Phased Array Radar Polarimetry for Weather Sensing: A Theoretical Formulation for Bias Corrections

Guifu Zhang, Senior Member, IEEE, Richard J. Doviak, Life Fellow, IEEE, Dusan S. Zrnić, Fellow, IEEE, Jerry Crain, David Staiman, Member, IEEE, and Yasser Al-Rashid

Abstract—It is becoming widely accepted that radar polarimetry provides accurate and informative weather measurements, while phased-array technology can shorten data updating time. In this paper, a theory of phased array radar (PAR) polarimetry is developed to establish the relation between electric fields at the antenna of the PAR and the fields in a resolution volume filled with hydrometeors. It is shown that polarimetric measurements with an electronically steered beam can cause measurement biases that are comparable to or even larger than the intrinsic polarimetric characteristics of hydrometeors. However, these biases are correctable if the transmitted electric fields are known. A correction to the measured scattering matrix that removes biases in meteorological variables is derived. The challenges and opportunities for weather sensing with a polarimetric PAR are discussed.

Index Terms—Atmospheric measurements, phased array radar (PAR), polarimetry, remote sensing, scattering.

I. INTRODUCTION

PHASED array radar (PAR) technology has been successfully utilized in the military for more than five decades [1]. Its agile electronic beam steering capability enables fast tracking of multiple targets. Recent advancements in microwave technology have made phased-array antennas more affordable, and the weather radar community has started to invest time and resources into phased-array technology [2]–[4]. Through a joint effort by a government/university/industry team, the nation’s first phased-array weather radar, the National Weather Radar Testbed (NWRT), was developed in Norman, OK [3]. It has been demonstrated that its pulse-to-pulse beam steering capability enables accurate meteorological measurements in a shorter dwell time than with a mechanically steered beam, resulting in faster data updates [4]. The higher temporal resolution (< 1 min versus 6 min) than that obtained scanning a storm with a mechanically steered beam has revealed detailed evolutions of severe storm phenomena such as the microburst [4]. Given this advantages, it is envisioned that polarimetric PAR (PPAR) has the potential to significantly advance weather observations.

Meanwhile, weather radar polarimetry has matured to a point that it is being implemented on the national network of WSR-88D Doppler radars [5]–[9]. The polarimetric WSR-88D (i.e., WSR-88DP) will simultaneously transmit and receive (i.e., the simultaneously transmitting and simultaneously receiving (STSR) mode) horizontally (H) and so-called “vertically” (V) polarized waves (the V electric field lies in a vertical plane, but is only vertical at the 0° elevation angle). The WSR-88DP measures the following polarimetric variables: the H-reflectivity factor ($Z_h$) associated with H-polarized backscatter for the transmitted H-polarized wave; the V-reflectivity factor ($Z_v$) (from which is derived the differential reflectivity $Z_{DR}$); the total differential phase ($\phi_{DP}$) (from which is derived the specific differential phase $\kappa_{DP}$); and the copolar cross-correlation coefficient ($\rho_{hv}$). Polarimetric radars that alternately transmit but simultaneously receive (i.e., alternately transmitting but simultaneously receiving (ATSR)) H and V electric fields can measure all components of the backscattering covariance matrix including the linear depolarization ratio (LDR) and the co-/cross-polarization correlations [5, Sec. 8.5.2]. However, this is done at the expense of increased errors in estimates of the polarimetric variables [8]. It is not clear at this time whether the PPAR should simultaneously or alternately transmit H, V waves. This is the subject of an ongoing investigation.

Polarimetric radar variables depend on the size, shape, orientation, density, and composition of the hydrometeors, allowing for better characterization of precipitation types and amounts. Polarimetric measurements have been successfully used to classify hydrometeor species and nonweather objects using fuzzy-logic algorithms [10]–[12]. The accuracy of quantitative precipitation estimation is significantly improved from an estimation error of about 40% when using only reflectivity (i.e., $Z_h$ or $Z_v$); henceforth, the reflectivity factor will be called reflectivity, not to be confused with the backscattered cross section per unit volume, which is also called reflectivity) to an estimation error of less than 15% when using polarimetric radar data [13]–[15]. Furthermore, the added polarimetric information
enables better weather forecasts through improved microphysical parameterization and initialization of numerical weather prediction models [16]–[18]. By combining polarimetric information with the fast-scan capabilities of the PAR, it becomes possible to provide scientists with data that could lead to a more detailed understanding of the microphysics and dynamics of clouds and precipitation [19]. A theoretical study has been performed to better understand the properties of PPAR [20]. Engineering efforts are also under way to design a prototype PPAR [21], [22].

Meteorologically useful information is obtained from the polarimetric variables $Z_{\text{DR}}$, $K_{\text{DP}}$, and $\rho_{\text{hv}}$ only because of the differences (amplitude and phase) of the H and V electric fields scattered by hydrometeors illuminated with H and V waves. The differences are usually very small because hydrometeor shapes projected onto the plane of polarization are, on average, almost circular. Thus, a high-accuracy measurement of polarimetric variables is required to provide meaningful information for reliable hydrometeor classification and improved quantitative precipitation estimation. For example, $Z_{\text{DR}}$ values range only from about 0.1 dB for drizzle and dry snow to 3–4 dB for heavy rain and large drops. Thus, it is desirable that the measurement error for $Z_{\text{DR}}$ be on the order of 0.1 dB. In addition, because copolar cross-correlation coefficients change little from one species of hydrometeors to another (0.99 for rain and 0.95 for melting snow), it is desirable that $\rho_{\text{hv}}$ error be less than 0.01.

