Simultaneous attenuation correction and rain estimation

- Attenuation correction algorithms developed were based on H-B solution which uses a A-Z relation with or without adjustment
- Rain estimation through empirical relations and DSD retrieval.
- DSD retrieval successful for non-attenuated data
- Integral equation approach used in dual-freq. technique

Analogy between dual-frequency and dual-polarization radar techniques

- Fundamentally the same with two most important measurements
  - Z and DWR for dual-freq.
  - Z and Z\textsubscript{DR} for dual-pol.
- Weak attenuation case
  - Un-attenuated Z and DWR linearly related to attenuation
  - Z and Z\textsubscript{DR} used to solve two parameter DSD
- Strong attenuation
  - Attenuation correction and DSD retrieval need to be done simultaneously
  - A-Z relation and integral equation
- References
Integral equations

- Reflectivity measured by a radar
  \[
  Z_{pp}^{(m)}(r, \lambda) = Z_{pp}(r, \lambda) e^{-0.46 \int_0^{\Lambda} A_p(\lambda, \lambda') d\lambda}
  \]

- Intrinsic reflectivity and attenuation
  \[
  Z_{pp}(r, \lambda) = \frac{4 \lambda^4}{\pi^4 |K_w|^2} \int |f_{pp}(\pi, D)|^2 \ N(D) dD
  \]
  \[
  A_p(r, \lambda) = 8.686 \lambda \int \text{Im} [f_{pp}(0, D)] N(D) dD
  \]

- Differential reflectivity
  \[
  Z_{DR}^{(m)}(r, \lambda) = 10 \log \frac{Z_{hh}^{(m)}(r, \lambda)}{Z_{vv}^{(m)}(r, \lambda)} = 10 \log \frac{Z_{hh}(r, \lambda)}{Z_{vv}(r, \lambda)} + 2 \int_0^\Lambda [A_H(l, \lambda) - A_V(l, \lambda)] dl
  \]

- Dual-wavelength ratio (DWR)
  \[
  DWR = 10 \log \frac{Z_{pp}^{(m)}(r, \lambda_1)}{Z_{pp}^{(m)}(r, \lambda_2)} = 10 \log \frac{Z_{pp}(r, \lambda_1)}{Z_{pp}(r, \lambda_2)} + 2 \int_0^\Lambda [A_p(l, \lambda_2) - A_p(l, \lambda_1)] dl
  \]

General equations for two parameter DSD

- Constrained gamma DSD model
  \[
  N(D) = N_0 D^\mu \exp(-\Lambda D)
  \]

  Fix \( \mu \) or use a \( \mu-\Lambda \) relation

- Dual-pol. integral equations to solve \( \Lambda \) and \( N_0 \)
  \[
  Z_{DR}(r) = Z_{DR}^{(m)}(r) + 2 \int_0^\Lambda N_0(l) [I_{AH}(l) - I_{AV}(l)] dl = 10 \log \left( \frac{I_{zh}(\Lambda)}{I_{zh}(\Lambda)} \right)
  \]
  \[
  Z_{hh}(r) = Z_{hh}^{(m)}(r) e^{-0.46 \int_0^\Lambda N_0(l) I_{ah}(l) dl} = N_0 C_Z I_h(\Lambda)
  \]

- Where
  \[
  I_{ah}(r) = 8.686 \lambda \int \text{Im} [f_{pp}(0, D)] D^\mu \exp(-\Lambda D) dD
  \]
  \[
  C_Z = \frac{4 \lambda^4}{\pi^4 |K_w|^2} \quad I_{zh}(\Lambda) = \int |f_{hh}(\pi, D)|^2 D^\mu \exp(-\Lambda D) dD
  \]
Forward and backward recursions

- **Forward recursion**
  \[ r: \text{starts from } r_0 \text{ to } r_N \]
  \[
  Z_{DR}(r) = Z_{DR}^{(m)}(r) + 2\int_{r_0}^{r} N_0(l) \left[ I_{AH}(l) - I_{AV}(l) \right] dl
  \]
  \[
  Z_{hh}(r) = N_0 C_{ZI} I_{ZI}(\Lambda)
  \]
  *Unstable due to error accumulation*

- **Backward recursion**
  \[ r: \text{starts from } r_N \text{ to } r_0 \]
  \[
  Z_{DR}(r) = Z_{DR}^{(m)}(r) + \left[ PIA_H - PIA_V \right] - 2\int_{r}^{r_0} N_0(l) \left[ I_{AH}(l) - I_{AV}(l) \right] dl
  \]
  \[
  Z_{hh}(r) = N_0 C_{ZI} I_{ZI}(\Lambda)
  \]
  *Stable due to PIA constrain*
Extended H-B solution

- Radar measurements with attenuation
  \[ Z(r) = Z^{(m)}(r) \left[ 1 - 0.46b \varepsilon \int_0^l aZ^{(m)b}(l)dl \right]^{-1/b} \]
  \[ \varepsilon = \left[ 1 - \exp(-0.23b \times PIA) \right] \left[ 0.46b \int_0^\infty aZ^{(m)b}(l)dl \right] \]

- Extended to dual-pol.
  \[ Z_{DR}(r) = Z_{DR}^{(m)}(r) - 10 \log \left[ Q_h^{1/h} / Q_v^{1/v} \right] \]
  \[ Z_{hh}(r) = N_0 C_Z I_{2h}(\Lambda) \]
  \[ Q = 1 - \left[ 1 - \exp(-0.23b \times PIA) \right] \int_0^r aZ^{(m)b}(l)dl \int_0^\infty aZ^{(m)b}(l)dl \]

Measurement bias effect

Fig. 4. Rain-rate estimates from the (top) kZ and (bottom) integral equation formulations for offset errors in the radar reflectivity factors.

Fig. 5. Estimates of median mass diameter from the (top) kZ and (bottom) integral equation formulations for offset errors in the radar reflectivity factors.
Conclusions

• Integral equation approach accounts for
  – Attenuation correction and
  – DSD variations
  – More accurate in the presence in DSD model or Z measurement error

• A-Z relation based solution similar to self-consistent method

• Dual-pol. and dual-freq. share the same methodology and equations