METR 5423
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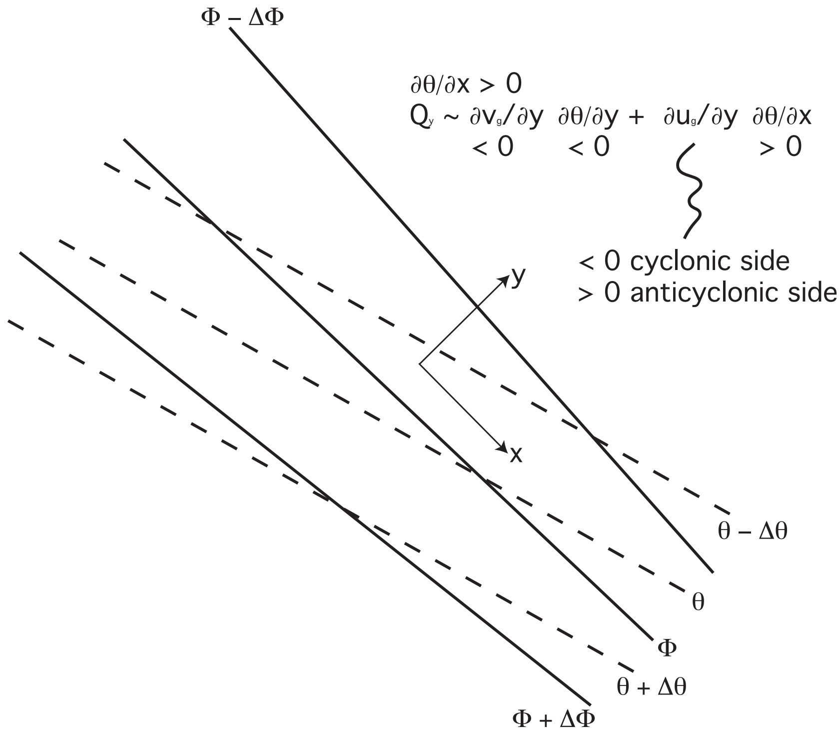
On the coordinate system used to diagnose the Sawyer-Eliassen equation when considering middle- and upper-tropospheric frontogenesis

The analysis of middle- and upper-tropospheric frontogenesis using the Sawyer-Eliassen (SE) equation as it appears in Vol. II (pp 375 – 377) could use some more explanation and updating, and someday I will prepare a revised version. In the meantime, I will try to clarify some aspects of the treatment as follows:

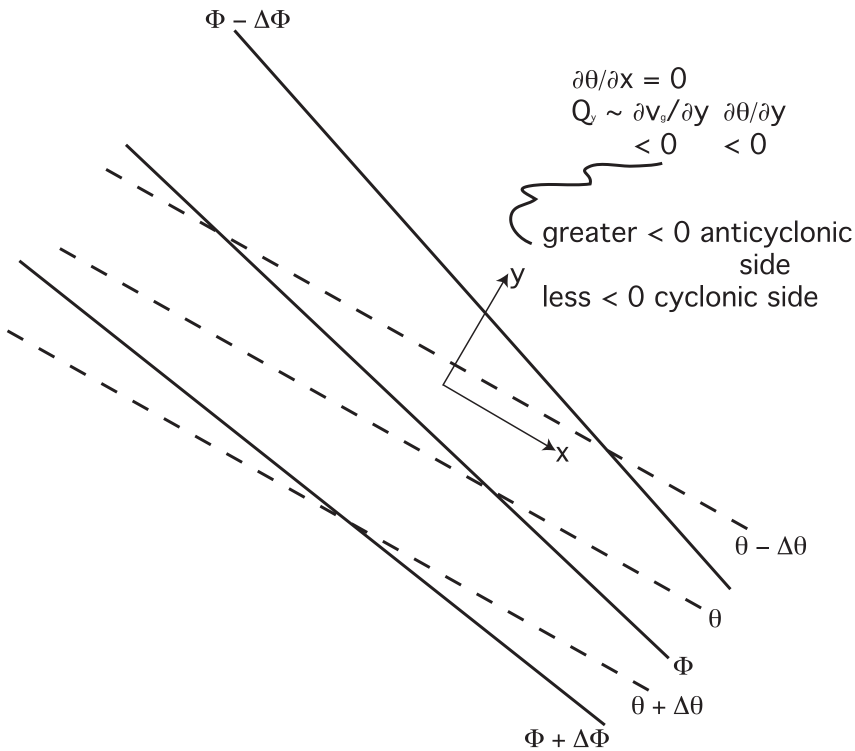
The case we consider is that when there is confluent geostrophic flow upstream from an upper-level trough, downstream from an upper-level ridge. In addition, there is some cold advection, as when a baroclinic wave is amplifying.