There are other challenges unique to the PPAR. For example, cross-polar coupling of H, V in beam directions away from broadside (i.e., normal to the array face) can cause errors larger than the values specified previously. This is not a problem for radars with mechanically steered antennas that maintain their sense of polarization (i.e., circular, linear, etc.) irrespective of the beam direction [21]. Therefore, it is important to quantify the polarization characteristics of the PPAR and hydrometeors so that we can separate their respective effects to obtain data useful and meaningful to meteorologists. This procedure can also be useful in correctly interpreting data from polarimetric synthetic aperture radars [23].

In Section II, a theory for PPAR is formulated, and the relation between the electric fields at the array element and at the hydrometeors is formulated based on the assumption that radiation from each element is equivalent to that generated by a pair of orthogonal dipoles. By removing the effects of cross-polar coupling for beams directed away from broadside, the correction for biases is developed in Section III to convert the measured apparent scattering matrix to the hydrometeor’s intrinsic scattering matrix. In Section IV, the PPAR variables of reflectivity ($Z_{\text{DR}}^{(p)}$), differential reflectivity ($Z_{\text{DR}}^{(p)}$), LDR ($LDR_{h,v}^{(p)}$), and copolar cross-correlation coefficient ($\rho_{\text{hv}}^{(p)}$) are introduced. The polarimetric variables’ biases are quantified and compared with those variables for polarimetric radar having no cross-polar coupling. The conclusion of this paper and discussions for future research and implementation are provided in Section V.

II. FORMULATION

A phased-array antenna is composed of elements whereby each is treated as a pair of crossed dipoles. In this section, a theory relating the electric fields at the PPAR’s antenna elements and at the hydrometeors is formulated through the processes of radiation, scattering, and propagation.

A. Dipole Radiation

A coordinate system (Fig. 1) is chosen with a pair of Hertzian dipoles located at its origin and with the PPAR array face in the $y, z$ plane. A resolution volume $V_{0}$ [5, Sec. 4.4.4] filled with hydrometeors at range $r$ lies along the beam direction $(\theta, \phi)$. Unit vectors $a_{r}$, $a_{\theta}$, and $a_{\phi}$ (denoted by bold text) form a local orthogonal system at $r$ that can be expressed in terms of the Cartesian coordinates as

$$a_{r} = a_{x} \sin \theta \cos \phi + a_{y} \sin \theta \sin \phi + a_{z} \cos \theta$$

$$a_{\theta} = a_{x} \cos \theta \cos \phi + a_{y} \cos \theta \sin \phi - a_{z} \sin \theta$$

$$a_{\phi} = -a_{x} \sin \phi + a_{y} \cos \phi.$$  

The electric field radiated by a dipole of moment $\vec{M}$ is [24, Sec. 2.4]

$$E_{q}(r) = -\frac{k^{2}e^{-jkr}}{4\pi\varepsilon r} \left\{ a_{r} \times [a_{r} \times \vec{M}] \right\},$$

in volts per meter

where $k = 2\pi/\lambda$, $\lambda$ is the radar wavelength, $\varepsilon$ is the permittivity for an assumed uniform precipitation-free atmosphere, and the dipole moment

$$\vec{M}_{q} = a_{q} A_{q} \exp(j\phi_{q}),$$

in coulomb meters

has amplitude $A_{q}$, phase $\phi_{q}$, and direction $a_{q}$, which is the unit vector along which the dipole moment is directed. The subscript $q$ (1 or 2) denotes the $q$th dipole.

Writing (2) in terms of spherical components

$$\vec{E}_{q}(r) = a_{0} E_{\theta q} + a_{\phi} E_{\phi q},$$

where $E_{\theta q}$ (i.e., the V wave) and $E_{\phi q}$ (i.e., the H wave) are the two spherical components of the wave radiated by the $q$th dipole.

The horizontal dipole ($\vec{M}_{1} = M_{1} a_{y}$) is aligned with the $y$-axis, and the vertical dipole ($\vec{M}_{2} = M_{2} a_{x}$) is aligned with the $z$-axis. After applying a vector identity to the cross-products in (2), the electric field at $\vec{r}$ from $\vec{M}_{1}$ and that from $\vec{M}_{2}$ are, respectively

$$\vec{E}_{1} = E_{11} [a_{y} - (a_{x} \sin \theta \cos \phi + a_{y} \sin \theta \sin \phi + a_{z} \cos \theta) \sin \theta \sin \phi] = E_{11} \vec{e}_{1}$$

$\vec{a}$
\[ E_2 = E_{12} \left[ a_x \sin^2 \theta - (a_x \cos \phi + a_y \sin \phi) \sin \theta \cos \theta \right] \]

where

\[ E_{12} = \frac{k^2 e^{-jkr}}{4\pi \varepsilon r} M \]

is the magnitude of the electric field transmitted along the normal to the array broadside. Vector \( \vec{e}_1 \) gives the direction of \( M_1 \)'s electric field propagating along the \((\theta, \phi)\) direction, and its magnitude \( |\vec{e}_1| \) is the fraction of the corresponding electric field propagating along the normal to the array.

Projections of \( \vec{e}_1, \vec{e}_2 \) onto the local horizontal \((a_h = a_0)\) and “vertical” \((a_v = -a_0)\) directions yield

\[ a_h \cdot \vec{e}_1 = a_0 \cdot \vec{e}_1 = \cos \phi \]

\[ a_v \cdot \vec{e}_1 = -a_0 \cdot \vec{e}_1 = -\cos \theta \sin \phi \]

\[ a_h \cdot \vec{e}_2 = a_0 \cdot \vec{e}_2 = 0 \]

\[ a_v \cdot \vec{e}_2 = -a_0 \cdot \vec{e}_2 = \sin \theta. \]

