NUMERICAL MODEL OF A NON-STEADY ATMOSPHERIC PLANETARY BOUNDARY LAYER, BASED ON SIMILARITY THEORY

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Abstract. A numerical model of a non-stationary atmospheric planetary boundary layer (PBL) over a horizontally homogeneous flat surface is derived on the basis of similarity theory. The two most typical turbulence regimes are reproduced: one corresponding to a convectively growing PBL and another corresponding to a stably stratified quasi-equilibrium PBL. The PBL is treated as a unit. Hence, its evolution is characterized by temporal changes of intrinsic parameters, such as the PBL depth, the vertically averaged values of potential temperature, specific humidity and components of wind velocity, the near-surface values of heat, water vapor and momentum fluxes. The internal structure of the PBL is considered self-similar. This allows one to represent the interaction between the air flow and the underlying surface by means of universal heat/mass transfer and resistance laws. Numerical experiments on the diurnal variations of meteorological fields in the lower 2 km layer confirm the ability of the model to reproduce the main features of the phenomena, known from observations.

1. Introduction

This paper describes a part of the microclimate modeling program outlined by Zilitinkevich (1990a), viz., the part devoted to an horizontally homogeneous planetary boundary layer (PBL) subjected to diurnal variations.

The typical pattern of diurnal changes in meteorological fields over land during clear weather is as follows. Soon after sunrise radiational heating of the terrestrial surface starts. Then heat exchange between the surface and the atmosphere results in warming of the lower air layer and development of unstable stratification which generates convection and facilitates intensive mixing. If in this case the wind is not too strong, the PBL can be considered in the first approximation as a layer of mainly convective turbulence. Its depth grows as long as the vertical buoyancy flux from the underlying surface to the atmosphere remains positive. This flux increases during the morning and decreases in the afternoon. As soon as it becomes zero, convection ceases. As a result, turbulence in the upper part of the PBL degenerates, so the PBL collapses, and there is a very fast decrease of the PBL depth. Then the PBL undergoes gradual changes depending first of all on the increase of stability in the lower air layer caused by radiational cooling of the underlying surface.

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Over water bodies that are not too shallow, the pattern can be opposite. Due to the great heat capacity of the upper mixed layer, the surface water temperature changes but slightly diurnally. As a first approximation, it can even be taken constant. Therefore, radiational cooling in the over-water PBL during the evening and night hours makes it cooler than the underlying surface. This results in the development of convection. On the other hand, radiational heating after sunrise leads to warming of the near-surface layer and, consequently, to stable stratification, degeneration of turbulence in the upper part of the turbulent air layer, and collapse of the PBL.

In both cases, over land as well as over water, a mathematical model of the PBL diurnal cycle should be able to provide a realistic description of the two main regimes; (i) essentially non-stationary convective development and (ii) formation and evolution of stable stratification when the dynamical phenomena can be considered in quasi-equilibrium.

Typical values of PBL depth are less than 10^2 m in strongly stable conditions and more than 10^3 m in the final stage of convection. In this latter case, quite a deep sublayer participates in considerable diurnal variations (the atmospheric active layer). During the remaining period, turbulence is weak or non-existent in the upper part of the sublayer. It is understood that very strong interactions must take place between the sublayer and the PBL.

There are many other interactions governing PBL structure and evolution. Figure 1 shows those taken into account in our model.

Notwithstanding its great importance, the effects of regular vertical motions (atmospheric upwelling and downwelling) on vertical transfer processes is not accounted for. Certainly, the vertical advection terms could be introduced artificially into model equations, if the vertical velocity w were known. It is clear, however, that vertical motions can hardly be treated realistically within the framework of any one-dimensional model.

A representation of the PBL diurnal cycle, as given above, is shown in Figure 2. Here, z is height, t is time, h is the PBL depth, and h_m is the depth of the atmospheric active layer. The same scheme is adopted in our model. Specifically the vertical structure of the atmospheric active layer, is parameterized using two alternative ways of calculating the PBL parameters: the similarity theory for the PBL with time-dependent depth during convection, and equilibrium-type similarity theory during stable stratification. Accordingly, an instantaneous fall in PBL depth is adopted at the moment of transition from unstable to stable conditions.

In support of the latter simplification, we refer to turbulence measurements carried out during the Øresund Experiment, May and June 1984 (Gryning, 1985). Figure 3 shows the dissipation rate of turbulence energy ϵ measured by an airplane in a traverse at 270-m over the Øresund and surrounding land on 5 June 1984. The sea temperature was several degrees cooler than the air, so that stable stratification developed over the Øresund. The wind passed across the sound from Barsebäck to Gladsaxe at a speed of the order of 10 m/s. The decrease in ϵ as the air passed Øresund is clearly seen. According to the diagram, the turbulence degeneration

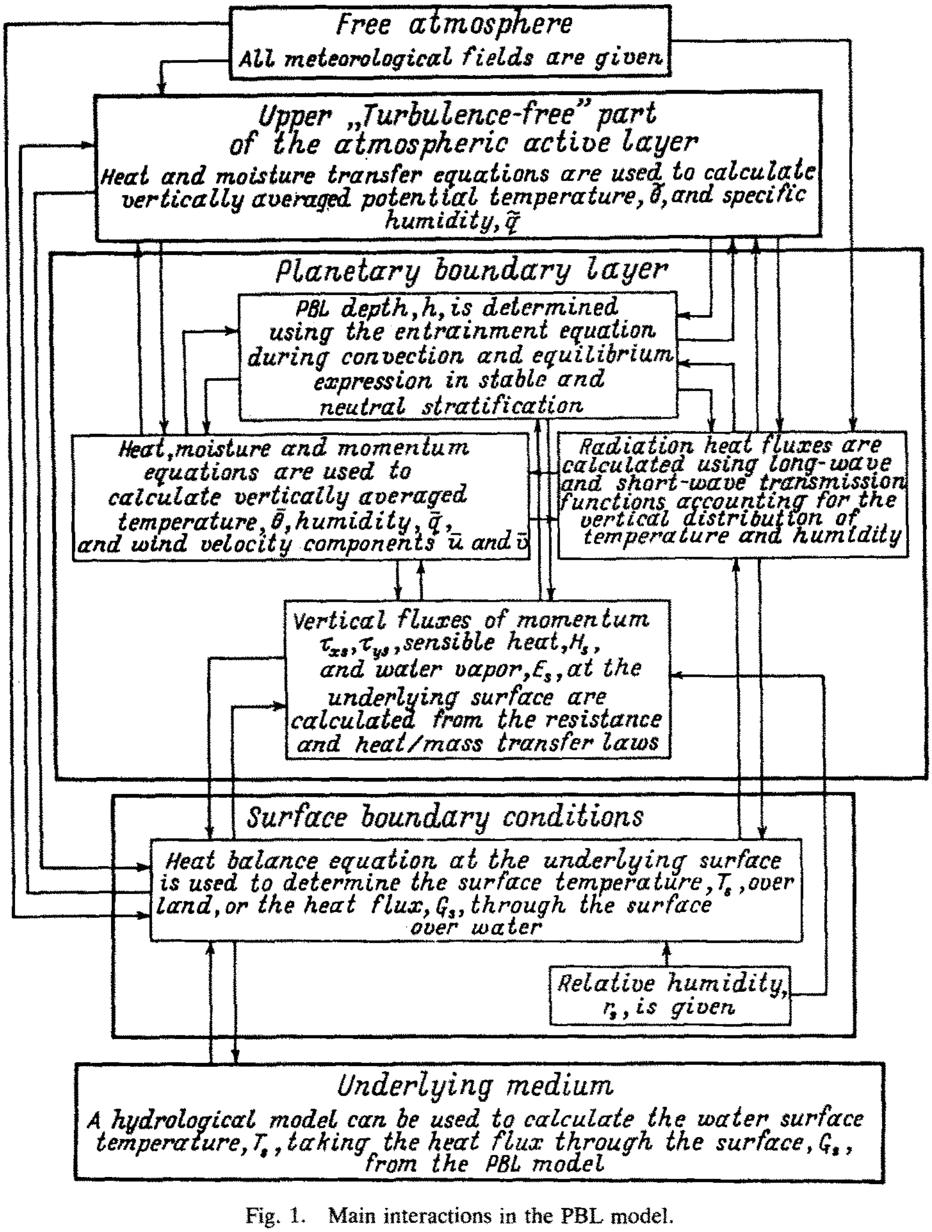

Fig. 1. Main interactions in the PBL model.

period did not exceed half an hour. Therefore the PBL collapse can really be represented in the first approximation as an instantaneous fall.

