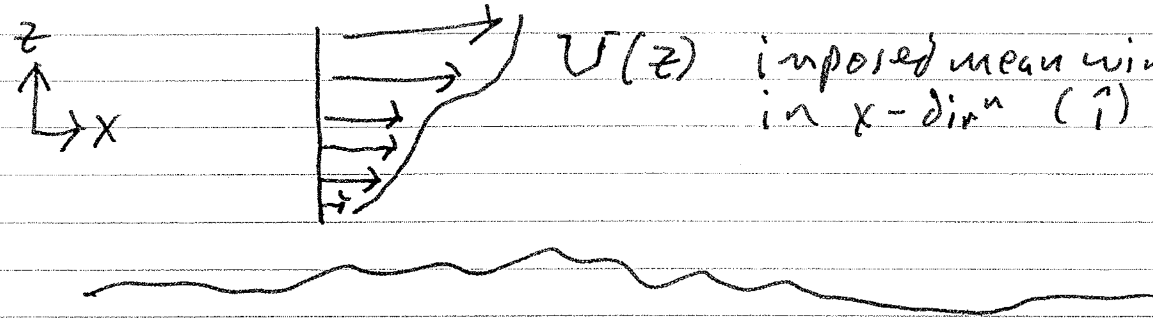
Stratified flow over Topography (lee waves).



Start w/ inviscid Boussinesq eq^{ns} of motion:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i} - g\frac{\rho'}{\rho_0}\delta_{i3}$$

~~Let~~ $u_i = U(z)\,\delta_{i1} + u_i'$ ρ_0 is a const ref density value.

where the <u>perturbation velocity</u> components u_1', u_2', u_3' are <u>small</u>.

$\rho = \bar{\rho}(z) + \rho'$ [so ρ' is deviation from <u>mean density</u>]

$p = \bar{p}(z) + p'$ [where $\dfrac{d\bar{p}}{dz} = -\bar{\rho} g$ defines $\bar{p}$]

<u>Linearize the eq^{ns} of motion</u> (neglect <u>products of perturbations</u> i.e. products of small quantities).

(1) $\dfrac{\partial u}{\partial t} + U\dfrac{\partial u}{\partial x} + w\dfrac{\partial U}{\partial z} = -\dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial x}$

(2) $\dfrac{\partial v}{\partial t} + U\dfrac{\partial v}{\partial x} = -\dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial y}$

(3) $\dfrac{\partial w}{\partial t} + U\dfrac{\partial w}{\partial x} = -\dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial z} - g\dfrac{\rho'}{\rho_0}$

where $u = u_1'$ $v = u_2'$ $w = u_3'$ so u, v, w <u>are</u> perturbation

Also use the incompressibility (and^n)

$$\nabla \cdot \overrightarrow{\underset{\text{full wind vector}}{U}} = 0$$

$$\therefore \nabla \cdot (U\hat{i} + \vec{u}') = 0$$

$$\therefore \underbrace{\frac{\partial U}{\partial x}}_{\to 0 \text{ since } U = U(z)} + \nabla \cdot \vec{u}' = 0 \qquad \therefore \nabla \cdot \vec{u}' = 0$$

$$\therefore (4) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

[u, v, w are the perturbation; Drop prime notation]

Thermo energy eq^n for non-diffusive liquid:

$$\frac{D\rho}{Dt} = 0$$

$$\therefore \frac{\partial}{\partial t}(\bar{\rho} + \rho') + (U + u)\frac{\partial}{\partial x}(\bar{\rho} + \rho') + v\frac{\partial}{\partial y}(\bar{\rho} + \rho') + w\frac{\partial}{\partial z}(\bar{\rho} + \rho') = 0$$

<u>Linearize</u> it and use fact that $\bar{\rho} = \bar{\rho}(z)$ [no x, y or t in it], get

$$\frac{\partial \rho'}{\partial t} + U\frac{\partial \rho'}{\partial x} + w\frac{d\bar{\rho}}{dz} = 0 \qquad \left[\frac{d\bar{\rho}}{dz} = \frac{\partial \bar{\rho}}{\partial z} \text{ since } \bar{\rho} = \bar{\rho}(z)\right]$$

introduce $N^2 \equiv -\dfrac{g}{\rho_0}\dfrac{d\bar{\rho}}{dz}$

$$\therefore (5) \quad \frac{\partial \rho'}{\partial t} + U\frac{\partial \rho'}{\partial x} - w\frac{N^2 \rho_0}{g} = 0$$

So we have 5 eq^{ns} in 5 unknowns. Lets rewrite it as 1 eq^n in 1-unknown (say w). Steps are similar to what we did for internal gravity waves <u>without</u> an imposed mean wind.