Note that 1) the intensities of the H [from (6a)] and V [from (6b) and (6d)] waves are functions of beam direction \((\theta, \phi)\) and 2) if the beam is directed away from the intersection of the equatorial planes of the respective dipoles, the horizontal dipole \( M_1 \) produces a “vertical” component given by (6b). This is the reason why the PPAR transmits cross-polar components, increasing in intensity with direction from the cardinal planes. This yields polarization biases that are not significant when using a mechanically steered beam. Thus, the incident electric fields \( \vec{E}_1 \) and \( \vec{E}_2 \) generated by dipoles 1 and 2 and projected onto the local H and V directions, give the incident H, V electric fields

\[ \vec{E}_1 = \begin{bmatrix} E_{1h} \\ E_{1v} \end{bmatrix} = P \begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} = P \vec{E}_1 \]  

where \( P \) is the matrix

\[ P = \begin{bmatrix} a_h \cdot \vec{e}_1 & a_h \cdot \vec{e}_2 \\ a_v \cdot \vec{e}_1 & a_v \cdot \vec{e}_2 \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 \\ -\cos \theta \sin \phi & \sin \theta \end{bmatrix} \]

that projects the oblique \( \vec{E}_1 \) and \( \vec{E}_2 \) onto the local H and V coordinates. Because there is no conversion of one polarized field into the other in a precipitation-free atmosphere, the projection matrix can be applied at any range. However, in precipitation-filled atmospheres, there is conversion, and it is important that the projection matrix be applied before the wave enters the precipitating medium. As we will see later, “local” will imply at the beginning of the precipitation.

### B. Backscattering Matrix

The backscattered electric field \( \vec{E}_s \) at the receiving array element (assumed to be the same as the transmitting array element) can be expressed as [5, Sec. 8.5.2.1]

\[ \vec{E}_s \equiv \begin{bmatrix} E_{sh} \\ E_{sv} \end{bmatrix} = S^{(b)} \vec{E}_1 \times \frac{\exp(-jkr)}{r} \]

where \( S^{(b)} \) is the intrinsic backscatter matrix of a hydrometeor

\[ S^{(b)} = \begin{bmatrix} s_{hh}^{(b)} \\ s_{hv}^{(b)} \\ s_{vh}^{(b)} \\ s_{vv}^{(b)} \end{bmatrix}. \]

Although (8a) gives the H, V electric fields at the receiving array element, we need to determine the fields parallel to the respective dipole axes because these fields enter the H, V channels of the receiver. The fields parallel to the dipole axes are obtained by projecting \( \vec{E}_s \) onto the respective dipole directions, and these projections are given by

\[ \vec{E}_t \equiv \begin{bmatrix} E_{1t} \\ E_{2t} \end{bmatrix} = \begin{bmatrix} \cos \phi & -\cos \theta \sin \phi \\ 0 & \sin \theta \end{bmatrix} \begin{bmatrix} E_{sh} \\ E_{sv} \end{bmatrix} = P^t \vec{E}_s \]

where \( P^t \) is the transpose of \( P \). By combining (7)–(9), \( \vec{E}_t \) can be expressed as

\[ \vec{E}_t = P^t \vec{E}_s = P^t S^{(b)} P \vec{E}_1 \times \frac{\exp(-jkr)}{r} \]

\[ = S^{(p)} \vec{E}_t \times \frac{\exp(-jkr)}{r} \]

where \( S^{(p)} = P^t S^{(b)} P \) is the PPAR scattering matrix, including the polarimetric effects associated with the hydrometeor as well as that due to the PPAR beam direction.

### C. Propagation and Cross-Polarization Effects

If differential attenuation, differential phase shift, and attenuation along the path are significant, differential transmission effects need to be accounted for. For wave propagation in the atmosphere containing hydrometeors, the cross-coupling of the H, V fields is usually negligible, and the transmission matrix accounting for the extra phase shift and attenuation due to the hydrometeors is [5], [7], [25]

\[ T = \begin{bmatrix} \exp \left\{ -j \int_0^r \delta k_h(r) dr \right\} & 0 \\ -j \int_0^r \delta k_v(r) dr & \exp \left\{ -j \int_0^r \delta k_v(r) dr \right\} \end{bmatrix} \]

\[ = \begin{bmatrix} T_{hh} & 0 \\ 0 & T_{vv} \end{bmatrix} \]

where

\[ \delta k_{h,v} = \frac{2\pi}{k} \int_0^{\infty} n(D) s_{h,v}(D) dD, \] per meter

are the increments of the one-way specific differential wavenumbers (radians per meter) added to the propagation wavenumber \( k \) of the precipitation-free atmosphere, \( n(D) \) is the hydrometeor size distribution, and \( s_{h,v}(D) \) are the complex forward scattering coefficients of the hydrometeor having an equivalent volume diameter \( D \) [5, Sec. 8.5.2.4]. In this case, (10), extended to include the transmission matrix, should be written as

\[ \vec{E}_t = P^t T S^{(b)} P \vec{E}_1 \times \frac{\exp(-jkr)}{r} \]

By combining the transmission matrix \( T \) with \( S^{(b)} \) (i.e., \( S' = TS^{(b)} T \)), (12) reduces to

\[ \vec{E}_t = P^t S' \vec{E}_1 \times \frac{\exp(-jkr)}{r} \equiv S^{(p)} \vec{E}_1 \times \frac{\exp(-jkr)}{r} \]
where \( \mathbf{S}^{(p)} \), shown at the bottom of the page, is the backscatter matrix for the H and V channels of the PPAR; \( \mathbf{S}^{(p)} \) now includes cross-coupling of the fields during propagation as well as in backscatter. The elements of the scattering matrix \( \mathbf{S}' \) (i.e., with propagation effects) are related to the elements of \( \mathbf{S}^{(p)} \) as:

\[
\mathbf{S}' = \begin{bmatrix}
T_{hh}^2 s_{hh}^{(b)} & T_{hh} T_{V V} s_{hv}^{(b)} \\
T_{hh} T_{V V} s_{hv}^{(b)} & T_{VV}^2 s_{VV}^{(b)}
\end{bmatrix} \equiv \begin{bmatrix}
s_{hh}' & s_{hv}' \\
s_{hv}' & s_{VV}'
\end{bmatrix}.
\] (14b)

Therefore, the propagation effects of attenuation and extra phase shifts are taken into account by using the transmission matrix \( \mathbf{T} \), which is a function of \( \delta k_{hv} \), with a real part related to the extra phase shift and an imaginary part associated with the attenuation.