2. Vertical Profiles of Temperature and Humidity

We confine ourselves to the lower layer of the atmosphere, from the surface to some height h_m taken above the maximum value that the PBL reaches in its diurnal course. In this way, the atmosphere is divided into three layers: (i) the

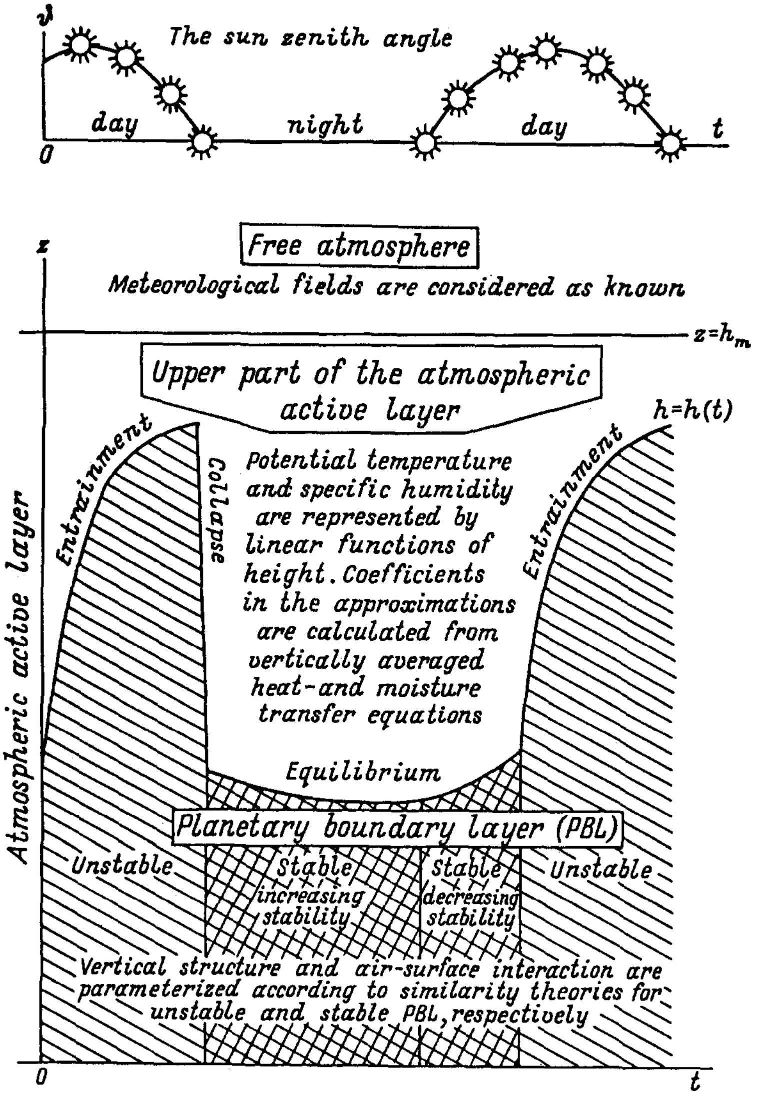

Fig. 2. General scheme of the diurnal evolution of the PBL over land.

Fig. 3. Dissipation rate of turbulent energy ϵ over Øresund and surroundings at 270-m height of 5 June 1984 (from Gryning, 1985).

PBL, $0 < z < h$, whose depth h changes considerably with time; (ii) the atmospheric thermocline, i.e., the upper, turbulence-free part of the atmospheric active layer, $h < z < h_m$, which interacts strongly with the PBL; (iii) an overlying layer of the free atmosphere, $z > h_m$.

In the free atmosphere, vertical profiles of atmospheric pressure p, potential temperature θ and specific humidity q are assumed known. They should be taken either from climate data or from a numerical weather forecast. We designate values of these parameters at the level $z = h_m$ by p_m, θ_m and q_m, respectively.

According to the ideal gas state equation, air density ρ is expressed by

$$\rho = p/RT, \quad T = \theta(p/p_0)^{R/c_p}, \tag{1}$$

where R is the gas constant of air, c_p is its specific heat at constant pressure, $p_0 = 1000hPa$ is the reference pressure, and T is the absolute temperature.

In the atmospheric thermocline, we assume (as the simplest approximation) that θ and q change linearly with height:

$$\theta = \theta_m + \Gamma_\theta(z - h_m), \quad q = q_m + \Gamma_q(z - h_m) \quad \text{for } h < z < h_m, \tag{2}$$

where Γ_θ and Γ_q are parameters involved in the diurnal cycle that are to be determined.

In the PBL the shapes of the profiles of θ and q depend on the character of stratification (Figure 4).

A. *Unstable stratification:* When the buoyancy flux B_s is directed from the underlying surface to the atmosphere, the vertical profile of potential temperature is as follows. Close to the surface, θ sharply decreases with height, then the function $\theta(z)$ levels off and remains practically constant with height up to the PBL upper boundary ($z = h$). Near this boundary, θ grows sharply across a comparatively small height increment $h - \Delta h/2 < z < h + \Delta h/2$, where Δh is of the order of one- to two-tenths of h, and the temperature increment $\Delta\theta$ reaches a degree or even several degrees C. The vertical profile of specific humidity has a similar shape.

The following expressions for θ and q profiles based on similarity theory (Monin and Obukhov, 1954) are suitable in the lower part of PBL:

$$\theta - \theta_s = \begin{cases} \theta_* \ln(z/z_{0T}) & \text{for } z_{0T} < z \leq \zeta_\theta|L| \\ \theta_*[a_\theta + C_\theta(z/L)^{-1/3} + \ln(|L|/z_{0T})] & \text{for } \zeta_\theta|L| \leq z < h \end{cases}, \tag{3}$$

$$q - q_s = \begin{cases} q_* \ln(z/z_{0q}) & \text{for } z_{0q} < z \leq \zeta_q|L| \\ q_*[a_q + C_q(z/L)^{-1/3} + \ln(|L|/z_{0q})] & \text{for } \zeta_q|L| \leq z < h \end{cases}, \tag{4}$$

where z_{0T} and z_{0q} are roughness lengths of the underlying surface for temperature

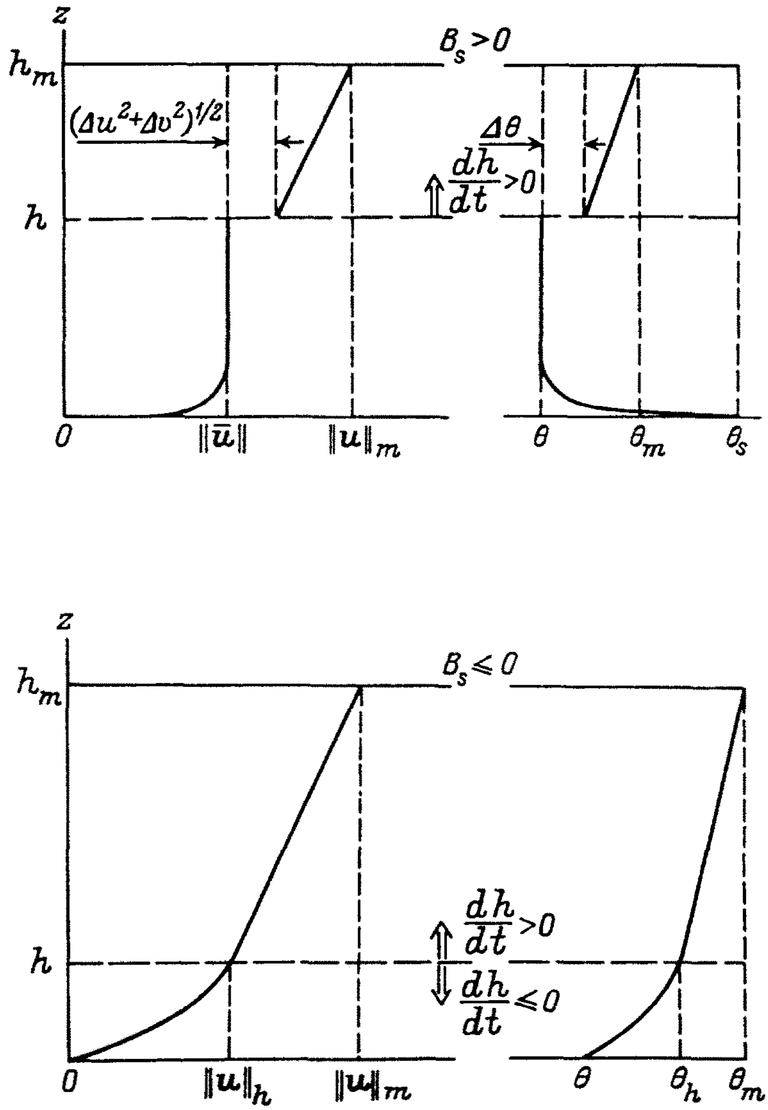

Fig. 4. Parameterized vertical profiles of the wind velocity modulus $\|\mathbf{u}\|$ and potential temperature θ for unstable ($B_s > 0$) and stable ($B_s < 0$) stratification.