Take $\dfrac{\partial}{\partial x}(1) + \dfrac{\partial}{\partial y}(2)$:

$$\frac{\partial}{\partial t}\underbrace{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{-\frac{\partial w}{\partial z}\text{ from (4)}} + U\frac{\partial}{\partial x}\underbrace{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{-\partial w/\partial z} + \frac{\partial w}{\partial x}\frac{dU}{dz} = -\frac{1}{\rho_0}\underbrace{\left(\frac{\partial^2 p'}{\partial x^2}+\frac{\partial^2 p'}{\partial y^2}\right)}_{\nabla_H^2 p'}$$

multiply by $-\rho_0$, get:

(6) $\therefore$ $$\nabla_H^2 p' = \rho_0\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial z} - \rho_0\frac{\partial w}{\partial x}\frac{dU}{dz}$$

Take $\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)$ (3):

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)w = -\frac{1}{\rho_0}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial p'}{\partial z} - \frac{g}{\rho_0}\underbrace{\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\rho'}_{\text{from (5)}}$$

mult by $-\rho_0$ and rearrange it, get:

from (5) This is $\dfrac{w N^2 \rho_0}{g}$

(7) $$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial p'}{\partial z} = -\rho_0\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2 w - \rho_0 N^2 w$$

(6) and (7) are 2 eq^{ns} in 2 unknowns p', w
Lets eliminate p'.

Take ∇_H^2 (7):

sub in from (6)

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial}{\partial z}\boxed{\nabla_H^2 p'} = -\rho_0\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla_H^2 w - \rho_0 N^2\nabla_H^2 w$$

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left\{\frac{\partial}{\partial z}\left[\rho_0\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial z} - \rho_0\frac{\partial w}{\partial x}\frac{dU}{dz}\right]\right\}$$

$$= -\rho_0\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla_H^2 w - \rho_0 N^2\nabla_H^2 w$$

Note that ρ_0 is common to every term. $\therefore$ $\div$ by ρ_0 to get rid of it expand out $\left\{\frac{\partial}{\partial z}[\cdots]\right\}$ get

cancellation

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left[\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial^2 w}{\partial z^2}+\frac{dU}{dz}\frac{\partial^2 w}{\partial x\partial z}-\frac{\partial^2 w}{\partial z\partial x}\frac{dU}{dz}\right.$$

$$\left.-\frac{\partial w}{\partial x}\frac{d^2U}{dz^2}\right]=-\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla_H^2 w-N^2\nabla_H^2 w$$

$$\therefore\ \left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\frac{\partial^2 w}{\partial z^2}-\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left(\frac{\partial w}{\partial x}\frac{d^2U}{dz^2}\right)$$

$$+\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla_H^2 w+N^2\nabla_H^2 w=0$$

$$\therefore\ \boxed{\begin{gathered}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla^2 w+N^2\nabla_H^2 w\\ -\frac{d^2U}{dz^2}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial x}=0\\ \text{where } U=U(z),\quad N=N(z)\end{gathered}}$$

(if $U(z)=0$, this eq^n reduces to our previous eq^n for internal gravity waves).

4th order linear homogeneous p.d.e.

Coefficients are ~~constant in~~ independent of x, y, t but <u>not</u> z.

$\therefore$ pde should permit separable sol^{ns} of the form:

$$w=W(z)\exp[i(kx+ly-\omega t)]$$

i.e. $w=W(z)\exp[i(\vec{K}_H\cdot\vec{x}-\omega t)]$ where $\vec{K}_H=k\hat{i}+l\hat{j}$ is horiz wavenumber vector.

Lets suppose topography varies only in x-dir^n (same dir^n as mean wind U). Then we only care about wavenumber in x-dir^n. So $\vec{K}_H = k\hat{i}$.