Further analysis of (14a) indicates that the PPAR not only causes bias in copolar measurements (diagonal terms) but also generates extra cross-polarization terms (i.e., the off-diagonal terms). As a simple example, consider the case where \( s_{hv} = s_{vh} = 0 \), whereby hydrometeors do not introduce cross-coupling, either along the propagation path or in backscatter. In this case, the cross-term \( s_{hv}^{(p)} = -s_{vh}^{(p)} \), with a real part related to the extra phase shift and an imaginary part associated with the attenuation.

III. SCATTERING MATRIX CORRECTIONS

There are two polarimetric transmitting/receiving modes currently in use by the weather radar community. They are 1) the ATSR and 2) the STSR modes. Using these as examples, we demonstrate the bias correction to the scattering matrix measured with a PPAR.

A. ATSR Mode

In this mode, all four elements of \( \mathbf{S}^{(p)} \) can be calculated using (13) and measurements of the transmitted and received fields. For a mechanically steered beam, \( \mathbf{P} \) is a unit matrix, and \( \mathbf{S}' = \mathbf{S}^{(p)} \) is directly recovered. If a PPAR is used, \( \mathbf{S}' \) can still be recovered, but in this case, it is necessary to multiply the calculated \( \mathbf{S}^{(p)} \) with the inverse of the projection matrices. That is

\[
\mathbf{S}' = (\mathbf{P}^t)^{-1} \mathbf{S}^{(p)} \mathbf{P}^{-1}.
\] (15)

Equation (15) shows that \( \mathbf{S}' \), obtained with a PPAR, is mathematically equivalent to a mechanically steered beam. Such equivalency allows us to use results obtained for mechanically steered beams to calculate \( \mathbf{S}^{(p)} \), the matrix that is of prime interest. Write (15) as

\[
\mathbf{S}' = \mathbf{C} \mathbf{S}^{(p)} \mathbf{C}
\] (16)

where we have defined a correction matrix

\[
\mathbf{C} = \mathbf{P}^{-1} \begin{bmatrix}
\frac{1}{\cos \phi} & 0 \\
\frac{\sin \phi}{\sin \phi} & 1
\end{bmatrix}.
\] (17)

Substituting (14a) and (17) into (16) yields \( \mathbf{S}' \), indicating that the polarization biases incurred by PPAR are fully correctable using the matrix \( \mathbf{C} \).

B. Simultaneous Transmission

Because we measure both components of \( \vec{E}_t \) when both components of \( \vec{E}_r \) are simultaneously transmitted, we cannot determine, using STSR, all four elements of \( \mathbf{S}^{(p)} \). Nevertheless, if there is negligible coupling between the H and V waves, as is common, then we can determine the two main diagonal components of \( \mathbf{S}' \). Writing (13) in matrix form, we have (18) or (19), shown at the bottom of the page. Because both \( \vec{E}_t \) and \( \vec{E}_r \) are known through measurement and calibration, the two equations in (19) are used to solve \( s_{hv} \) and \( s_{vh} \).

It has been shown that the polarization bias can be corrected by rectifying the scattering matrix in either the ATSR or STSR mode. Although it is not necessary to force the transmitted H and V waves to be equal, they do need to be known through measurement and calibration. That is, corrections can be made if the transmitted field magnitudes and phases are known, and received fields are accurately measured along with the electronic beam position.

IV. POLARIMETRIC VARIABLES

To obtain a quantitative assessment of the effects that an electronically steered beam has on weather radar measurements, we compare the polarimetric radar variables measured by a PPAR with those by radar using a mechanically steered beam. In the following sections, the intrinsic scattering matrix is assumed to be diagonal, a valid assumption for most meteorological conditions. However, the developed formulation in Sections II and III describes the more general case.
A. Reflectivity Factor

The H, V intrinsic reflectivity factors (i.e., that for backscatter from a resolution volume $V_0$ illuminated with incident H and V waves) are, in the Rayleigh approximation, given by [5, Sec. 8.5]

$$Z_{h,v} = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{hh,vv}^{(b)} \right|^2$$  \hspace{1cm} (20)

where

$$\left| s_{hh,vv}^{(b)} \right|^2 = \int p(D) \left| s_{hh,vv}^{(b)}(D) \right|^2 dD.$$  \hspace{1cm} (21a)

$N$ is the number density of scatters per unit volume, and $p(D)$ is the probability density of scatterers’ diameters. Although $p(D)$ has the same functional dependence on $D$ as does the drop size distribution $n(D)$, it is not equivalent to it; $p(D)$ is normalized so that the integral of $p(D)$ over all possible values of $D$ is one, whereas $n(D)$ is normalized to the drop diameter and unit volume [5, Sec. 4.4]. The drop size distribution, however, can be used to compute $p(D)$. That is

$$p(D) = \frac{n(D)}{\int_0^\infty n(D)dD} = \frac{n(D)}{N}.$$  \hspace{1cm} (21b)

If precipitation fills the resolution volumes along the propagation path, the H, V reflectivity factors $Z_{h,v}^\prime$ measured by the radar using a mechanically steered beam are

$$Z_{h,v}^\prime = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{hh,vv}^{(b)} \right|^2.$$  \hspace{1cm} (22)

It can be shown that the intrinsic reflectivity factors $Z_{h,v}$ and those $Z_{h,v}^\prime$ that include propagation effects are related by

$$Z_{H,V} = Z_{H,V} - 2 \int_0^r A_{H,V}(r)dr \text{ (dBZ).}$$  \hspace{1cm} (23)

The one-way specific attenuation $A_{H,V}(r)$ (in decibels per meter) is related to $\delta k_{h,v}$ as $A_{H,V} = 8.686 \text{ Im}[\delta k_{h,v}]$ (lowercase subscripts are used to designate a linear scale, whereas the uppercase subscripts are used to denote a decibel scale).