and humidity, respectively, θ_s and q_s are near-surface values of temperature and humidity, $a_\theta \approx a_q$, $C_\theta \approx C_q$, $\zeta_\theta \approx \zeta_q$ are dimensionless constants, θ_* and q_* are temperature and humidity scales for the near-surface layer and L is the Monin-Obukhov length scale. The last three values are determined by

$$\theta_* = -H_s/kc_p\rho_s u_*, \quad q_* = -E_s/k\rho_s u_*,$$
$$L = -u_*^3/kB_s = -u_*^3/k(\beta_s H_s/c_p\rho_s + 0.608gE_s/\rho_s)\,. \tag{5}$$

Here, $k = 0.4$ is the von Karman constant, g is the acceleration of gravity, u_* is the friction velocity, H_s, E_s and B_s are the near-surface values of the vertical fluxes of heat, water vapor and buoyancy, respectively, $\beta_s = g/T_s$ and $\rho_s r$ are near-surface values of the buoyancy parameter with respect to temperature, and the air density. In the case of well-developed convection, $|L| < h$. Since $\zeta_\theta \approx \zeta_q \approx 10^{-1}$ (section 1.2.4 in Zilitinkevich, 1970), formulas (3) and (4) correspond to the approximate constancy of potential temperature and specific humidity with height throughout almost the whole PBL (except a thin near-surface layer), and hence, lead to the following approximate relations:

$$\theta(z) \approx \bar{\theta} \approx \theta_{h-0} \approx \theta_s + \theta_*[\alpha_\theta + \ln(|L|/z_{0T})] , \tag{6}$$

$$q(z) \approx \bar{q} \approx q_{h-0} \approx q_s + q_*[a_q + \ln(|L|/z_{0q})] , \tag{7}$$

where the overbar denotes averaging over the PBL.

Formulas (3) and (4) will not be required hereafter in calculations of the PBL parameters. Therefore the numerical values of constants C_θ, C_q, ζ_θ, ζ_q will not be required either. Also, we shall not be worried about the power of z in the dependencies of θ and q on z for $\zeta_\theta|L| \leq z < h$. It makes no difference whether this power should be considered equal to $-1/3$ according to Monin and Obukhov (1954) or $-1/2$ according to Businger *et al.* (1971). This will not affect Equations (6) and (7).

Together with (5) these equations represent the bulk laws of heat and moisture transfer, expressing θ_* and q_*, i.e., fluxes of heat H_s and water vapor E_s, in terms of air-surface differences of temperature $\bar{\theta} - \theta_s$ and humidity $\bar{q} - \vartheta_s$. More general derivation of the same laws and empirical estimates of the dimensionless constants, $a_\theta \approx a_q 1 \approx 0.1$, are given by Zilitinkevich (1975).

The adopted parameterization (Figure 4) provides for discontinuity of θ and q at the upper boundary of the convective PBL:

$$\Delta\theta \equiv \theta_{h+0} - \theta_{h-0} = [\theta_m - \Gamma_\theta(h_m - h)] - \bar{\theta} , \tag{8}$$

$$\Delta q \equiv q_{h+0} - q_{h-0} = [q_m - \Gamma_q(h_m - h)] - \bar{q} , \tag{9}$$

and hence, for a discontinuity in buoyancy:

$$\Delta b \equiv b_{h+0} - b_{h-0} = \beta_h \Delta\theta + 0.608 g \Delta q , \tag{10}$$

where β_h is buoyancy parameter at $z = h$. The buoyancy increment determined by (10) should be positive, otherwise the model loses physical sense.

B. *Stable and neutral stratification* ($B_s \leq 0$): In this case, the potential temperature θ and the specific humidity q change considerably with height, not only near the surface but within the whole PBL. We use the following log-polynomial expressions:

$$\theta(z) = \theta_s + \theta_* \left[\ln \frac{z}{z_{0T}} + (c_0 + C_h \beta_\theta \mu^{1/2}) \frac{z}{h} - \right.$$

$$\left. - \frac{1}{2}(1 + c_0 + C_h \beta_\theta \mu^{1/2}) \left(\frac{z}{h}\right)^2 \right], \tag{11}$$

$$q(z) = q_s + q_* \left[\ln \frac{z}{z_{0q}} + (c_0 + C_h \beta_\theta \mu^{1/2}) \frac{z}{h} - \right.$$

$$\left. - \frac{1}{2}(1 + c_0 + C_h \beta_\theta \mu^{1/2}) \left(\frac{z}{h}\right)^2 \right], \tag{12}$$

where μ is a dimensionless stratification parameter,

$$\mu = ku_* / |f| L \; ; \tag{13}$$

f is the Coriolis parameter ($f = 2\Omega \sin \phi$, Ω is angular velocity of the earth rotation, ψ is latitude); $c_0 = -4$, $C_h = 0.85$ and $\beta_\theta = 9$ are dimensionless constants (Zilitinkevich, 1989b).

Averaging (11) and (12) over the PBL, we obtain

$$\bar{\theta} = \theta_s + \theta_* \left(\frac{1}{3} c_0 - \frac{7}{6} + \frac{1}{3} C_h \beta_\theta \mu^{1/2} + \ln \frac{h}{z_{0T}} \right), \tag{14}$$

$$\bar{q} = q_s + q_* \left(\frac{1}{3} c_0 - \frac{7}{6} + \frac{1}{3} C_h \beta_\theta \mu^{1/2} + \ln \frac{h}{z_{0q}} \right). \tag{15}$$

Just as in (6) and (7), these heat and moisture transfer laws can be used to calculate the fluxes of heat H_s and water vapor E_s in terms of differences of potential temperature $\bar{\theta} - \theta_s$ and specific humidity $\bar{q} - q_s$.

In this case, our parameterization (Figure 4) provides for no discontinuities at the PBL upper boundary:

$$\Delta\theta \equiv \theta_{h+0} - \theta_{h-0} = 0, \quad \Delta q \equiv q_{h+0} - q_{h-0} = 0 \, . \tag{16}$$

Accordingly, the temperature and humidity profiles of type (2) in the layer $h < z < h_m$ appear to be fully determined by the PBL parameters θ_{h-0} and q_{h-0} which are obtained from (11) and (12), and by the free atmosphere parameters θ_m and q_m.

The parameterization of temperature and humidity profiles is not complete until the corresponding roughness lengths z_{0T} and z_{0q} are determined.

Over land we adopt $z_{0T} = z_{0q}$ and use the relation between roughness lengths with respect to temperature z_{0T} and wind z_{0u}, obtained by interpretation of the published data of heat-transfer experiments (Section 1.2.2 in Zilitinkevich, 1970). Representing formula (1.78) from that reference in terms of z_{0T}, we obtain

$$z_{0T} = z_{0q} = z_{0u} \exp[-(0.84 z_{0u} u_* / \nu_a)^{0.45}] , \tag{17}$$

where $\nu_a = 1.4 \times 10^{-5}$ m^2/s is kinematic air viscosity. The wind roughness length z_{0u} is approximately expressed by

$$z_{0u} = \begin{cases} 0.135 \nu_a / u_* & \text{for} \quad z_0 u_* / \nu_a \leq 0.135 \\ z_0 & \text{for} \quad z_0 u_* / \nu_a > 0.135 \end{cases} , \tag{18}$$

where z_0 is the limiting value of z_{0u} corresponding to the regime of fully-developed roughness. For most types of land surface, z_0 is more or less known (Table 1.1 in Zilitinkevich, 1970). A detailed discussion on roughness length over water has been given recently by Brutsaert (1982), Greenaert (1990), and Mironov (1991).

3. Budget and Heat and Moisture

The temporal changes of temperature and humidity in the PBL are described by the equations:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial z} \frac{H + F}{c_p \rho} , \quad \frac{\partial q}{\partial t} = - \frac{\partial}{\partial z} \frac{E}{\rho} . \tag{19}$$

Here, t is time, H and F are turbulent and radiation vertical fluxes of heat and E is vertical flux of water vapor. These fluxes are considered positive when they are directed upward.