To bring out the main ideas of the motion, consider the simplest case of <u>sinusoidal</u> topography (<u>monochromatic</u> wave topography) ∿∿∿∿∿∿∿∿
So topo height is $\xi = Re[\,a e^{ikx}\,]$ → which is $a\cos kx$. More general topography (e.g. isolated ridge) can be written as a Fourier Integral. The solution to that more general problem can be written as an <u>integral</u> (over k) of the solution to the single-wave problem.

or cosine

We'll also restrict attention to <u>steady state</u> flow. So $\omega = 0$. [Note: flow appears steady in frame of ref fixed w.r.t. mountain but appears unsteady in frame of ref moving w/ the wind].

$$So \quad w = W(z)\, e^{ikx} \quad \text{(well, real part)}$$

<u>Lower boundary cond^n</u> (kinematic):

Impermeability cond^n: $\vec{v}\cdot\hat{n} = 0$ where $\hat{n}$ is normal to topo

This cond^n also means that a parcel on the mountain streamline <u>stays on the mountain</u> streamline (flow can be <u>tangent</u> to mtn but not ⊥ to mtn, on the mtn stl).

$$\therefore \; w_{\substack{\text{parcel}\\ \text{on mtn}\\ \text{streamline}}} = \frac{D\, z_{\substack{\text{parcel}\\ \text{on mtn}\\ \text{streamline}}}}{Dt} = \frac{D}{Dt}\, a e^{ikx}$$

$$= \left[\frac{\partial}{\partial t} + (U+u)\frac{\partial}{\partial x}\right] a e^{ikx}$$

$\boxed{u \text{ is pert.}}$

$$= (U+u)\, aik\, e^{ikx}$$

$\therefore\ W(z_{\substack{mtn\\ streamline}})e^{ikx} = (U+u)aike^{ikx}$ $\div$ by e^{ikx}

and linearize this cond^n. 2 linearizing steps: neglect u term, and also approximate $W(z_{\substack{mtn\\ streamline}})$ by $W(0)$. See my notes on grav waves (Dyn I) or Kundu for discussion of this second linearizing approx.

Also U here is U at $z=0$.

$\therefore$ $W(0) = ikaU$ lower b.c.

For an upper b.c. we want w to be finite and want the wave energy to be propagating upward, not downward (radiation condition).

Now plug $w = W(z)e^{ikx}$ into the p.d.e., get

$$(Uik)^2\left[(ik)^2 + \frac{d^2}{dz^2}\right]W + N^2(ik)^2W - \frac{d^2U}{dz^2}(ikU)ikW = 0$$

$$\therefore\ U^2k^4W - U^2k^2\frac{d^2W}{dz^2} - N^2k^2W + U\frac{d^2U}{dz^2}k^2W = 0$$

$\div\ -k^2U^2$

$$\therefore\ \frac{d^2W}{dz^2} + \left(\frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2} - k^2\right)W = 0$$

or:
$$\frac{d^2W}{dz^2} + (\ell^2 - k^2)W = 0$$

where $\ell^2 \equiv \dfrac{N^2}{U^2} - \dfrac{1}{U}\dfrac{d^2U}{dz^2}$ is the Scorer parameter.

If $l^2 - k^2 > 0$ sol^{ns} oscillate in z

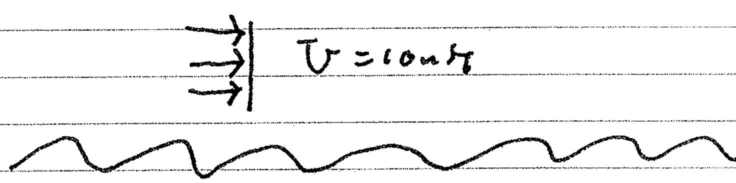
If $l^2 - k^2 < 0$ sol^{ns} are monotonic (exponential type).

For fixed environment (l), get <u>oscillatory</u> behavior for small k (big wavelength i.e. <u>wide</u> mountains). Conversely, narrow mtns yield <u>evanescent</u> waves (rapidly decaying w/ z).

If $U(z) \to 0$ at some height then $l^2 \to +\infty$, (positive infinity. Use definition of l^2 to see why.)

so $\left|\frac{d^2 W}{dz^2}\right| \to \infty$ $\therefore$ oscillations became increasingly rapid (large vertical wavenumber). Level where $U(z) \to 0$ is a <u>critical layer</u>.