Following the same definitions as (20) and (22), the reflectivity factors for the PPAR are

$$Z_{h}^{(p)} = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{hh}^{(p)} \right|^2 = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{hh} \cos^2 \phi + s_{hv} \cos^2 \theta \sin^2 \phi \right|^2$$

$$= Z_{h}^{\prime} \cos^4 \phi + Z_{v}^{\prime} \cos^4 \theta \sin^4 \phi + \frac{1}{2} \left[ Z_{h}^{\prime} Z_{v}^{\prime} \text{Re} \left[ \rho_{hv} \right] \cos^2 \theta \sin^2 2\phi \right.$$

$$\left. + Z_{h}^{\prime} Z_{v}^{\prime} \text{Im} \left[ \rho_{hv} \right] \sin^2 \theta \sin^2 2\phi \right]$$  \hspace{1cm} (24a)

for horizontal polarization ($\rho_{hv}$ is defined in Section IV-C) and

$$Z_{v}^{(p)} = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{vv}^{(p)} \right|^2 = \frac{4\lambda^4 N}{\pi^4 |K_w|^2} \left| s_{vv} \right|^2$$

$$= Z_{v}^{\prime} \sin^4 \theta$$  \hspace{1cm} (24b)

for vertical polarization. It can be seen that reflectivity factors are biased low compared to those for a mechanically steered beam. This is due to the fact that scatterers located along directions other than the direction to the intersection of the dipoles’ equatorial planes experience weaker fields. The bias correction for $Z_{v}^{(p)}$ is simpler than that for $Z_{h}^{(p)}$ which depends on both $Z_{h}^{\prime}$ and $Z_{v}^{\prime}$ as well as $\rho_{hv}$.

B. Differential Reflectivity

Differential reflectivity ($Z_{DR}$) is a good indicator of hydrometeor oblateness. The differential reflectivity measured with a mechanically steered beam is the intrinsic differential reflectivity minus the path-integrated two-way differential attenuation [6], [7]

$$Z_{DR}^{\prime} = 10 \log \left\{ \frac{\left| s_{hh}^{(b)} \right|^2}{\left| s_{vv}^{(b)} \right|^2} \right\}$$

$$= 10 \log \left\{ \frac{\left| s_{hh}^{(b)} \right|^2}{\left| s_{hv}^{(b)} \right|^2} \right\} \exp \left\{ -2 \int_0^r \alpha_{dp}(r)dr \right\}$$

$$= Z_{DR} - 2 \int_0^r 4.343 \alpha_{dp}(r)dr \hspace{1cm} (25a)$$

where it can be shown that

$$\alpha_{dp}(r) = 2 \text{ Im}[\delta k_{h} - \delta k_{v}] \hspace{1cm} (25b)$$

is the one-way specific differential attenuation. The factor 2 in the exponent is required because the measured reflectivity is attenuated by losses along the path to and from the resolution volume. Substitution of (11b) into (25) gives

$$Z_{DR}^{\prime} = Z_{DR} - 2 \int_0^r A_{DP}(r)dr \hspace{1cm} (25c)$$

where

$$A_{DP} = 8.686 \text{ Im}[\delta k_{h} - \delta k_{v}]$$

$$= 8.686 \lambda \int_0^\infty n(D) \text{ Im} [s_{h}(0,D) - s_{v}(0,D)] dD \hspace{1cm} (25d)$$

is the one-way specific differential attenuation in decibels per meter.

The differential reflectivity measured with PPAR is

$$Z_{DR}^{(p)} = 10 \log \left\{ \frac{\left| s_{hh}^{(b)} \right|^2}{\left| s_{vv}^{(b)} \right|^2} \right\} \hspace{1cm} (26)$$
In the case of the ATSR mode, the PPAR measured differential reflectivity $Z_{\text{DR}}$ (ATSR) is related to the differential reflectivity $Z'_{\text{DR}}$ measured with a mechanically steered beam as

$$Z_{\text{DR}} \, (\text{ATSR}) = Z'_{\text{DR}} + 10 \log \frac{a^2 + b^2 Z_{dr}^{-1} + 2ab Z_{dr}^{-1/2} Re(\rho_{hv})}{c^2} \tag{27a}$$

$$Z_{\text{DR}} \, (\text{ATSR}) = Z'_{\text{DR}} + Z_{\text{DR}} \, \text{Bias(ATS)} \tag{27b}$$

where $a = \cos^2 \phi$, $b = \cos^2 \theta \sin^2 \phi$, and $c = \sin^2 \theta$. Thus, biases incurred with PPAR measurements using the ATSR mode can be corrected by subtracting $Z_{\text{DR}} \, \text{Bias(ATS)}$ from the measured $Z_{\text{DR}}$ (ATSR), yielding the conventional (i.e., that measured with a mechanically steered beam) differential reflectivity $Z'_{\text{DR}}$. However, $Z'_{\text{DR}}$ is coupled with $\rho_{hv}$ in (27), and they need to be solved jointly. This is mathematically more complicated than directly correcting the scattering matrix presented in Section III.

Fig. 2 shows $Z_{\text{DR}} \, \text{Bias(ATS)}$. This bias is plotted as a function of the azimuth angle $\phi$ using $\theta$ as a parameter. The negative bias increases for $|\phi| > 0$, and the bias can be positive for small azimuth and zenith angles. The biases are due to the fact that H, V radiation from an array element changes for small azimuth and zenith angles. The biases are due to the fact that H, V radiation from an array element changes for small azimuth and zenith angles. The biases are due to the fact that H, V radiation from an array element changes for small azimuth and zenith angles.

