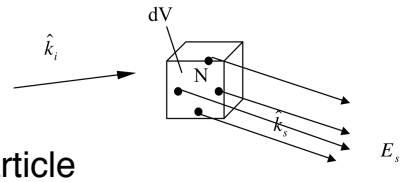
Independent scattering

- Total scattered field

$$E_s = \sum_{j=1}^{N} E_{sj}$$



- Scattered wave field by the jth particle

$$E_{sj} = \frac{e^{ik|\vec{r}_0 - \vec{r}_j|}}{|\vec{r}_0 - \vec{r}_j|} f_j(\hat{k}_s, \hat{k}_i) e^{i\vec{k}_i \bullet \vec{r}_j} \approx \frac{e^{ikr_0}}{r_0} e^{i(\vec{k}_i - \vec{k}_s)\bullet \vec{r}_j} f_j(\hat{k}_s, \hat{k}_i)$$

- Cross terms

$$\left\langle E_{sj} E_{sl}^* \right\rangle = \frac{1}{r_0^2} \left\langle f_j(\hat{k}_s, \hat{k}_i) f_l^*(\hat{k}_s, \hat{k}_i) e^{i\vec{k}_d \bullet (\vec{r}_j - \vec{r}_l)} \right\rangle \to 0$$

- Independent scattering

$$\left\langle |E_s|^2 \right\rangle = \sum_{j=1}^{N} \left\langle |E_{sj}|^2 \right\rangle = \frac{1}{r_0^2} \sum_{j=1}^{N} \left\langle \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 \right\rangle = \frac{1}{r_0^2} N \left\langle \left| f_j(\hat{k}_s, \hat{k}_i) \right|^2 \right\rangle$$

Time correlation

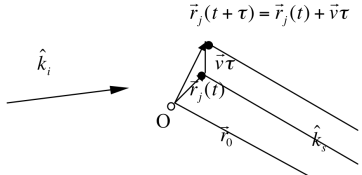
- Correlation function

$$C(\tau) = \left\langle E_s(t+\tau) E_s^*(t) \right\rangle$$

$$= \sum_{j=1}^{N} \left\langle E_{sj}(t+\tau) E_{sj}^*(t) \right\rangle = \frac{1}{r_0^2} \sum_{j=1}^{N} |f_j(\hat{k}_s, \hat{k}_i)|^2 \left\langle e^{i\vec{k}_d \bullet \vec{v}\tau} \right\rangle$$

- Mean and turbulent motion

$$\vec{v} = \vec{v}_0 + \vec{v}_1$$



- Time-correlation cross section

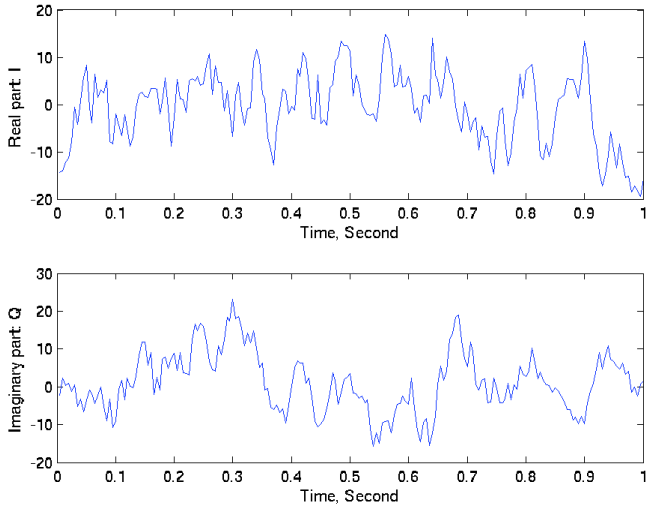
$$\left\langle \sigma_d(\hat{k}_s, \hat{k}_i; \tau) \right\rangle = \sigma_d(\hat{k}_s, \hat{k}_i) \left\langle e^{i\vec{k}_d \bullet \vec{v}\tau} \right\rangle = \sigma_d(\hat{k}_s, \hat{k}_i) e^{-k_d^2 \sigma_v^2 \tau^2 / 2} e^{i\vec{k}_d \bullet \vec{v}_0 \tau}$$

- Estimation of Doppler velocity and spectrum width

$$v_r = \angle C(T_s) / (k_d T_s)$$

$$\sigma_v = \left(2\ln(|C(0)|/|C(T_s)|)\right)^{1/2} / (k_d T_s)$$

Simulation of time-series radar data



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Simulation procedures for wave scattering by randomly distributed particles

- Specify radar constants
- Specify scattering amplitudes
- Generate random positions for each particle
- Calculate scattered field of each particle
- Calculate total scattered field
- Generate realization/time-series data
- Average to obtain coherent and incoherent intensity
- Correlation analysis to obtain Doppler velocity and spectrum width

Generation of Random Numbers (RN)

- By experiment
- By computer
  - Uniformly distributed RNs x(0,1)
  - rand(size): $z_n=az_{n-1}$ mod m $x_i=z_i/m$
  - Gaussian distributed RNs x(0,1)
  - randn(size)
  - Other RN generation

Realization and time sequences of random positions

- Realization: event

  $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

  Many events:

  $[\vec{r}]_1,[\vec{r}]_2,...,[\vec{r}]_{Nr}$
- Time: pulse

  $\left[\vec{r}\right]_{n+1} = \left[\vec{r}\right]_n + (\vec{v}_0 + \vec{v}_1)T_s$
- Averaging

Matlab code

- Try the matlab code and run you own simulations!