Consider simplest case of sinusoidal topography ($\xi = a e^{ikx}$) with $U = \text{const}$ (not 0) and $N = \text{const}$.



So $$\frac{d^2 W}{dz^2} + \left(\frac{N^2}{U^2} - k^2\right) W = 0$$

Hit ridges at time intervals (periods) of

$T = \frac{\lambda}{U} = \frac{2\pi}{kU}$. So <u>frequency</u> of encounter

is $\frac{2\pi}{T} = kU$.

→ handout, pg 36 of Turner's "Buoyancy effects in fluids"

Case (a). Suppose frequency of encounter $> N$

i.e. $(kU)^2 > N^2 \quad \therefore \quad \dfrac{N^2}{U^2} < k^2 \quad \therefore \quad \dfrac{N^2}{U^2} - k^2 < 0$

So get exponential (not oscillatory) sol^{ns}. Evanescent wave.

try $W = e^{\gamma z} \quad \therefore$ ode says: $\gamma^2 + \left(\dfrac{N^2}{U^2} - k^2\right) = 0$

$$\gamma^2 = -\underbrace{\left(\frac{N^2}{U^2} - k^2\right)}_{-} = \underbrace{k^2 - \frac{N^2}{U^2}}_{+}$$

$$\gamma = \pm\sqrt{k^2 - N^2/U^2}$$

$\therefore$ general sol^n is: $W = A\exp\left(\sqrt{k^2 - N^2/U^2}\, z\right) + B\exp\left(-\sqrt{k^2 - N^2/U^2}\, z\right)$

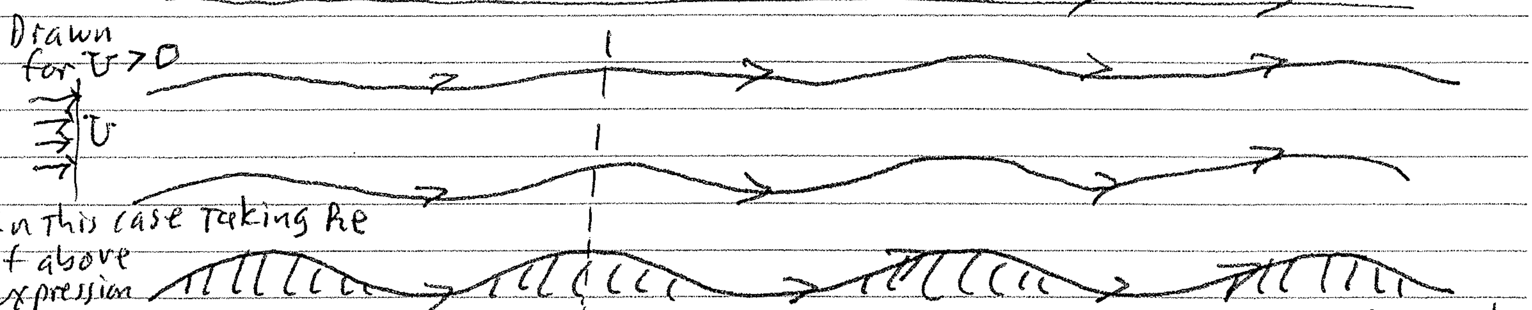
From finiteness cond^n must take $A = 0$.

Then, from lower b.c. $W(0) = ikaU$ must have

$$ikaU = B$$

$$\therefore \quad W(z) = ikaU\exp\left(-\sqrt{k^2 - N^2/U^2}\, z\right)$$

$$\therefore \quad w = ikaU\exp\left(-\sqrt{k^2 - N^2/U^2}\, z\right)e^{ikx}$$



In this case taking Re of above expression yields $w \sim -\sin kx$, ↑ minus!

$x = 0$; keep in mind our assumed terrain was $\zeta = a\cos kx$

disturbance is in phase with topography and dies out with z.

Scale height h of this particular disturbance is height at which disturbance drops to e^{-1} of it's sfc value.

$$\therefore \quad \exp\left[-\sqrt{k^2 - N^2/U^2}\; h\right) = \exp(-1)$$

$$\therefore \quad h = \frac{1}{\sqrt{k^2 - N^2/U^2}} = \frac{1}{k\sqrt{1 - \frac{N^2}{U^2 k^2}}} > \frac{1}{k} = \frac{\lambda}{2\pi}$$

So scale height is generally on the order of the wavelength (or larger).