If the STSR mode is used, biases are different and more complicated than those for the ATSR mode. The measured differential reflectivity can be obtained by using (19) with the transmitted wave fields related by $E_{\epsilon2} = \gamma E_{\epsilon1} e^{i\psi}$, where $\gamma$ is the amplitude ratio of the transmitted electric fields and $\psi$ is their relative phase. Then, $Z_{\text{DR}} \, (\text{STSR})$ is defined as shown at the bottom of the page, where $b_s = b - \gamma e^{i\phi} d$, $c_s = \gamma e^{i\psi} \sin^2 \theta - d$, and $d = \sin \theta \cos \theta \sin \phi$. $Z_{\text{DR}} \, \text{Bias(STS)}$ depends not only on the beam direction but also on the amplitude ratio and the relative phase of transmitted waves, as well as the hydrometeor characteristics.

Fig. 3 shows $Z_{\text{DR}} \, \text{Bias(STS)}$. Results for transmitting equal H and V amplitudes (i.e., $\gamma = 1$) are shown in Fig. 3(a). It can be seen that $Z_{\text{DR}} \, \text{Bias(STS)}$ is not symmetric about the $x-z$ plane because the V dipole contributes to the $E_{\epsilon2}$ field, if $\theta < \pi/2$, in phase with that of the H dipole for $-\pi/2 < \phi < 0$, whereas the contribution is out of phase with the H dipole’s contribution for $0 < \phi < \pi/2$, as shown by (14a) and (19). However, the projection of the V dipole field into the $E_{\epsilon2}$ direction has the same magnitude for symmetrical azimuths, and that of the H dipole is different. Thus, $Z_{\text{DR}}$, proportional to the ratio of $E_{\epsilon1}$ to $E_{\epsilon2}$, is different for negative $\phi$’s from that for equal positive $\phi$’s. The results for unequal transmitted H and V fields are shown in Fig. 3(b) and (c) for various amplitude ratios $\gamma$ and phase differences $\psi$. The amplitude ratios of 0.9441, 0.9716, 1.0292, and 1.0593 correspond to $-0.5$,$-0.25$,$0.25$, and 0.5-dB differences in power. The power imbalance causes $Z_{\text{DR}} \, \text{Bias(STS)}$ in a similar way as that of the elevation angle because they both yield relative differences in the projection of the transmitted fields to the local polarization directions. It is interesting to note that $Z_{\text{DR}} \, \text{Bias(STS)}$ also depends on the relative phase of the transmitted fields, and it becomes antisymmetric when $\psi = 180^\circ$. This is because the polarization of the transmitted fields changes, depending on the relative phase. The differential reflectivity can also be corrected through the PPAR estimated value $Z_{\text{DR}}^{\text{est}}$ by inverting (28) to solve for $Z'_{\text{DR}}$ if $\rho_{hv}$ is known, but the power imbalance needs to be known within 0.1 dB and the relative phase within a few degrees.

C. Correlation Coefficient

The copolar cross-correlation coefficient is defined as

$$\rho_{hv} = \frac{\left\langle s_{hh}^{(b)} s_{vv}^{(b)} \right\rangle}{\sqrt{\left\langle s_{hh}^{(b)} \right\rangle^2 \left\langle s_{vv}^{(b)} \right\rangle^2}} \tag{29}$$

When propagation effects are taken into account, the copolar cross-correlation coefficient measured with a mechanically
steered beam is

$$
\rho_{hv}' = \frac{(s_{hh}' s_{vv}')}{(s_{hh} s_{vv})} = e^{j\phi_{DP}} \rho_{hv}
$$

with the contribution from the differential phase \( \varphi_{DP} = 2 \int_0^r \text{Re}[\delta k_h(r) - \delta k_v(r)]dr. \)

In the case of an ATSR PPAR, the computation of the correlation coefficient produces

$$
\rho_{hv}(\text{ATSR}) = \frac{\langle s_{hh} \rangle \langle s_{vv} \rangle}{\sqrt{\langle s_{hh}^2 \rangle \langle s_{vv}^2 \rangle}}
\frac{\langle (s_{hh}' \cos^2 \phi + s_{vv}' \cos^2 \theta \sin^2 \phi) s_{vv}' \sin^2 \theta \rangle}{\sqrt{\langle |s_{hh}'|^2 \rangle \langle |s_{vv}'|^2 \rangle}}
= \frac{a \rho_{hv} + b (Z_{dr}')^{-1/2}}{\sqrt{a^2 + b^2 (Z_{dr}')^{-1/2}} + 2ab(Z_{dr}')^{-1/2}} \text{Re}(\rho_{hv}')
$$

where \( a \) and \( b \) are defined after (27). Fig. 4 shows the \( \rho_{hv} \) (ATSR) dependence on the electronically steered beam direction. This figure shows that the bias for an intrinsic \( \rho_{hv} = 0.9 \) is smaller than 0.02. Calculations, not given here, show that the bias is even smaller if \( \rho_{hv} \) is larger, as is the case for most precipitation.

If the STSR mode is used, the correlation coefficient is

$$
\rho_{hv}(\text{STSR}) = \frac{\langle (as_{hh}' + bs_{vv}') s_{vv}' \rangle \langle (as_{hh}' + bs_{vv}') \rangle}{\sqrt{\langle |as_{hh}' + bs_{vv}'|^2 \rangle \langle c_s s_{vv}'^2 \rangle}}
= \frac{a \rho_{hv} + b (Z_{dr}')^{-1/2}}{\sqrt{a^2 + b^2 (Z_{dr}')^{-1/2}} + 2ab(Z_{dr}')^{-1/2}} \text{Re}(b \rho_{hv}')
$$

where \( a, b_s, \) and \( c_s \) are given after (28). It can be seen that the bias of \( \rho_{hv} \) (STSR) depends on the beam direction, relative
amplitude, and phase of the transmitted H, V fields and on the scattering properties of the hydrometeors.

