\begin{aligned} u_a(\tau) &= u_a(0)\cos f\tau + v_a(0)\sin f\tau \\ v_a(\tau) &= -u_a(0)\sin f\tau + v_a(0)\cos f\tau \end{aligned}}$$

solⁿ for ageostrophic wind components

(*) Note: Taking $u(0)=0$ is an idealization. There is cross-isobar transport, ~~but~~ ($u \neq 0$) but u is much smaller than v for case considered.

full sol^n is :

$$\boxed{\begin{aligned} u &= u_G + u_a(0)\cos f\tau + v_a(0)\sin f\tau \\ v &= v_G - u_a(0)\sin f\tau + v_a(0)\cos f\tau \end{aligned}}$$

Sol^n is periodic with period T satisfying $fT = 2\pi$ i.e. $\boxed{T = 2\pi/f}$ → about 21 hrs in central Okla.

This sol^n takes the form of a <u>circle</u> on the hodograph plane (u, v axes) centered at point $u = u_G$, $v = v_G$. The radius of the circle is the (const) ageostrophic wind speed.

Draw the hodograph for case of a <u>southerly</u> geostrophic wind (so $u_G = 0$ and $v_G > 0$). Consider two different air parcels: parcel P, and parcel Q which is at a <u>lower elevation</u> than P. [draw picture indicating pgf, pressure, and flow]

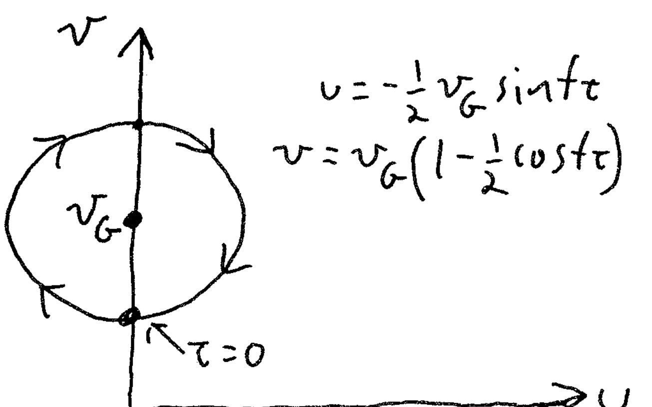
Suppose parcel P has initial wind components $u(0)=0$, $v(0) = \frac{1}{2}v_G$. Then its initial ageostrophic wind components are:

Parcel P: $u_a(0) = u(0) - u_G = 0 - 0 = 0$, $v_a(0) = v(0) - v_G = \frac{1}{2}v_G - v_G = -\frac{1}{2}v_G$

And suppose parcel Q has initial wind comp's: $u(0) = 0$, $v(0) = \frac{1}{4}v_G$. [Note that parcel Q, being closer to the ground than P has <u>smaller</u> wind speed than P]. So parcel Q's initial ageostrophic wind comp's are:

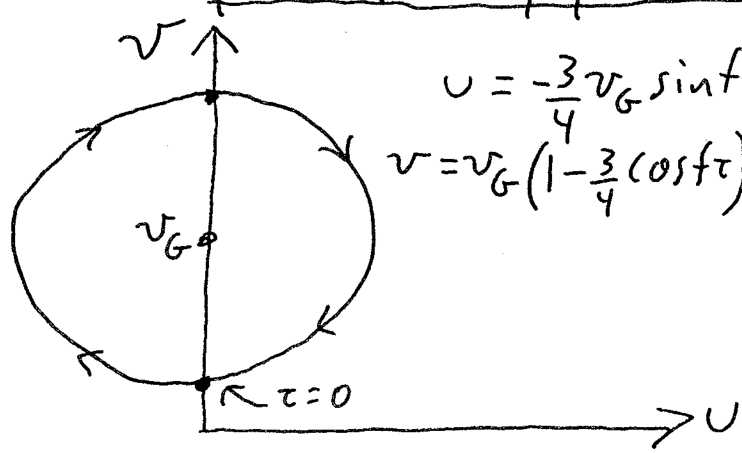
Parcel Q: $u_a(0) = 0$, $v_a(0) = \frac{1}{4}v_G - v_G = -\frac{3}{4}v_G$

<u>So parcel P's hodograph looks like:</u>



$u = -\frac{1}{2}v_G \sin f\tau$
$v = v_G(1 - \frac{1}{2}\cos f\tau)$

while <u>parcel Q's hodograph looks like:</u>



$u = -\frac{3}{4}v_G \sin f\tau$
$v = v_G(1 - \frac{3}{4}\cos f\tau)$

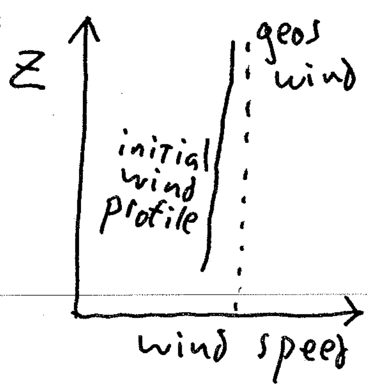
The parcel closer to the ground (Q) has initial winds that differ the most from geostrophic winds (so larger ageostrophic wind) and so is more unbalanced when the sun sets. ∴ Bigger hodograph circle.

→ This idea is illustrated schematically in Fig. 10 of Blackadar (1957). See handout.

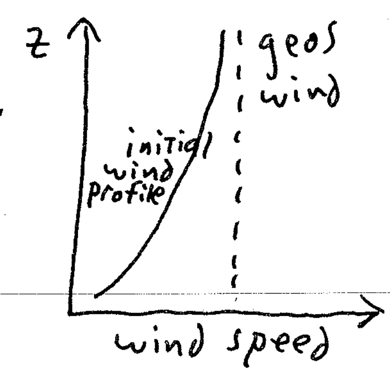
But there is a problem with the Blackadar I.O. theory for the LLJ. The I.O. mechanism is probably operating in many LLJs and may account for many of them but it can't explain cases where LLJs ~~are~~ have wind speeds that are strongly supergeostrophic (only marginally so).

The problem is that under the usual scenario where LLJs form after an afternoon of strong dry-convective mixing, the initial (sunset) wind profile is usually much more uniform in the vertical than indicated in Blackadar's Fig. 10.

So, more like this:



Than like this:



An initial wind speed at the 300–500m level (corresponding to the height at which the LLJ wind maximum often is found) might be 80% or so of the geostrophic wind speed, which would yield an LLJ peak wind speed that is supergeostrophic by only 20% more than geostrophic. Can't explain how LLJ can be supergeostrophic by 100% or 200% or more of geos wind.