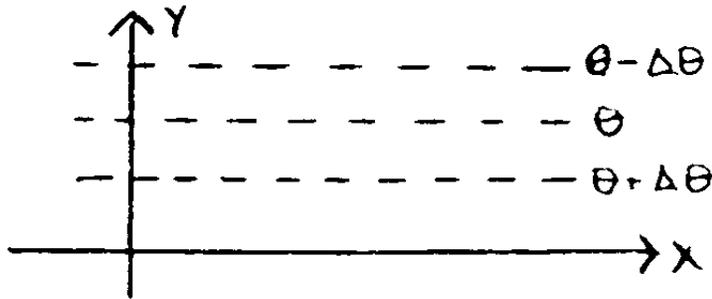


2.22



$$F = \frac{1}{2} |\nabla_p \theta| (D \cos 2b - \delta) = 0$$

$$\text{so, } D \cos 2b = \delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

axis of dilatation is perpendicular to axis of contraction $\implies b=90^\circ$

$$b = 90^\circ = \frac{1}{2} \tan^{-1} \frac{D_2}{D_1} \implies \tan^{-1} \frac{D_2}{D_1} = 180^\circ \implies D_2 = 0$$

$$D = \sqrt{D_1^2 + D_2^2} = 3 \times 10^{-5} \text{ s}^{-1} = \pm D_1$$

Since the axis of contraction is parallel to the x-axis, $D_1 < 0$.

$$\implies D_1 = -3 \times 10^{-5} \text{ s}^{-1} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

$$D \cos 2b = (3 \times 10^{-5} \text{ s}^{-1}) \cos 180^\circ = -3 \times 10^{-5} \text{ s}^{-1}$$

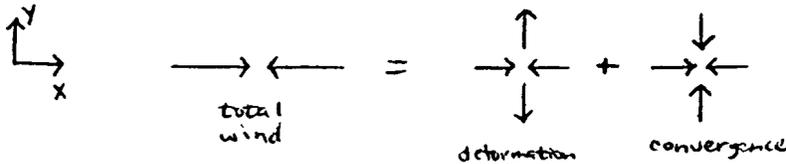
$$-3 \times 10^{-5} \text{ s}^{-1} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$-3 \times 10^{-5} \text{ s}^{-1} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

$$-6 \times 10^{-5} \text{ s}^{-1} = 2 \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = -3 \times 10^{-5} \text{ s}^{-1}$$

$$\frac{\partial v}{\partial y} = -3 \times 10^{-5} \text{ s}^{-1} - \frac{\partial u}{\partial x} = -3 \times 10^{-5} \text{ s}^{-1} + 3 \times 10^{-5} \text{ s}^{-1} = 0$$



effects of convergence counterbalance effects of deformation!

2.23

Let the x axis be oriented along the isotherms; let the y axis point toward the cold air. Then,

$$F = \frac{D}{Dt} |\nabla_p \theta| = \frac{D}{Dt} \left(-\frac{\partial \theta}{\partial y} \right)_p = \left(\frac{\partial v}{\partial y} \right)_p \left(\frac{\partial \theta}{\partial y} \right)_p + \left(\frac{\partial \omega}{\partial y} \right)_p \frac{\partial \theta}{\partial p} - \frac{1}{c_p} \left(\frac{p_0}{p} \right)^\kappa \left(\frac{\partial}{\partial y} \right)_p \left(\frac{dQ}{dt} \right)$$

$$= 0 + \left(\frac{\partial \omega}{\partial y} \right)_p \frac{\partial \theta}{\partial p} + 0$$

$$\frac{D}{Dt} |\nabla_p \theta| = \frac{(30 \text{ K} / 500 \text{ km}) - (10 \text{ K} / 500 \text{ km})}{1 \text{ day}} \times \frac{1 \text{ day}}{86,400 \text{ s}} = 4.63 \times 10^{-7} \text{ K s}^{-1} \text{ km}^{-1}$$

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa$$

$$\frac{\partial \theta}{\partial z} = \frac{\partial \theta}{\partial p} \frac{\partial p}{\partial z} = -\rho g \frac{\partial \theta}{\partial p} = -\frac{p g}{RT} \frac{\partial \theta}{\partial p}$$

$$\frac{\partial \theta}{\partial p} = -\frac{RT}{p g} \frac{\partial \theta}{\partial z} = -\frac{RT}{p g} \left[\frac{\partial T}{\partial z} \left(\frac{p_0}{p} \right)^\kappa - T \kappa \left(\frac{p_0}{p} \right)^\kappa \frac{1}{p} \frac{\partial p}{\partial z} \right]$$

$$= -\frac{RT}{p g} \left[0 + T \kappa \left(\frac{p_0}{p} \right)^\kappa \frac{1}{p} \frac{p g}{RT} \right]$$

$$= -\frac{\kappa \theta}{p} = \frac{(0.286)(308 \text{ K})}{500 \text{ mb}} = -0.176 \text{ K mb}^{-1}$$

$$\frac{\partial \omega}{\partial y} = \frac{4.63 \times 10^{-7} \text{ K s}^{-1} \text{ km}^{-1}}{-0.176 \text{ K mb}^{-1}} \times \frac{1000 \text{ mb}}{1 \text{ km}} = -2.6 \text{ } \mu\text{b s}^{-1} (1000 \text{ km})^{-1}$$