Fig. 5 shows $\rho_{hv}$ (STSR) as functions of beam direction as well as amplitude and phase imbalance. The $\rho_{hv}$ (STSR) measurement can be substantially biased as is $Z'_{dr}$. The bias can be either positive or negative, depending on the beam position, balanced factors, and hydrometeor properties; this is due to the change in the relative strength of horizontal and vertical wave fields. The bias, however, can be corrected through either the scattering matrix or polarimetric variables. The bias correction for the polarimetric variables is done by jointly solving (28) and (32) for $Z'_{dr}$ and $\rho'_{hv}$.

D. LDR

The LDR cannot be measured if the STSR mode is used. However, if observations are made with the ATSR mode, we can calculate LDR and relate it to the backscattering matrix elements $s_{hv}^{(b)}$ and $s_{vh}^{(b)}$. By assuming $s_{hv}^{(b)} = s_{vh}^{(b)} = 0$, the measured LDR is strictly the LDR bias for PPAR. Thus, it can be shown that $\text{Bias}(\text{LDR})$ is

$$
\text{Bias}(\text{LDR})_{hv} \equiv 10 \log \left( \frac{d^2}{c^2} \right)
$$

Bias($\text{LDR}$)$_{hv}$ (Fig. 6) increases as the beam points away from broadside, meaning large system effect. Bias($\text{LDR}$)$_{hv}$ is a few decibels larger than Bias($\text{LDR}$)$_{v}$ because the copolar power ($Z_{hv}^{(p)}$) for horizontal polarization is lower than that for vertical polarization in the directions of a large $|\phi|$. The system (LDR) can be up to $-15$ to $-10$ dB for the $25^\circ$ elevation and $45^\circ$ azimuth span needed for weather observations. This is much larger than that required to make meaningful meteorological observations (i.e., typically hydrometeor LDRs $<-20$ dB). Fortunately, this is amendable through calibrating the scattering matrix.

E. Polarization Correction

The aforementioned biases are correctable either by applying the correction matrix $C$ and its transpose to the scattering matrix

$$
\text{Bias}(\text{LDR})_{h,v} \equiv 10 \log \frac{d^2}{c^2}
$$

(33a)

Bias($\text{LDR}$)$_{h,v}$
The off-diagonal term in the polarization of the dipoles to the so-called vertical and horizontal polarization respectively, compensating the projection loss of wave field by known PPAR characteristics. The correction matrix elements is straightforward, and the correction induced by the horizontal dipole. The calculation of the correction matrix tailored to fit the radiation patterns of the actual array element. Corrections can be performed using a correction matrix (Section III) or the estimated polarimetric variables (Section IV), provided that the amplitudes and the relative phase of the transmitted wave fields are known. The procedure can be extended to correct any system bias/coupling by replacing the projection matrix (7) with a corresponding error matrix defined in [26] and [27]. For radars using mechanically steered beams and transmitting equal amplitude H, V waves, knowledge of the relative transmitter phase is not necessary. However, in general, correcting the transmitter power in each of the two polarizations to balance the two is not necessary if both amplitude and phase are known.

For radars that alternately transmit but simultaneously receive (ATSR) both polarizations, the intrinsic scattering matrix can be fully recovered, allowing for the measurement of the full covariance matrix. The correction matrix method for PPAR calibration presented here for a dipole element has also been extended to a whole array consisting of many elements [22]. However, the angular dependence of the correction matrix should be the same if all elements of the full array are identical. For radars that invoke the simultaneous transmission and reception (STSR) of waves of both polarizations, the two main diagonal elements of the scattering matrix can be obtained, yielding copolar data. This analysis indicates that the ATSR is a favorable polarization configuration for PPAR, but practical issues are being addressed and will be reported in the future.

The effects of attenuation, differential attenuation, and differential phase shift along the propagation path are included in the formulation by introducing the transmission matrix. Since the main diagonal terms of the transmission matrix represent the wave propagation for horizontal and vertical polarizations, respectively, there are no added biases for the PPAR caused by propagation effects. Once the scattering matrix is corrected, the propagation effects of differential attenuation and differential phase can be determined using the same procedures as those used for mechanically steered beams.

An alternative to making corrections for each of the thousands of beams is to restrict the beams to lie in the broadside vertical plane and to limit electronic steering to the vertical direction. Azimuthal steering would be accomplished with a turntable that would provide a continuous azimuth scan of the four faces of the PPAR. This would have other advantages in addition to simplifying the correction of bias. For example, the size of the array could be reduced by 40%, and the number of active elements for beam steering could be reduced by a factor of almost a hundred, resulting in considerable cost savings. The NWRT has dual-scanning capability, has been used for

V. Conclusion and Discussions

It has been shown that biases in polarimetric variables introduced by the electronically steered beam of a phased-array antenna can be comparable to or larger than the intrinsic polarimetric value for hydrometeors. There would be no biases, if the H, V radiation fields of the array element were equal over the entire angular sector typically used for weather radar observations (i.e., \(-45^\circ \leq \phi \leq +45^\circ; -10^\circ \leq \theta \leq +10^\circ\) for each face of a four-faced antenna). Matching fields to within 0.1 dB over such a large angular sector is a challenging task but is required because polarimetric information depends on small changes in hydrometeor characteristics. We used a pair of Hertzian dipoles to simulate the array H, V element radiation patterns, even though the use of Hertzian dipoles could magnify the biases. Nevertheless, matching is not necessary if the procedures outlined herein are implemented with a correction matrix tailored to fit the radiation patterns of the actual array element. Corrections can be performed using a correction matrix (Section III) or the estimated polarimetric variables (Section IV), provided that the amplitudes and the relative phase of the transmitted wave fields are known. The procedure can be extended to correct any system bias/coupling by replacing the projection matrix (7) with a corresponding error matrix defined in [26] and [27]. For radars using mechanically steered beams and transmitting equal amplitude H, V waves, knowledge of the relative transmitter phase is not necessary. However, in general, correcting the transmitter power in each of the two polarizations to balance the two is not necessary if both amplitude and phase are known.