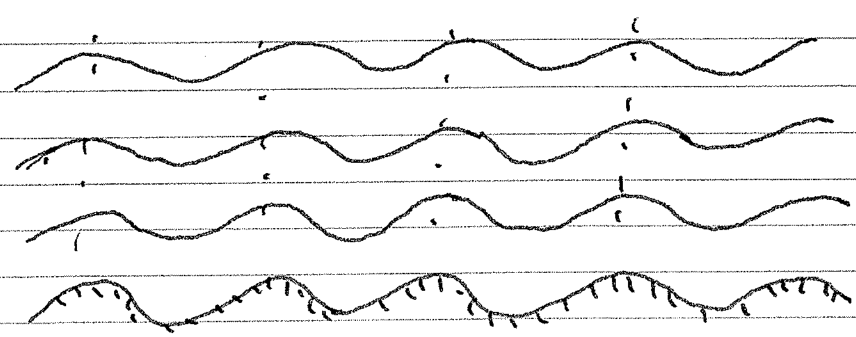
So Get <u>decay</u> with z, <u>not</u> <u>wave propagation</u> for freq of encounter > Brunt-Väisälä frequency.

Case (b) <u>As $N \to kU$ (freq of encounter $\to N$)</u>, <u>the z-dependence drops out of the sol^n</u>:

$$\frac{d^2\hat{w}}{dz^2} = 0$$

$\therefore \quad \hat{w} = \cancel{a}\,c + bz$. Finiteness cond^n (at ∞) yields $b=0$
Lower b.c. then yields $c = ikaU$

$$\therefore \quad w = ikaU\, e^{ikx}$$



- no attenuation
- disturbance is in phase w/ topography.

$\boxed{\text{case (c)}}$ $k^2U^2 < N^2$ (frequency of encounter $< N$)

$$\therefore \quad \frac{N^2}{U^2} - k^2 > 0$$

define $\beta \equiv \sqrt{\frac{N^2}{U^2} - k^2}$ so β is real and pos.

$$\frac{d^2 W}{dz^2} + \beta^2 W = 0$$

$$\therefore \quad W = Ae^{i\beta z} + Be^{-i\beta z}$$

lower b.c.: $W(0) = iakU \longrightarrow iakU = A + B \rightarrow A = iakU - B$

$$\therefore \quad W = (iakU - B)e^{i\beta z} + Be^{-i\beta z}$$

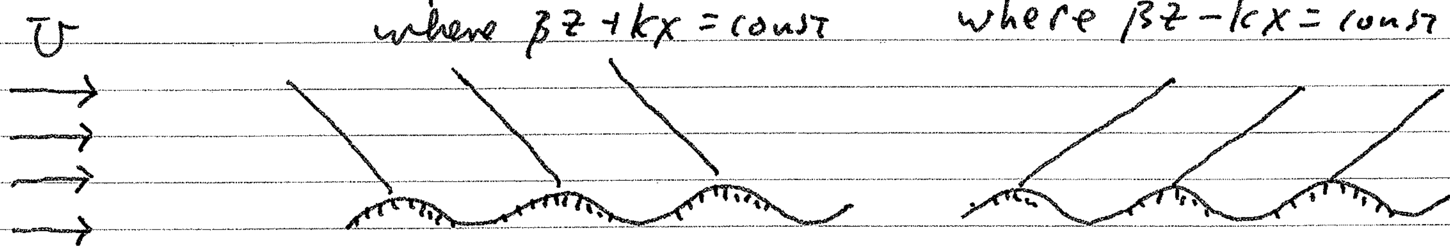
$$\therefore \quad w = (iakU - B)e^{i(\beta z + kx)} + Be^{-i(\beta z - kx)}$$

what is value of B? finiteness cond^n doesn't help.

