## static stability

$$\sigma = -\frac{RT}{p}\frac{\partial \ln\theta}{\partial p} = \frac{R}{p}\left(\frac{R}{c_p}\frac{T}{p} - \frac{\partial T}{\partial p}\right)$$

can be easily shown using simple algebra

$$\frac{\partial T}{\partial t} = -\vec{V}\cdot\vec{\nabla}_p T + \underbrace{\omega\sigma\frac{p}{R}}_{-\omega\frac{\partial T}{\partial p} + \omega\frac{RT}{c_p p}}$$

vert. adv. compression/expansion

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$$\underbrace{c_p\frac{D\ln\theta}{Dt}}_{\frac{Ds}{Dt}} = \frac{1}{T}\frac{dQ}{dt}$$

thermo. eqn. (another form)
spec. entropy

$$\frac{D\theta}{Dt} = \frac{\theta}{c_p T}\frac{dQ}{dt}$$

$$\frac{\partial\theta}{\partial t} + \vec{V}\cdot\vec{\nabla}_p\theta + \omega\frac{\partial\theta}{\partial p} = \frac{\theta}{c_p T}\frac{dQ}{dt}$$

* will appear later in EPV thinking section

$$-\frac{\partial}{\partial p}\quad \frac{\partial}{\partial t}\left(-\frac{\partial\theta}{\partial p}\right) = -\frac{\partial}{\partial p}\left(-\vec{V}_p\cdot\vec{\nabla}_p\theta\right) + \underbrace{\frac{\partial\omega}{\partial p}}_{-\delta}\frac{\partial\theta}{\partial p} - \omega\frac{\partial}{\partial p}\left(-\frac{\partial\theta}{\partial p}\right) + \frac{\partial}{\partial p}\left(\frac{\theta}{c_p T}\frac{dQ}{dt}\right)$$

another measure of static stability

$$-\frac{\partial}{\partial p}\left(-\vec{V}_g\cdot\vec{\nabla}_p\theta\right) - \cancel{\frac{\partial}{\partial p}\left(-\vec{V}_a\cdot\vec{\nabla}_p\theta\right)}^{\,0 \text{ in Q.G.}}$$

$$\cancel{\left(\frac{\partial\vec{V}_g}{\partial p}\cdot\vec{\nabla}_p\theta\right)}^{\,0} - \vec{V}_g\cdot\vec{\nabla}_p\left(-\frac{\partial\theta}{\partial p}\right)$$

thermal wind $\perp$ temp gradient

static stability eqn.

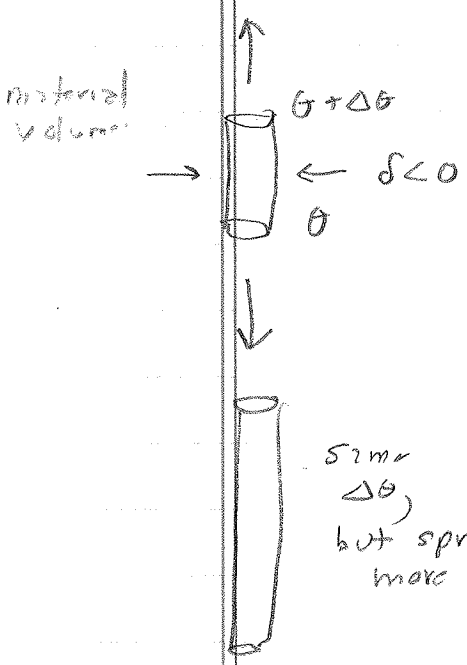
so, $$\frac{\partial}{\partial t}\left(-\frac{\partial\sigma}{\partial p}\right) = \underbrace{-\vec{V}_g\cdot\vec{\nabla}_p\left(-\frac{\partial\sigma}{\partial p}\right) - \omega\frac{\partial}{\partial p}\left(-\frac{\partial\sigma}{\partial p}\right)}_{\text{advection}}$$

$$\underbrace{+\delta\left(-\frac{\partial\sigma}{\partial p}\right)}_{\text{stretch or shrink air column}} \underbrace{-\frac{\partial}{\partial p}\left(\frac{\sigma}{C_pT}\frac{dQ}{dt}\right)}_{\text{differential diabatic heating}}$$

e.g.
- latent heat release
- evap. cooling
- cool air over warm water & vice versa
- radiation

$\delta < 0$ reduces static stability

$\delta > 0$ increases static stability




* vice versa

same $\Delta\sigma$, but spread out more vertically → less stability

## effective static stability

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Q.G. $\omega$-eqn. for latent-heat release only

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{R}{\sigma p}\nabla_p^2\left(\frac{1}{c_p}\frac{d\phi}{dt}\right)$$

$$\underbrace{(M_v + M_d)}_{\text{mass of "moist" air}}\, d\phi = \underbrace{-L_v\, \overbrace{dM_v}^{<0}}_{>0}$$

vapor → liquid

(stable) sat'd ascent

algebra $r = \frac{M_v}{M_d}$

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma_e}\frac{\partial^2}{\partial p^2}\right)\omega = 0$$

where $\sigma_e = \sigma - \frac{R}{p}\frac{L_v}{c_p}\underbrace{\frac{dr_s}{dp}}_{>0}$

recall $r_s(T, p)$

effect of diabatic heating put into static stability parameter

reduces $\sigma$

$\sigma > 0$, $\sigma_e < 0$ conditional instability → need "moist adjustment" or explicit — non QG.

$\sigma > 0$, $\sigma_e > 0$ OK!

$\sigma_e < 0$, $\sigma < 0$ absolute instability (gulp!)