Integrating (19) over z from 0 to h_m, we obtain the following heat and moisture budget equations for the entire atmospheric layer $0 < z < h_m$:

$$\frac{\mathrm{d}}{\mathrm{d}t} [h \bar{\theta} + (h_m - h) \tilde{\theta}] = \frac{H_s + F_s}{c_p \rho_s} - \frac{F_m}{c_p \rho_m} , \tag{20}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} [h \bar{q} + (h_m - h) \tilde{q}] = \frac{E_s}{\rho_s} . \tag{21}$$

Here, $\tilde{\theta}$ and $\tilde{q}$ are mean values of θ and q within the layer $h < z < h_m$; $\bar{\theta}$ and $\bar{q}$ are θ and q PBL means; and in the right-hand sides of the equations, the index "s" denotes values related to the level $z = 0$; the index "m" denotes those related to the level $z = h_m$. According to expression (2),

$$\tilde{\theta} = \theta_m - \Gamma_\theta (h_m - h)/2 , \quad \tilde{q}_0 = q_m - \Gamma_g (h_m - h)/2 . \tag{22}$$

Now we integrate (19) over z from 0 to h accounting for the variability of h to obtain

$$h \frac{\mathrm{d}\bar{\theta}}{\mathrm{d}t} = \Delta \theta \frac{\mathrm{d}h}{\mathrm{d}t} + \frac{H_s + F_s}{c_p \rho_s} - \frac{F_h}{c_p \rho_h} , \tag{23}$$

$$h \frac{\mathrm{d}\bar{q}}{\mathrm{d}t} = \Delta q \frac{\mathrm{d}h}{\mathrm{d}t} + \frac{E_s}{\rho_s} . \tag{24}$$

Equations (23) and (24) are used in the present model only during convection. In stable stratification, $\bar{\theta}$ and $\bar{q}$ are calculated by means of the conditions of continuity (16), using the heat and moisture budget equations (20), (21), the equilibrium PBL formulations (11), (12), (43) and parameterization (2) for the upper part of the atmospheric active layer. Equations (20), (21), (23), (24) contain turbulent fluxes of heat and water vapor H_s, E_s at the underlying surface. At the PBL upper boundary and at the upper boundary of the whole layer under consideration, these fluxes are assumed negligible because of the absence of developed turbulence outside the PBL.

4. Boundary Conditions on the Underlying Surface

We use the equation of the heat balance on the underlying surface:

$$F_s + H_s + L_E E_s - G_s = 0 , \qquad (25)$$

where L_E is the latent heat of vaporization and G_s is the near-surface value of the heat flux in soil or water, considered positive when it is directed upward.

On land, G_s is, as a rule, much less than any other term in (25). So, as a first approximation, $G_s = 0$, although sometimes it is taken equal to a small fraction of the net radiation at the surface F_s. Anyway, Equation (25) is used to calculate the surface temperature T_s.

In water bodies, it is not admissible to neglect the term G_s in (25). In this case, the absolute temperature of the water surface should be considered prescribed (being either measured or calculated by means of a hydrological model of the heat regime of the water body). Then G_s can be calculated with the help of Equation (25) and, if required, used in the hydrological model calculations.

The near-surface value of the specific humidity q_s is expressed according to

$$q_s = 0.622 r_s e_m(T_s)/p_s , \qquad (26)$$

where r_s is the near-surface value of relative humidity (a prescribed parameter), and e_m is the saturation vapor pressure which depends only on temperature and is determined by the Clausius-Clapeyron equation. According to Henderson-Sellers (1984), the following approximation can be used:

$$e_m(T) = 2.17 \times 10^{10} \exp[-4157/(T - 33.91)] , \qquad (27)$$

where e_m is measured in N/m^2.

In this way, the lower boundary conditions are as follows:

Over land, $r_s \leqslant 1$, G_s is prescribed, T_s is calculated.

Over a water body, $r_s = 1$, T_s is prescribed, G_s is calculated.

The latent heat of vaporization L_E can be taken constant (2.5×10^6 J/kg); however, it is not difficult to include its dependence on temperature, for instance after the Henderson-Sellers (1984) formula:

$$L_E(T) = 1.92 \times 10^6[T/(T-33.9)]^2\,, \tag{28}$$

where L_E is measured in J/kg.

Then the heat and moisture transfer laws (6), (7), (14), (15), together with the boundary conditions considered above, can be used to calculate the fluxes of heat and water vapor at the underlying surface H_s and E_s.

5. Radiation Heat Transfer

We represent the radiation heat flux in the form:

$$F = F_S + F_L = (F_S^{\uparrow} - F_L^{\downarrow}) + (F_L^{\uparrow} - F_L^{\downarrow})\,, \tag{29}$$

where F_S is the flux of short-wave (solar) radiation, F_L is the flux of long-wave radiation of the atmosphere and the earth, while each of these fluxes is made up of upward and downward components marked by corresponding arrows.

A. Long-wave radiation. We assume that its integral transmission function P_L, i.e., the share of radiation transmitted by a certain air layer $z_1 < z < z_2$, depends only on the effective mass of water vapor in this layer determined by

$$m(z_1, z_2) = \int_{z_1}^{z_2} qp\frac{p}{p_0}\left(\frac{T_0}{T}\right)^{1/2} \mathrm{d}z\,, \tag{30}$$

where $T_0 = 273$ K (Chapter 6 in Laikhtman, 1976). As one of many possible variants, we use the approximation of the function $P_L(m(z_1, z_2))$ given in the quoted book:

$$P_L(m) = 0.539\exp(-2.45\sqrt{1.66m}) + + 0.461\exp(-0.213\sqrt{1.66m})\,, \tag{31}$$

where the argument m is measured in kg/m^2.

Suppose there are clouds somewhere above the atmospheric active layer. Then long-wave radiation fluxes at an arbitrary level z situated below the cloud are expressed by means of the function P_L as follows:

$$F_L^{\uparrow}(x) = \sigma T_s^4 P_L(m(0,z)) + \int_0^z \sigma[T(z')]^4 \times \times \frac{\mathrm{d}}{\mathrm{d}z'} P_L(m(z',z))\,\mathrm{d}z'\,, \tag{32}$$

$$F_L^{\downarrow}(z) = -\int_z^{h_c} \sigma(T(z')]^4 \frac{\mathrm{d}}{\mathrm{d}z'} P_L(m(z,z'))\,\mathrm{d}z' - - (1-n)\int_{h_c}^{\infty} \sigma(T(z')]^4 \frac{\mathrm{d}}{\mathrm{d}z'} P_L(m(z,z'))\,\mathrm{d}z' + + n\sigma T_c^4 P_L(m(z,h_c))\,, \tag{33}$$

where σ is the Stephan-Boltzman constant, n is the cloud cover, h_c is the height of the cloud layer (it is assumed that the clouds are black and have a zero depth).

We consider as prescribed the cloud temperature T_c and effective temperatures T_{sc} and T_{ac} of the subcloud and abovecloud layers $h_m < z < h_c$ and $h_c < z < \infty$. The layers $0 < z < h$ and $h < z < h_m$ are characterized by their mean absolute temperatures $\bar{T}$ and $\tilde{T}$.

Then it is not difficult to express by formulas (32) and (33) the three long-wave fluxes that we need: $F_L(0)$, $F_L(h)$ and $F_L(h_m)$.

The effective masses of water vapor in the layers $0 < z < h$ and $h < z < h_m$, $m(0, h)$ and $m(h, h_m)$ respectively, are expressed through calculated parameters; and in the layers $h_m < z < h_c$ and $h_c < z < \infty$, they are considered as prescribed external parameters characterizing the free atmosphere: $m(h_m, h_c) = m_{sc}$, $m(h_c, \infty) = m_{ac}$. Through these parameters, it is easy to express the values of m for the other layers that we need. Thus, for the layer $0 < z < \infty$, we obtain

$$m(0, \infty) = m(0, h) + m(h, h_m) + m_{sc} + m_{ac}\,. \tag{34}$$

B. Short-wave radiation: We take into account reflection and absorption by clouds in the whole spectral range and absorption by water vapor only in the near-infrared part of the spectrum. Absorption by carbon dioxide (less effective by an order of magnitude) is not considered.

We use the short-wave radiation transmission function of the following type (Lacis and Hansen, 1974):

$$P_S(m) = 1 - \frac{0.29m}{(1 + 14.2m)^{0.635} + 0.59m}\,, \tag{35}$$

where m is measured is kg/m^2, as before.