Consider behavior of $e^{i(\beta z + kx)}$ and $e^{-i(\beta z - kx)}$

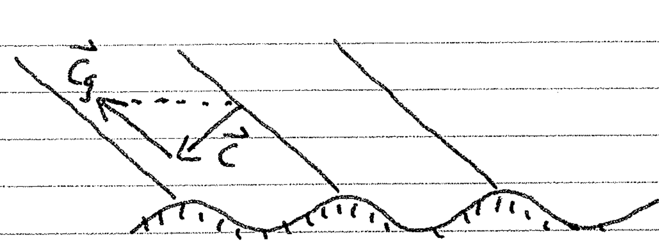
$\uparrow$ phase is const where $\beta z + kx = \text{const}$

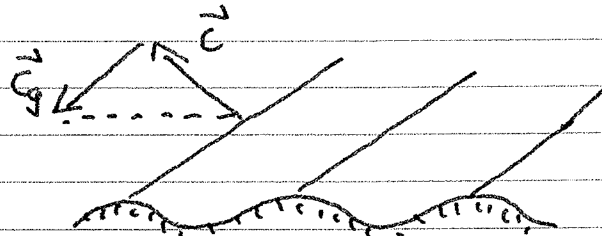
$\uparrow$ phase is const where $\beta z - kx = \text{const}$



If mean flow is from the left (like this: $\rightrightarrows$) then from <u>energy considerations</u>, the case on the right isn't physically realistic. Why? Consider a frame of ref moving with the mean flow. Then mountain (and phase lines) move in this direction $\longleftarrow$.

So the two scenarios (in moving frame) look like:

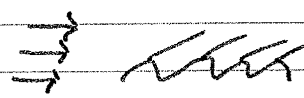




In this case energy moves upward, <u>away</u> from boundary.

In this case energy moves downward, <u>toward</u> boundary.

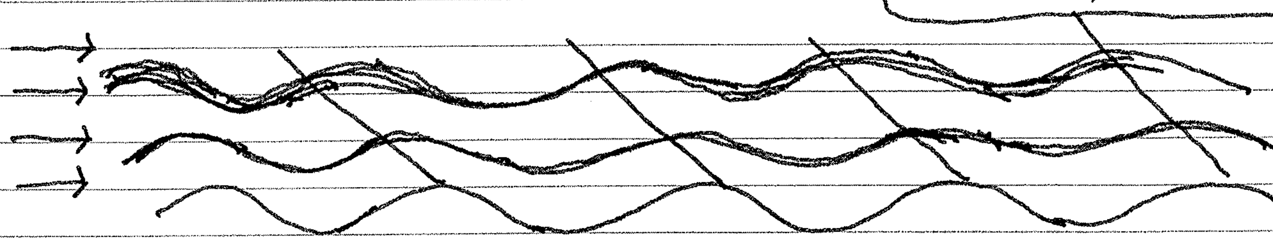
Radiation condition: energy flux must be <u>outward</u> from disturbance producing the wave motion (in this case, the ground).

So  scenario is not physical.

$\therefore$ set $\beta = 0$

$\therefore \quad w = i\,ak\bar{U}\,e^{i(\beta z + kx)}$

$\boxed{\text{So phase lines tilt upstream!}}$

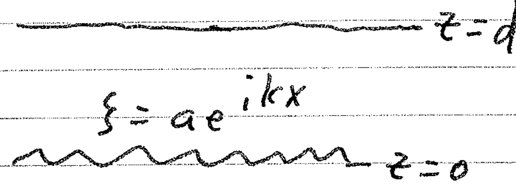


This sinusoidal topography is just 1 Fourier component. For isolated topography  (e.g. bell-shaped ridge), can construct linear (small-amplitude) solutions by Fourier methods. See handout, pg. 36 of Turner's book.

$\longrightarrow$ handout (pg. 36 Turner) showing lee waves over an isolated ridge.

If stratified flow is confined beneath an upper sfc, resonance is possible (e.g. in a numerical model with "rigid lid" condⁿ instead of radiation condⁿ).

e.g.:



