Independent scattering

- Total scattered field
  \[ E_s = \sum_{j=1}^{N} E_{sj} \]

- Scattered wave field by the jth particle
  \[ E_{sj} = \frac{e^{ik_0 \cdot \vec{r}_j}}{r_0} f_j(\hat{k}_s, \hat{k}_i) e^{ik_0 \cdot \vec{r}_j} f_j(\hat{k}_s, \hat{k}_i) \]

- Cross terms
  \[ \langle E_{sj} E_{sl}^* \rangle = \frac{1}{r_0^2} \sum_{j=1}^{N} \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 \langle \hat{k}_s \cdot \hat{k}_i \rangle \rightarrow 0 \]

- Independent scattering
  \[ \langle |E_s|^2 \rangle = \sum_{j=1}^{N} \langle |E_{sj}|^2 \rangle = \frac{1}{r_0^2} \sum_{j=1}^{N} \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 = \frac{1}{r_0^2} N \langle \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 \rangle \]

---

Time correlation

- Correlation function
  \[ C(\tau) = \langle E_s(t + \tau) E_s^*(t) \rangle \]
  \[ = \sum_{j=1}^{N} \langle E_{sj}(t + \tau) E_{sj}^*(t) \rangle = \frac{1}{r_0^2} \sum_{j=1}^{N} \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 \langle e^{ik_0 \cdot \vec{r}_j \cdot \vec{c}_0} \rangle \]

- Mean and turbulent motion
  \[ \vec{V} = \vec{V}_0 + \vec{V}_1 \]

- Time-correlation cross section
  \[ \langle \sigma_d(\hat{k}_s, \hat{k}_i; \tau) \rangle = \sigma_d(\hat{k}_s, \hat{k}_i) \langle e^{ik_d \cdot \vec{c}_0 \cdot \vec{c}_0} \rangle = \sigma_d(\hat{k}_s, \hat{k}_i) e^{-k_d^2 \sigma_{v_r}^2 \tau^2 / 2} e^{ik_d \cdot \vec{c}_0 \cdot \vec{c}_0} \]

- Estimation of Doppler velocity and spectrum width
  \[ v_r = \angle C(T_s) / (k_d T_s) \]
  \[ \sigma_v = \left( 2 \ln(1/C(0)) / 1/C(T_s) \right)^{1/2} / (k_d T_s) \]
Simulation of time-series radar data

Zhang, 1998

Simulation procedures for wave scattering by randomly distributed particles

- Specify radar constants
- Specify scattering amplitudes
- Generate random positions for each particle
- Calculate scattered field of each particle
- Calculate total scattered field
- Generate realization/time-series data
- Average to obtain coherent and incoherent intensity
- Correlation analysis to obtain Doppler velocity and spectrum width
Generation of Random Numbers (RN)

- By experiment
- By computer
  - Uniformly distributed RNs $x(0,1)$
  - $\text{rand(size)}$: $z_n = az_{n-1}$ mod $m$ \hspace{1cm} $x_i = z_i/m$
  - Gaussian distributed RNs $x(0,1)$
  - $\text{randn(size)}$
  - Other RN generation

Realization and time sequences of random positions

- Realization: event
  \[ \vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \]
- Many events:
  \[ [\vec{r}]_1, [\vec{r}]_2, ..., [\vec{r}]_{Nr} \]
- Time: pulse
  \[ [\vec{r}]_{n+1} = [\vec{r}]_n + (\vec{v}_0 + \vec{v}_1) T_s \]
- Averaging
Matlab code

• Try the matlab code and run you own simulations!