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multipattern experiments [28], and will be used to perform a test of mechanical scanning in azimuth and electronic steering in elevation.

Like radars using mechanically steered beams, the performance of a PPAR is also affected by system limitations, model deficiency, and estimation error. The contribution from cross-polarization might be needed to account for the case of aligned ice crystals or randomly oriented hailstones. However, these are the same problems for radars using mechanically steered beams and should be addressed in the future and in the scope of general radar polarimetry. Other issues such as the effects of coupling between the array elements, actual radiation field of the element, cross-polar pattern effects [27], and experimental verification are also of interest, but it is premature to perform a detailed analysis because a PPAR with known system characteristics is not yet available.

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REFERENCES


Richard J. Doviak (S’56–M’57–SM’72–F’90–LF’97) received the B.S. degree in electrical engineering from Rensselaer Polytechnic Institute, Troy, NY, in 1956 and the M.S. and Ph.D. degrees from the University of Pennsylvania, Philadelphia, in 1959 and 1963, respectively.

He is currently a Senior Engineer with the National Severe Storms Laboratory, National Oceanic and Atmospheric Administration, Norman, OK, and also an Adjunct Professor with the School of Electrical and Computer Engineering and the School of Meteorology, University of Oklahoma, Norman. He is a coauthor of the text Doppler Radar and Weather Observations. He was an Associate Editor of the Journal of Atmospheric and Oceanic Technology and the Journal of Applied Meteorology.

Dr. Doviak was the recipient of IEEE’s Harri Diamond Memorial Award in 1988 for his outstanding technical contributions in the field of government service in any country. He is a Fellow of the American Meteorological Society (AMS) and the Cooperative Institute for Mesoscale Meteorological Studies of the University of Oklahoma. He has given short courses on radar meteorology at the U.S. National Radar Conferences (April 2006) and at the AMS Annual meetings. He was Editor of the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING.

Jerry Crain received the B.S. degree in electrical engineering from Wichita State University, Wichita, KS, in 1964 and the M.S. and Ph.D. degrees in electrical engineering from the University of Colorado, Boulder, in 1966 and 1970, respectively.

He has over 40 years of experience in the research, design, and development of phased-array radars and antennas. He is currently a Professor of electrical and computer engineering with the University of Oklahoma (OU), Norman. He came to OU in 1994 after more than 25 years in research, design, and development programs with Texas Instruments’ Advanced Technology and Radar Divisions. The Active Electronically Scanned Array work sponsored by the Department of Defense, the Federal Aviation Administration, and the National Aeronautics and Space Administration includes early prototype systems such as the Reliable Advanced Solid-State Phased-Array Radar, Discrete Airborne Beacon System, the Microwave Landing System, and Solid State Phased Array Program and culminated with the D&D of the F-22 airborne radar system.

He has been engaged in the development and extension of the National Weather Radar Testbed, a phased-array weather radar based in Norman. He was a member of the initial government-industry-university collaborative design team. He is now on the team funded by a National Science Foundation-sponsored Major Research Instrumentation grant to OU/National Severe Storms Laboratory to modify the single channel receiver system to eight simultaneous beam receive channels. This extension will enable this system to support research in the application of advanced/adaptive radar signal processing to weather radar.

David Staiman (M’62) was born in Paris, France in 1939. He received the B.S.E.E. degree from the City College of New York in 1962, the M.S.E.E. degree from the Polytechnic Institute of Brooklyn in 1965 and the Ph.D. Degree from the University of Pennsylvania, Philadelphia in 1974.

He was a member of the microwave antenna engineering staff at Wheeler Laboratories in Smythtown, Long Island in New York from 1962 to 1965 where he participated in the design and test of flash missile antennas for the early Atlas, Titan and Thor-Agena and Thor-delta booster missiles. In 1965, he joined the Defense Electronics Division of RCA, Moorstreet, NJ, and held increasing responsible positions through its transition to GE Aerospace, Martin Marietta and Lockheed Martin rising to his final position as Director of the Microwave/RF Analog Signal Processing and Antenna Design Center from which he retired in 2001. He has been responsible or played a significant roles in the AEGIS AN/SPY-1A, and AND/SPY-1B Antennas, the MOTR (multiple object tracking radar) incorporating a 8000 element phased array, and the COBRA (counter battery radar antenna) that incorporates the first fielded solid state phased array antenna, and the RAN_UP high power X-band solid state transmitter. He is responsible for the development of the first algorithms for near-field alignment of the production version of the AN/SPY-1A antenna. He has presented or published over 12 papers and holds two patents.

Dr. Staiman has continued to consult several companies including Lockheed Martin and Basic Commerce and Industries (BCI) where he has developed element level digital beamforming radar system and hardware level concepts for the MPAR (Multi-function phased array radar) program, since his retirement.

Yasser Al-Rashid received the B.S. degree in electrical and computer engineering and the Ph.D. degree in digital signal processing and communication from Wichita State University, Wichita, KS, in 1991 and 1998, respectively.

From 1998 to 2000, he was with the radar signal processing design group at Raytheon Electronic Systems, Sudbury, MA. He is currently a Principal Member of the Engineering Staff with Lockheed Martin Corporation, Moorstown, NJ. His research interest is in the area of polarimetric phased-array system design and signal processing.