Consider $\frac{N^2}{U^2} - k^2 > 0$ (so wavy solⁿˢ)

$$\beta \equiv \sqrt{\frac{N^2}{U^2} - k^2}$$

$$\therefore \hat{w} = Ae^{i\beta z} + Be^{-i\beta z}$$

lower b.c. $\hat{w}(0) = ikaU$

$$\therefore ikaU = A + B \rightarrow A = ikaU - B$$

Top b.c. $\hat{w}(d) = 0$

$$\therefore 0 = Ae^{i\beta d} + Be^{-i\beta d} \rightarrow \text{so } A = -Be^{-2i\beta d}$$

sub in from above

$$\therefore (ikaU - B)e^{i\beta d} + Be^{-i\beta d} = 0$$

$$B(e^{-i\beta d} - e^{i\beta d}) = -ikaUe^{i\beta d}$$

$$\therefore B(e^{i\beta d} - e^{-i\beta d}) = ikaUe^{i\beta d}$$

$$2iB\underbrace{\frac{(e^{i\beta d} - e^{-i\beta d})}{2i}} = ikaUe^{i\beta d}$$

$$\therefore 2iB\sin\beta d = ikaUe^{i\beta d}$$

$$\therefore \boxed{B = \frac{kaUe^{i\beta d}}{2\sin\beta d}} \qquad \therefore \boxed{A = -\frac{kaUe^{-i\beta d}}{2\sin\beta d}}$$

$$\therefore\ W = \frac{kaU}{2\sin\beta d}\left(-e^{i\beta(z-d)} + e^{-i\beta(z-d)}\right)$$

mult by $\frac{i}{i}$ and rearrange

$$= \frac{ikaU}{\sin\beta d}\left(\frac{e^{i\beta(d-z)} - e^{-i\beta(d-z)}}{2i}\right)$$

$$= \frac{ikaU}{\sin\beta d}\sin[\beta(d-z)]$$

$$w = We^{ikx} = \frac{ikaU}{\sin\beta d}\sin[\beta(d-z)](\cos kx + i\sin kx)$$

Take real part

$$w = -\frac{kaU}{\sin\beta d}\sin[\beta(d-z)]\sin kx$$

see next pg for more about this solution.

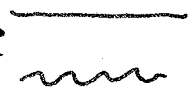
Since β is real and positive (we've assumed we're dealing with "wavy" sol^{ns}) there's a singularity when $\sin\beta d = 0$:

$$\beta d = \underbrace{\sqrt{\frac{N^2}{U^2} - k^2}\, d = n\pi}_{\text{resonance cond}^n}$$

Waves reflect off top boundary, arrive back at bottom with same phase. An integral number of waves from top to bottom.

Now back to non-const $U(z)$, $N(z)$:

$$\frac{d^2W}{dz^2} + \underbrace{\left(\frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2} - k^2\right)}_{\text{acts locally as "}m^2\text{", square of vert comp wavenumber.}} W = 0$$

More about
soln of this problem: $\exists$ 

is $w = -\dfrac{kaU}{\sin\beta d} \sin[\beta(d-z)] \sin kx$

<u>scratch paper</u>

$\cos(a+b) = \cos a \cos b - \sin a \sin b$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\therefore \cos(a-b) - \cos(a+b) = 2 \sin a \sin b$

$\therefore \sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

Now use this identity to rewrite our soln for w [use $a = \beta(d-z)$, $b = kx$]

end of scratch

$$\therefore w = -\frac{kaU}{2\sin\beta d}\left[\cos(\beta d - \beta z - kx) - \cos(\beta d - \beta z + kx)\right]$$

So get a sum of <u>2 waves</u>, one tilts forward", one tilts backward]. Energy propogates up from ~~[illegible]~~ wavy boundary and down from rigid lid.

The net effect is to get a <u>cellular</u>. pattern (in xz plane)

Two special ~~boundaries~~ levels in environment:

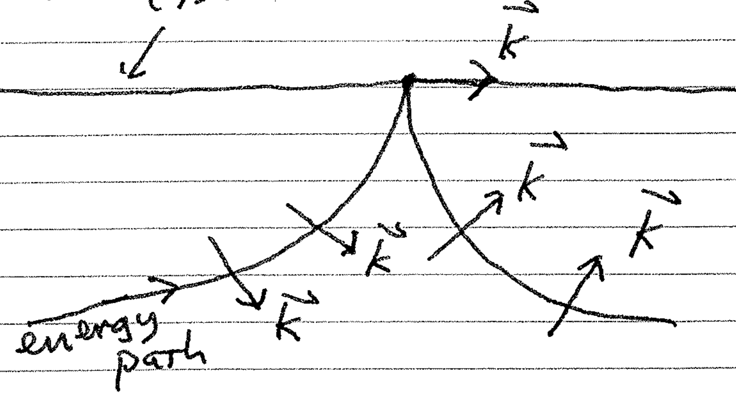
(a) <u>Turning points in solution</u>: region where

$$\left(\frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2} - k^2\right) \text{ changes sign.}$$

( ) < 0 decaying exponential-like solutions

( ) = 0 ∴ vertical part of wavenumber → 0.