The short-wave radiation flux at an arbitrary level z situated below the cloud level is expressed with the help of this function as follows:

$$\begin{aligned} F_S(z) = -I_0 \cos\vartheta\{(1-n)[P_S(m(z,\infty)) - A_s P_S(m(0,\infty) + \tfrac{5}{3}m(0,z))] + \\ + n(1-A_c)[P_S(m(h_c,\infty) + \tfrac{5}{3}m(z,h_c)) - A_s P_S(m(h_c,\infty) + \\ + \tfrac{5}{3}m(0,h_c) + \tfrac{5}{3}m(0,z))]\}\,. \end{aligned} \tag{36}$$

Here, A_s is the albedo of the underlying surface; A_c is the attenuation coefficient of clouds ($A_c = A_{c1} + A_{c2}$, A_{c1} is the albedo of the upper boundary of clouds, A_{c2} is the share of the short-wave radiation, absorbed by the cloud layer) I_0 is the reduced (sub-ozone) value of the solar constant; ϑ is the Sun zenith angle,

$$\cos\vartheta = \sin\phi \cdot \sin\delta + \cos\phi \cdot \cos\delta \cdot \cos\psi\,, \tag{37}$$

ϕ is latitude, δ and ψ are the Sun declination and hour angles.

The latter is determined by

$$\psi = 2\pi t/t_d \quad (-\tau_1 < t < t_1)\,, \tag{38}$$

where t is time counted from noon, $t_d = 86400s$ is the duration of the solar day, $2t_l$ is the duration of the light period of the day,

$$2t_l = \frac{t_d}{\pi}\psi_0 = \frac{t_d}{\pi}\arccos(-tg\phi \cdot tg\delta)\,, \tag{39}$$

ψ_0 is the hour angle of the Sun at the moment of sunset (at $\cos\vartheta = 0$).

For the declination angle of the Sun, we assume the approximation (Henderson-Sellers, 1984)

$$\delta = 0.409\sin[2\pi(d - 79.8)/365]\,, \tag{40}$$

where $d = 1, 2, \ldots, 365$ is a Julian date counted in the Northern hemisphere from January 1.

The values of $F_S(z)$ at levels $z = 0$, h, h_m are calculated after formulas (36)–(40) by substituting in them the effective masses of water vapor $m(0, h)$, $m(h, h_m)$, m_{sc} and m_{ac}. The values of the total radiation flux of interest are found after formula (29).

6. PBL Depth

As already mentioned, the turbulence regime in the PBL is very dependent on the character of the density stratification near the underlying surface. In unstable stratification, a non-stationary regime of turbulent penetrative convective exists. In stable and neutral stratification, an equilibrium quasi-stationary regime seems to be possible. These essential features of the PBL should be taken into account in any mathematical model of the diurnal PBL cycle claiming to be realistic.

A. Unstable stratification ($B_s > 0$): If there are no sharp changes in meteorological conditions in the free atmosphere, the turbulent regime in the PBL is governed mainly by the convective mechanism. The PBL thickness h in this case grows with time monotonously. This process can be calculated by a prediction equation for h, a purely convective variant of which is considered by Zilitinkevich (1991). In the presence of considerable wind shear, the equation obtained (already a rather complicated one) should be generalized with regard to shear generation of turbulence energy.

As a first step towards developing a parameterized model of the PBL, we calculate the effect of penetrative convection approximately, by a simplified version of the entrainment equation. The latter corresponds to the Betts (1973), Carson (1973), Tennekes (1973) model which is commonly accepted as quite satisfactory for weak winds. According to this model, the buoyancy flux caused by the entrainment process at the PBL upper boundary, i.e., the value of $\Delta b\, \mathrm{d}h/\mathrm{d}t$, makes up a standard share of the buoyancy flux at the underlying surface $B_s = \beta_s H_s/c_p\rho_s + 0.608gE_s/\rho_s$, so that the entrainment equation has the form

$$\mathrm{d}h/\mathrm{d}t = C_1 B_s/\Delta b\,, \tag{41}$$

where $C_1 = 0.2$ is an empirical dimensionless constant, Δb is the buoyancy jump, determined by formulas (8)–(10). The simplest phenomenological version of the equation generalizing (41) with regard to the velocity shear is

$$\mathrm{d}h/\mathrm{d}t = (C_1 B_s + C_* u_*^3/h)/\Delta b\,, \tag{42}$$

where $C_* = 5$ is another dimensionless constant (Driedonks and Tennekes, 1984).

More physically grounded, but more complicated, versions of the convective entrainment equation in the presence of wind shear were derived by Gryning and Batchvarova (1990), Batchvarova and Gryning (1990), Zilitinkevich (1990b), etc. Any one of these versions can be easily substituted in our model instead of Equation (42). Sensitivity of the model to the choice of an entrainment equation will be examined in future numerical experiments.

B. Stable and neutral stratification ($B_s \leqslant 0$): The PBL depth h can be estimated by

$$h = h_e = \frac{u_*}{f}\left(\frac{1}{\Lambda_0} + \frac{\mu^{1/2}}{kC_h}\right)^{-1}, \tag{43}$$

where $L = 0.3$, $k = 0.4$, $C_h = 0.85$ are dimensionless constants, μ is a dimensionless stratification parameter determined by (13) and h_e is the equilibrium PBL depth (Zilitinkevich, 1989a). Expression (43) for h_e is derived by interpolation between two known formulas, viz., Rossby and Montgomery (1935) for neutral stratification and Zilitinkevich (1972) for strongly stable stratification.

In more careful consideration of the PBL collapse and evolution during stable stratification, a prediction equation for the PBL depth should be used, e.g., $\mathrm{d}h/\mathrm{d}t = (h_e - h)/t_*$, where t_* is the adjustment time scale of the PBL. In the present model, we merely assume t_* to be negligibly small and adopt $h = h_c$.

Then, according to (43), the model provides for a drop of h by a jump in the course of a transition from unstable to stable conditions: as soon as the buoyancy flux at the underlying surface B_s becomes zero, integration of the prediction equation (42) stops, and (43) is used for determination of h. As was already mentioned, a fairly steep decrease of h (fast collapse) just as B_s passes through zero is an essential observed feature of the PBL.

On transition to the convective regime (after B_s becoming positive), an initial value of h is necessary for integrating the prediction equation (42). In our case, we use the last value of h obtained after (43). In this way, the model provides for realistic continuous changes of h in the course of replacement of a stable regime by an unstable one.

7. Wind Field and Surface Friction

A. Unstable stratification ($B_s > 0$): The PBL grows quickly. Its thickness can reach 2 km and more. Under such conditions, not only non-stationarity is important, but also the baroclinicity of the flow, i.e., the vertical variation of the geostrophic wind. A parameterized model of a non-stationary baroclinic PBL is proposed below.

The intensive mixing in the main part of the convective PBL (with the exception of a thin near-surface layer) results in approximate vertical homogeneity of not only potential temperature and specific humidity, but also wind speed and direction. This observation (Arya and Wyngaard, 1975) greatly facilitates the solution of the problem of the temporal evolution of the wind field during convection.

We use the momentum equations

$$\frac{\partial u}{\partial t} = f(v - V) + \frac{\partial \tau_x}{\partial z}, \quad \frac{\partial v}{\partial t} = -f(u - U) + \frac{\partial \tau_y}{\partial z}. \tag{44}$$

Here, u and v are wind velocity components along the x and y axes, τ_x and τ_y are momentum flux components, and U and V are geostrophic wind velocity components.

We assume U and V to be constant in time and linearly dependent on height:

$$U = U_s + \frac{\partial U}{\partial z}, \quad V = V_s + \frac{\partial V}{\partial z} z. \tag{45}$$

Here,

$$U_s = -\frac{1}{f\rho_s}\frac{\partial p_s}{\partial y}, \quad V_s = \frac{1}{f\rho_s}\frac{\partial p_s}{\partial x}, \quad \frac{\partial U}{\partial z} = -\frac{g}{fT}\frac{\partial T}{\partial y}, \quad \frac{\partial V}{\partial z} = \frac{g}{fT}\frac{\partial T}{\partial x}; \tag{46}$$

and in the last two equations, the vertically averaged horizontal temperature gradient components can be substituted for $\partial T/\partial x$ and $\partial T/\partial y$.

Let us introduce the following designations:

$$u = \|\mathbf{u}\| \cos \alpha, \quad v = \|\mathbf{u}\| \sin \alpha, \tag{47}$$

where $\|\mathbf{u}\|$ is the wind velocity modulus and α is the angle between its direction and the x-axis.