How should one orient the x- and y- coordinate systems? In class we chose to orient the x-axis along the *geostrophic flow*, not along the isotherms. If we can neglect curvature in the flow ($\partial v_g/\partial x \approx 0$) and $Dv_g/Dt \approx 0$, then the forcing function in the SE equation is $Q_2 \equiv Q_y \sim \partial \mathbf{v}_g/\partial y \bullet \nabla\theta = \partial v_g/\partial y \; \partial\theta/\partial y + \partial u_g/\partial y \; \partial\theta/\partial x$. In this case there is confluence, so $\partial v_g/\partial y < 0$, and there is cyclonic shear ($-\partial u_g/\partial y > 0$) on the northeast side of the jet and anticyclonic shear ($-\partial u_g/\partial y < 0$) on the southwest side of the jet. The first term is positive since $\partial\theta/\partial y < 0$. The second term is positive on the anticyclonic-shear side of the jet and negative on the cyclonic-shear side of the jet, since $\partial\theta/\partial x > 0$. The inquisitive student asks, "Why didn't you orient the x-axis along the isotherms and the y-axis in the direction opposite to that of the temperature-gradient vector?" The flustered instructor replies, "I could have done that, but the physical interpretation would have been a bit more difficult."



If the x-axis were oriented along the isotherms, with the cold air to the left, then $\partial\theta/\partial x = 0$, so that $Q_y \sim \partial v_g/\partial y\ \partial\theta/\partial y$ only. But, in this new coordinate system there is an addition of $\partial v_g/\partial y > 0$ on the cyclonic-shear side and $\partial v_g/\partial y < 0$ on the anticyclonic shear side, which adds frontogenetic forcing on the anticyclonic-shear side and frontolytic forcing on the cyclonic-shear side. So, the net effect of the forcing function on the vertical circulation is the same as if we had oriented the x-axis along the geostrophic wind. In effect, what appears as shear in one coordinate system shows up as confluence/diffluence in the other coordinate system. [Remember that the effects of deformation don't change if the coordinate system is changed, *but each component of deformation depends on how the coordinate system is oriented*. Consider a field of pure deformation (there is no translation, no vorticity, and no divergence) such that the axis of dilatation is oriented along the x-axis. Then $\partial u/\partial x > 0$, $\partial u/\partial y = 0$, $\partial v/\partial x = 0$, and $\partial v/\partial y < 0$, so that $D_1 = \partial u/\partial x - \partial v/\partial y > 0$, but $D_2 = \partial v/\partial x + \partial u/\partial y = 0$. However, if the axis of dilatation is oriented along the axis rotated 45^0 in a counterclockwise direction from the x axis, then $\partial u/\partial x = 0$,

$\partial u/\partial y > 0$, $\partial v/\partial x > 0$, and $\partial v/\partial y = 0$, so that $D_1 = \partial u/\partial x - \partial v/\partial y = 0$, but $D_2 = \partial v/\partial x + \partial u/\partial y > 0$.]



Now you know why I dislike the terms shearing and stretching deformation: They can change physical meaning depending on what coordinate system you use. The solutions to the SE equation should not depend on what coordinate system we use, and they don't!