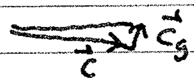
( ) > 0 oscillatory solutions



all these wavenumbers have <u>same</u> <u>horiz</u> comp.

At turning point local $\vec{k}$ becomes <u>purely horizontal</u>

∴ $\vec{c}$ is horizontal ∴ $c_g \to 0$ [since $\vec{c} + \vec{c}_g$ have to add up to a <u>horiz</u> vector]

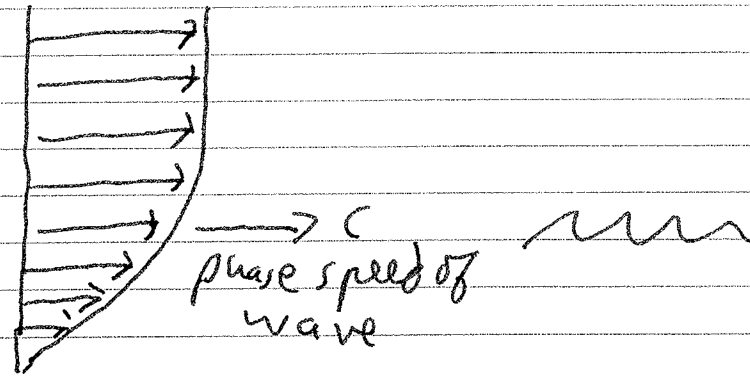


Get <u>reflection</u> at turning point.

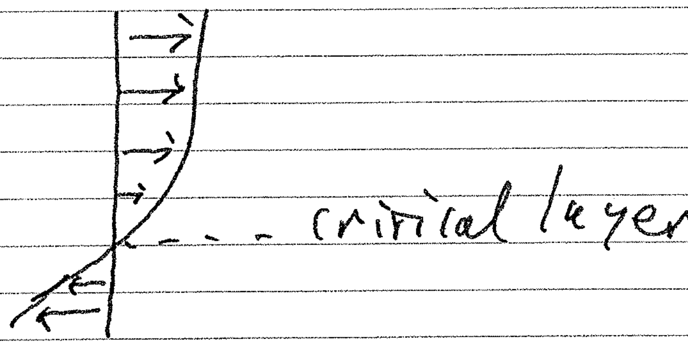
(b) <u>Critical layers</u>

In general, if a wave is propagating at phase speed c, e.g. $f(z)e^{ik(x-ct)}$ in an environment with mean velocity comp $U(z)$ then the level where $U(z) = c$ is called a <u>critical layer (level)</u>.

e.g.

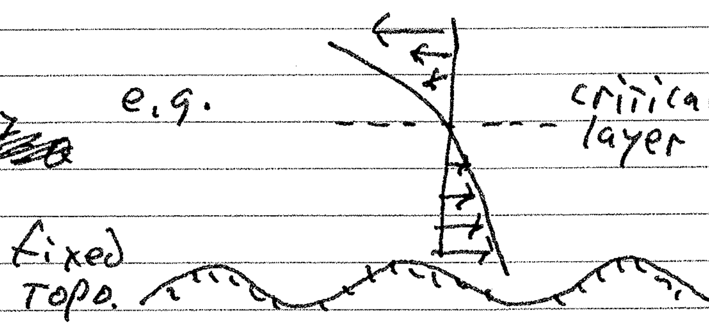


In frame of ref moving w/ wave, the wave appears stationary:



In our discussion of waves over topography, waves are stationary ∴ critical layer is where ~~U~~ $U = 0$.

e.g.



Local vert wavenumber $m^2 = \frac{N^2}{U^2} - \frac{1}{U}\frac{d^2U}{dz^2} - k^2$.

$\vec{k}$ has fixed horiz comp k but m varies. As critical layer is approached, $U \to 0$ so $m \to \infty$. So $\vec{k}$ becomes increasingly vertical (and infinite). ∴ vertical scale of wave becomes increasingly fine. Get extremely large wind shear. Get turbulence. So, wave energy radiates upward from mtn to critical layer where it gets absorbed.