As was already mentioned, the major vertical changes of the velocity components in the convective PBL are confined to a comparatively thin near-surface layer, whose thickness does not exceed several meters, i.e., it appears to be much less than the PBL thickness. Therefore, it is admissible to adopt the same type of the Monin-Obukhov (1954) similarity theory approximation for the vertical profile of wind velocity modulus as we have already used for potential temperature and specific humidity [see equations (3) and (4)]:

$$\|\mathbf{u}\| = \begin{cases} k^{-1}u_* \ln(z/z_{0u}) & \text{for } z_{0u} < z \leq \zeta_u|L| \\ k^{-1}u_*[a_u + C_u(z/L)^{-1/3} + \ln(|L|/z_{0u}] & \text{for } \zeta_u|L| \leq z < h \end{cases} . \tag{48}$$

Here, k, a_u, C_u and ζ_u are dimensionless constants; z_{0u} is the roughness parameter relative to the wind, u_* is friction velocity $[u_* = (\tau_{xs}^2 + \tau_{ys}^2)^{1/4}$, where τ_{xs} and τ_{ys} are near-surface values of τ_x and $\tau_y]$.

In the case of well developed convection, i.e., for $h \gg |L|$, there follows from (48) an approximate formula similar to (6) and (7):

$$\|\bar{\mathbf{u}}\| \approx \|\mathbf{u}\|_{h-0} \approx k^{-1}u_*[a_u + \ln(|L|/z_{0u})] . \tag{49}$$

This formula together with (5) compose a resistance law which allows one to determine the friction velocity u_* in terms of the mean wind velocity in the PBL $\|\bar{\mathbf{u}}\|$. The more general form of the resistance law is derived by Zilitinkevich (1975). It includes an equation for u_* having the formula (49) as a particular case, and the following expression for the angle of full turn of the wind in the PBL:

$$\sin \alpha_* = \sin(\alpha_s - \alpha_{h-0}) = \frac{a_\alpha}{k}\left(\frac{h}{|L|}\right)^{-1/3} \frac{u_*}{\|\bar{\mathbf{u}}\|} \operatorname{sign} f , \tag{50}$$

where α_s and a_{h-0} are the values of the angle α between the wind direction and x-axis at $z = 0$ and $z = h - 0$, respectively, and a_0 is a dimensionless constant. The empirical evaluations were obtained for unstably stratified PBL: $a_u = 1$ and $a_\alpha = 3$, taking the traditional value $k = 0.4$ for the von Kármán constant.

Mean values of the wind velocity components in the PBL are expressed, with the same accuracy as in (49), through $\|\bar{\mathbf{u}}\|$ and α_{h-0}:

$$\bar{u} = \|\bar{\mathbf{u}}\| \cos \alpha_{h-0}, \quad \bar{v} = \|\bar{\mathbf{u}}\| \sin \alpha_{h-0} ; \tag{51}$$

and the near-surface values of the vertical momentum flux components τ_{xs} and τ_{ys}, through u_* and α_s:

$$\tau_{xs} = u_*^2 \cos \alpha_s, \quad \tau_{ys} = u_*^2 \sin \alpha_s . \tag{52}$$

We shall no longer need expression (48) for the velocity profile in calculations of the PBL parameters. Neither shall we need the values of constants C_u and ζ_u; also the choice of the power in a power-law dependence of $\|\mathbf{u}\|$ on z for $\zeta_u|L| \leq z < h$, $-1/3$ after Monin and Obukhov (1954) or $-1/4$ after Businger *et al.* (1971), will be of no importance.

Near the PBL upper boundary, the wind velocity components can change sharply from values $\|\mathbf{u}\|_{h-0} \cos \alpha_{h-0}$ and $\|\mathbf{u}\|_{h-0} \sin \alpha_{h-0}$ characteristic for the PBL inner region, to geostrophic values $U_s + (\partial U/\partial z)h$ and $V_s + (\partial V/\partial z)h$. These increments are confined to the layer of finite depth Δh, but since $\Delta h \ll h$, it is admissible to approximate them as jumps:

$$\Delta u = U_s + \frac{\partial U}{\partial z} h - \|\bar{\mathbf{u}}\| \cos \alpha_{h-0} ,$$
$$\Delta v = V_s + \frac{\partial V}{\partial z} h - \|\bar{\mathbf{u}}\| \sin \alpha_{h-0} , \tag{53}$$

where, according to (49), $\|\mathbf{u}\|_{h-0} = \|\bar{\mathbf{u}}\|$ is adopted on the right-hand sides.

Due to such discontinuities, the convective development is accompanied by the appearance of the vertical momentum fluxes at the PBL upper boundary: $\Delta u \, \mathrm{d}h/\mathrm{d}t$ and $\Delta v \, \mathrm{d}h/\mathrm{d}t$ (see Deardorff, 1972). Integration of the momentum equations (44) over z from 0 to $h - 0$, accounting for these momentum fluxes at $z = h - 0$, leads to

$$\mathrm{d}M_x/\mathrm{d}t = fM_y - \tau_{xs}, \quad \mathrm{d}M_y/\mathrm{d}t = -fM_x - \tau_{ys} , \tag{54}$$

where M_x and M_y are components of the total ageostrophic current in the PBL:

$$M_x = h\left(\|\bar{\mathbf{u}}\| \cos \alpha_{h-0} - U_s - \frac{1}{2}\frac{\partial U}{\partial z} h\right),$$
$$M_y = h\left(\|\bar{\mathbf{u}}\| \sin \alpha_{h-0} - V_s - \frac{1}{2}\frac{\partial V}{\partial z} h\right). \tag{55}$$

Determination of function $h(t)$ was discussed earlier. The Monin–Obukhov length scale L is determined by the third formula of (5); the roughness parameter z_{0u} is found from expression (18) over land and according to Mironov (1991) over a water body. In this way, the system (49)–(55) is closed relative to the unknown quantities M_x, M_y, $\|\bar{\mathbf{u}}\|$, τ_{xs}, τ_{ys}, u_* and α_s. The initial conditions for the equations (54) are defined using the values of M_x and M_y (or $\|\bar{\mathbf{u}}\|$ and a_{h-0}) at the moment of neutral stratification preceding the start of convection.

B. Stable and neutral stratification ($B_s \leq 0$): The dynamic regime can be considered quasi-stationary. Besides, the stably stratified PBL is thin, so the baroclinicity effect is not very important. Therefore, calculation of the near-surface friction u_* and the angle α_* between the directions of the geostrophic wind and surface friction can be done by means of the resistance law and the expressions for the universal functions $A(\mu)$ and $B(\mu)$ derived earlier (Zilitinkevich, 1989a). Then the vertical profiles of wind velocity components are determined from the same type of log-polynomial approximation as was used to represent the temperature and humidity profiles in Section 2.

8. Additional Relations

We consider the meteorological fields above the atmospheric active layer, at $z \leq h$, as known. In particular, calculations with the proposed model imply prescription of the following meteorological parameters at $z = h_m$: potential temperature θ_m

and specific humidity q_m in (22); atmospheric pressure p_m (it will be required later); air density ρ_m in (20) and (21) (it is found from (1) at $p = p_m$ and $\theta = \theta_m$).

Formulas (5), (10) and equations (20), (21), (23), (24) contain the buoyancy parameter $\beta = g/T$ and air density $\rho = p/RT$ at the levels $z = 0$ and $z = h$; β_s, β_h, ρ_s and ρ_h, which depend on the atmospheric pressure p and absolute temperature T values at these levels: p_s, p_h, T_s and T_h. Considering a temperature discontinuity at $z = h$, we adopt $T = (T_{h-0} + T_{h+0})/2$. The boundary condition for specific humidity (26) contains the near-surface values of the atmospheric pressure and absolute temperature, p_s and T_s. Calculation of the radiation fluxes implies values of the absolute temperature, averaged over the layers $0 < z < h$ and $h < z < h_m$, $\bar{T}$ and $\tilde{T}$ respectively, and the effective content of water vapor in these layers, $m(0, h)$ and $m(h, h_m)$. These quantities should be expressed through the prescribed parameters and calculated PBL characteristics.

In an unstable stratification, $\theta(z)$ is expressed in the PBL by formula (3) according to which $\theta \approx \bar{\theta}$ in the greater part of this layer. In stable and neutral stratification, θ changes in height throughout the PBL in accordance with formula (11); but the stronger the stability, the thinner the PBL. Therefore, it is admissible in this case to substitute $\bar{\theta}$ for $\theta(z)$ in the estimates hereafter when calculating integrals of expressions containing $\theta(z)$. In other words, the approximation $\theta = \bar{\theta}$ is justified in a thick convective PBL because it is actually fulfilled over most of the layer, while in a thin stably or neutrally stratified PBL, it is justified because the contribution of such a PBL to the integrals we need is comparatively small and, therefore, the requirements for approximation accuracy are reduced.

We use the state equation (1) and the hydrostatic equation

$$\partial p/\partial z = -g\rho \,. \tag{56}$$

In accordance with the comment given above, we approximate potential temperature θ and specific humidity q in the PBL as height-constant ($\theta = \bar{\theta}$ and $q = \tilde{q}$ for $0 < z < h$) and take Equations (2) and (22) to express θ and q in the upper part of the atmospheric active layer ($h < z < h_m$). Then the vertical profiles of absolute temperature T, atmospheric pressure p and air density ρ, as well as the effective content of the water vapor in the considering layers, $m(0, h)$ and $m(h, h_m)$, are easily determined by means of integration of equation (56).

In this way, introducing the designations

$$\xi_s = (p_s/p_0)^{R/c_p}, \quad \xi_m = (p_m/p_0)^R/c_p \,, \tag{57}$$

we obtain

$$\xi_s \approx \xi_m + \frac{\gamma_\alpha h}{\bar{\theta}} + \frac{\gamma_\alpha}{\theta_m}(h_m - h)\left(2 - \frac{\bar{\theta}}{\theta_m}\right), \tag{58}$$

$$T_s = \theta_s \xi_s, \quad p_s = p_0 \xi_s^{c_p/R} \,, \tag{59}$$

$$\bar{T} = \bar{\theta}\xi_s - \tfrac{1}{2}\gamma_\alpha h\,, \tag{60}$$

$$m(0,h) = \frac{p_0 \xi_s^{\lambda+1}\bar{q}}{R_{\gamma\alpha}(\lambda+1)}\left[\frac{T_0}{\bar{\theta}}\right]^{1/2}\left[1 - \left(1 - \frac{\gamma_\alpha h}{\xi\bar{\theta}}\right)^{\lambda+1}\right], \tag{61}$$

$$T_h = (\bar{\theta} + \Delta\theta/2)\left(\xi_s - \frac{\gamma_\alpha h}{\bar{\theta}}\right), \quad p_h = p_0\left(\xi_s - \frac{\gamma_\alpha h}{\bar{\theta}}\right)^{c_p/R}, \tag{62}$$

$$\tilde{T} \approx \tilde{\theta}\xi_m + \gamma_\alpha(h_m - h)/2\,, \tag{63}$$

$$m(h,h_m) \approx \frac{p_0 T_0^{1/2}(h_m - h)\xi_m^\lambda}{R\theta_m^{3/2}}\{\tilde{q} + (\eta + 3\lambda\kappa/2)(\tilde{q} - q_m/4) + \\ + [15\eta^2/16 + \lambda\kappa\eta + \lambda(\lambda-1)\kappa^{2/4}](\tilde{q} - q_m/3)\}\,. \tag{64}$$

where

$$\eta = 2\left(1 - \frac{\tilde{\theta}}{\theta_m}\right), \quad \kappa = \frac{\gamma_\alpha}{\xi_m\theta_m}(h_m - h), \\ \lambda = 2\frac{c_p}{R} - \frac{3}{2} = 5.46\,, \tag{65}$$

and $\gamma_a = g/c_p$ is the adiabatic temperature gradient. Approximations (58), (63) and (64) correspond to a square approximation with respect to the small parameters η and κ, i.e., they are true to within a few tenths of a percent.

9. Results of Calculations

Some numerical experiments with the proposed model have been carried out on an IBM PC AT-286.

Figure 5 shows the simulated day-time variations of the PBL depth h and the convective velocity scale $w_* = (B_s h)^{1/3}$ as compared with experimental data obtained during the Summer 1983 Boundary Layer Experiment BLX83. The model run corresponds to 15 June 1983. External parameters used in the calculations ($z_0 = 2$ cm, $A_s = 0.15$, $r_s = 0.5$, $h_m = 2000$ m, $\theta_m = 295$ K, $q_m = 0.006$, $n = 0.2$, $\phi = 35°$ NH, $U_s = 7$ m/s, $V_s = 0$, $\partial U/\partial z = 0.001\ \mathrm{s}^{-1}$, $\partial V/\partial z = 0$) tally with weather conditions of that day presented in Nelson *et al.* (1989). The data for h and w_* for comparison are also taken from that paper.

The model seems to be quite realistic in reproducing the scales and general character of the calculated variations. At the same time, however, the predicted behavior of the PBL depth h appears to be not perfectly correct; for example, the calculated h continues growing after w_* reaches its maximum and starts to decrease. According to the observations, the growth of h weakens after noon, so that the $h(t)$ curve levels off. This discrepancy may be caused by superfluous

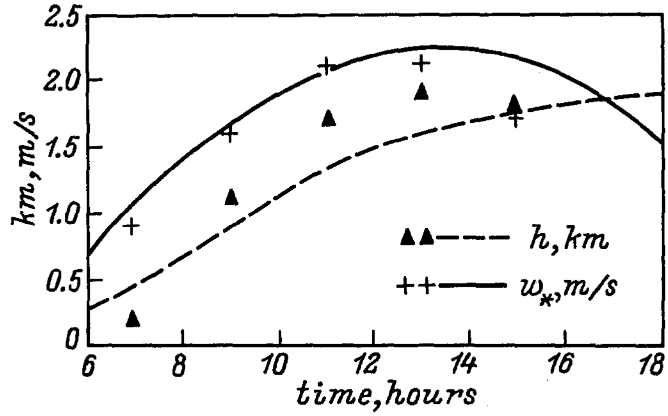

Fig. 5. Simulated and observed day-time variations of the PBL depth h and the convective velocity scale $w_* = (B_s h)^{1/3}$ during 15 June 1983 over the Great Plains, Oklahoma. Triangles and crosses represent measurement data (from Nelson *et al.*, 1989).

simplification of the entrainment equation (42). Thus, further improvement of the model should comprise a more complete description of entrainment.

The simulated diurnal cycle of the meteorological fields using the same external parameters as in Figure 5, i.e., the expected diurnal cycle for 15 June 1983 over the Great Plains, Oklahoma, is shown in Figures 6–8.

Figure 6 shows the diurnal variations of the PBL depth h and parameters characterizing the vertical profiles of potential temperature θ and specific humidity q, viz., the near-surface values θ_s and q_s, the PBL means values $\bar{\theta}$ and $\bar{q}$, and the upper layer means $\tilde{\theta}$ and $\tilde{q}$. The humidity curves appear to be very similar to the same curves for temperature. To a great extent, this is stipulated by taking a fixed value of the near-surface relative humidity r_s. In further studies, the model should include an additional sub-model, one that simulates the diurnal variations of moisture and temperature in the soil.

Figure 7 shows the variations of the friction velocity u_*, the angle of full turn of the wind α_* and the wind velocity modulus at the PBL top $\|\mathbf{u}\|_{h-0}$. All curves look quite reasonable. For example, maximum values of α_* and minima of u_* refer to night hours. During the day, they change synchronously, α_* decreasing, u_* increasing in the morning and day-time and vice versa in the evening.

The diurnal cycle of the surface heat balance components is shown in Figure 8. The heat flux in the soil G_s is taken equal to 10% of the radiation balance $F_s = F_s(0) + F_L(0)$ in the day time and to 30% of it during night.

Figures 9 and 10 demonstrate the ability of the model to reproduce the observed vertical structure in stable and unstable stratification, respectively.

10. Conclusions

i. The suggested one-dimensional, time-dependent PBL model based on simi-

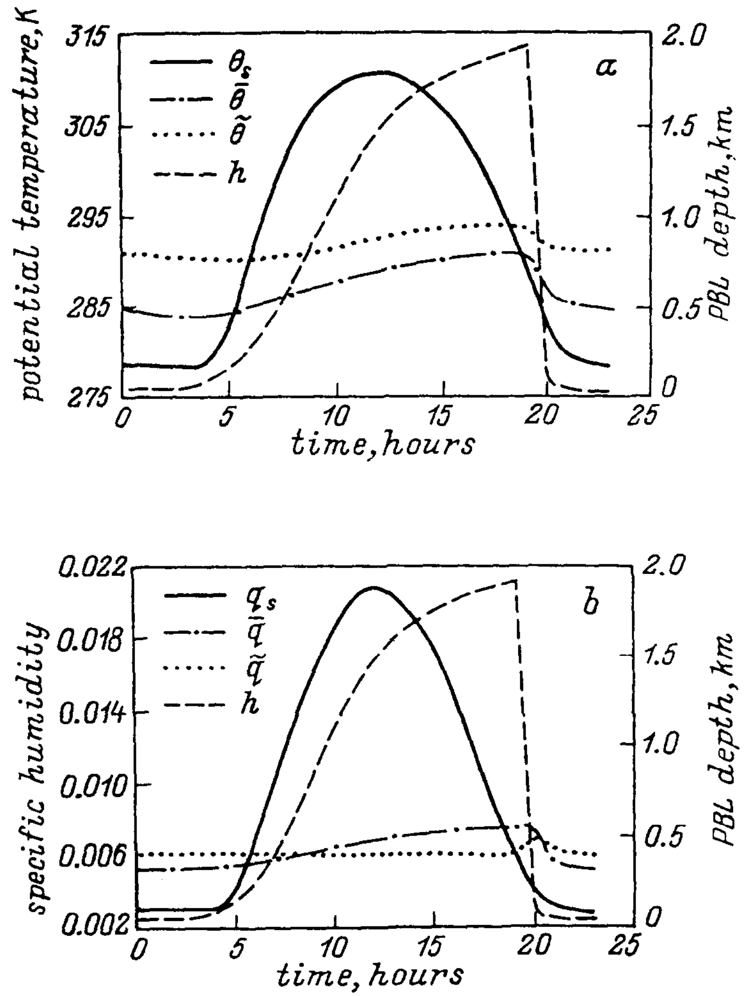

Fig. 6. Simulated diurnal variations of (a) potential temperature θ and (b) specific humidity q, for 15 June 1983 over the Great Plains, Oklahoma. Near-surface values are marked with the subscript "s"; PBL means values with any overbar; the inversion sublayer mean values with tilda. h is the PBL depth.

larity theory reproduces reasonably well the main features of the PBL diurnal cycle found from observations.

ii. The similarity approach provides a satisfactory realistic simulation of such regimes as the PBL collapse after sunset and its convective growth during the morning and day-time hours, which are not easily reproduced in turbulence closure models.

iii. Parameterization of the vertical structure simplifies greatly the numerical calculations, so that the model is appropriate to run on PCs (10-day simulation needs only 8 minutes of IBM PC AT-286 processor time).

iv. The model is a convenient tool for testing different turbulence parameteriz-

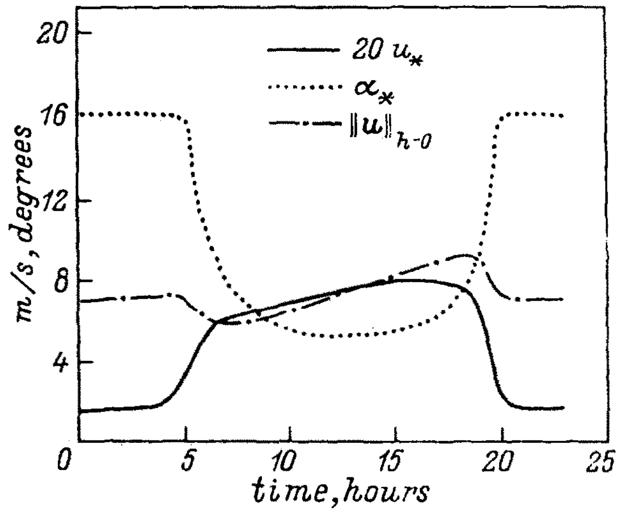

Fig. 7. Same as in Fig. 6 for the PBL dynamic characteristics: friction velocity u_*, the angle of full turn of wind α_* and the wind velocity modulus at the PBL top $\|\mathbf{u}\|_{h-0}$.

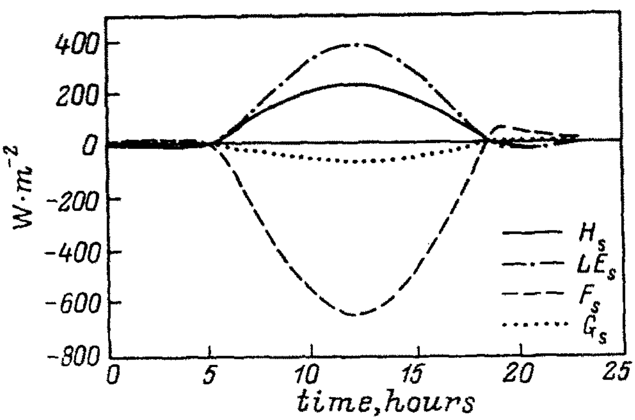

Fig. 8. Diurnal cycle of the surface heat balance components: radiation balance F_s, heat flux in the soil G_s, sensible and latent heat fluxes H_s and $L_E E_s$.

ation schemes, e.g., alternative versions of the convective entrainment equation, taking into account the wind shear, as well as alternative formulations for the equilibrium PBL depth in stable stratification.

v. The model will be equipped with a soil/vegetation sub-model for calculation of the near-surface relative humidity and heat flux in soil.

vi. The following physical problems should be given priority in further improvement of the PBL model:

- transition processes in the PBL, first of all, time scales of the PBL collapse and of its variations during stable stratification;

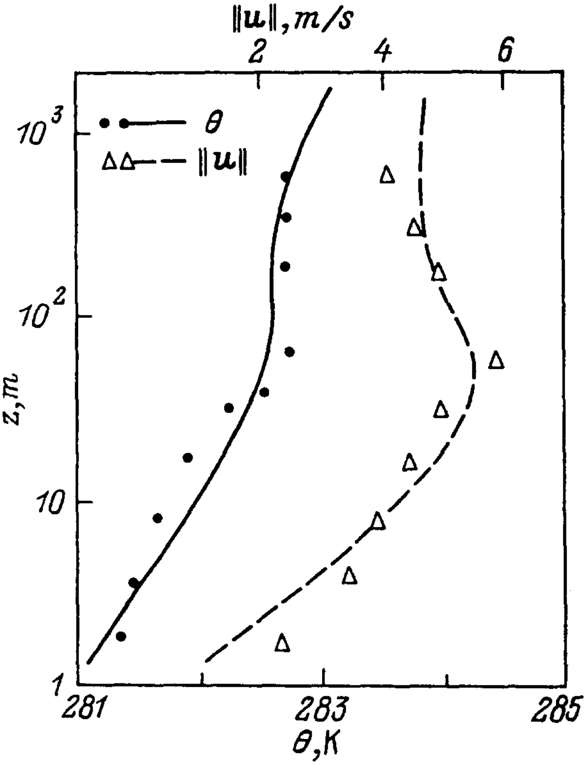

Fig. 9. Simulated and observed vertical profiles of potential temperature θ and wind velocity modulus $\|\mathbf{u}\|$ in nocturnal stable stratification (experimental points from Caughey *et al.*, 1979).

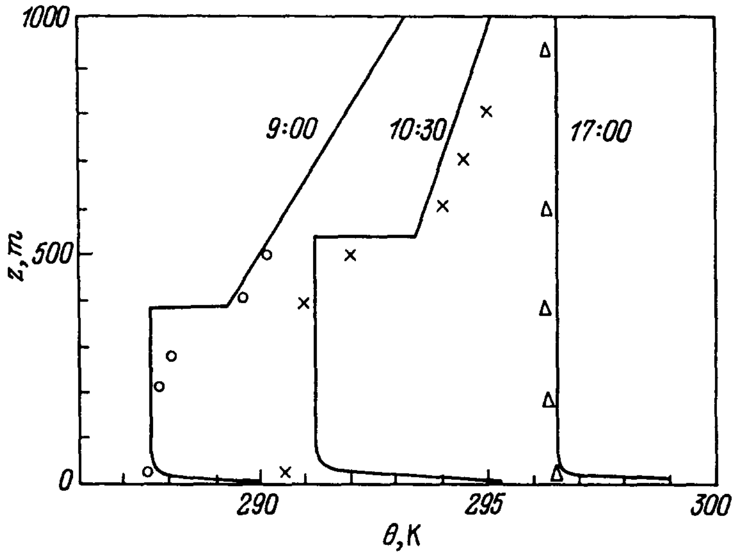

Fig. 10. Simulated and observed consequence of the potential temperature profiles, at 9.00, 10.30 and 17.00, during day-time convection (experimental points from Chorley *et al.*, 1975).

- effects of baroclinicity and non-stationarity (e.g., inertial oscillations) in stably stratified PBL;
- heat, mass and momentum transfer in the "turbulence-free" sublayer, capping the PBL.

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