2.36

$$\frac{D\theta}{Dt} = 0 = \frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + \omega\frac{\partial\theta}{\partial p}$$

take $\frac{\partial}{\partial p}$:

$$\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial p}\right) + \frac{\partial u}{\partial p}\frac{\partial\theta}{\partial x} + u\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial p}\right) + \frac{\partial v}{\partial p}\frac{\partial\theta}{\partial y} + v\frac{\partial}{\partial y}\left(\frac{\partial\theta}{\partial p}\right) + \frac{\partial\omega}{\partial p}\frac{\partial\theta}{\partial p} + \omega\frac{\partial}{\partial p}\left(\frac{\partial\theta}{\partial p}\right) = 0$$

$$\frac{D}{Dt}\left(\frac{\partial\theta}{\partial p}\right) = 0 = -\frac{\partial u}{\partial p}\frac{\partial\theta}{\partial x} - \frac{\partial v}{\partial p}\frac{\partial\theta}{\partial y} + \delta\frac{\partial\theta}{\partial p}$$

$$\implies \frac{\partial\vec{v}}{\partial p} \cdot \nabla_p \theta = \delta\frac{\partial\theta}{\partial p}$$

$$\frac{\partial\vec{v}_g}{\partial p} \cdot \nabla_p \theta = 0, \text{ so } \frac{\partial\vec{v}_a}{\partial p} \cdot \nabla_p \theta = \delta\frac{\partial\theta}{\partial p}$$

$$\left| \frac{\partial\vec{v}_a}{\partial p} \right| \sim \left| \frac{\delta\frac{\partial\theta}{\partial p}}{\nabla_p \theta} \right| \sim \frac{(10^{-6} \text{ s}^{-1})\left(\frac{20 \text{ K}}{40 \text{ kPa}}\right)}{\left(\frac{10 \text{ K}}{10^5 \text{ m}}\right)}$$

$$= 5 \times 10^{-3} \text{ m s}^{-1} \text{ kPa}^{-1} = \frac{0.5 \text{ m s}^{-1}}{100 \text{ kPa}}$$

2.37

$$\left| \frac{\partial\vec{v}}{\partial p} \right| = \left| \frac{\partial\vec{v}_g}{\partial p} \right| = \frac{R}{f_o p} |\nabla_p T|$$

$$= \frac{287 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}}{(8.34 \times 10^{-5} \text{ s}^{-1})(50 \text{ kPa})} \frac{10 \text{ K}}{10^5 \text{ m}} = 6.88 \text{ m s}^{-1} \text{ kPa}^{-1} = \frac{68.8 \text{ m s}^{-1}}{10 \text{ kPa}^{-1}}$$

$$\left| \frac{\partial\vec{v}_g}{\partial z} \right| = \left| \frac{\partial\vec{v}_g}{\partial p} \right| (\rho g)$$

$$= (6.88 \text{ m s}^{-1} \text{ kPa}^{-1}) \left(\frac{50 \text{ kPa}}{287 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1} \times 252 \text{ K}} \right) (9.8 \text{ m s}^{-2})$$

$$= 4.66 \times 10^{-2} \text{ s}^{-1} = \frac{46.6 \text{ m s}^{-1}}{\text{km}}$$

$$\theta = T \left(\frac{p_o}{p} \right)^{\frac{R}{c_p}}$$

$$g \frac{\partial(\ln\theta)}{\partial z} = g \left(\frac{1}{T} \frac{\partial T}{\partial z} - \frac{R}{c_p p} \frac{\partial p}{\partial z} \right)$$

$$= g \left(0 + \frac{g}{c_p T} \right) = \frac{g^2}{c_p T}$$

$$= \frac{(9.8 \text{ m s}^{-2})^2}{(1004 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1})(252 \text{ K})} = 3.8 \times 10^{-4} \text{ s}^{-2}$$

$$Ri = \frac{g \frac{\partial(\ln\theta)}{\partial z}}{\left| \frac{\partial \vec{v}}{\partial z} \right|^2} = \frac{3.8 \times 10^{-4} \text{ s}^{-2}}{(4.66 \times 10^{-2} \text{ s}^{-1})^2} = 0.17$$

2.44

$$F = \frac{\partial v_g}{\partial y} \frac{\partial \theta}{\partial y} + \frac{\partial \omega}{\partial y} \frac{\partial \theta}{\partial p}$$

(a)

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa$$

$$\Delta \theta = \Delta T \left(\frac{p_0}{p} \right)^\kappa$$

$$\frac{\partial v_g}{\partial y} \equiv \frac{-6.08 \text{ m s}^{-1}}{200 \times 10^3 \text{ m}} = -3.04 \times 10^{-5} \text{ s}^{-1}$$

$$\frac{\partial \theta}{\partial y} \equiv \frac{-5 \text{ K} \left(\frac{1000}{500} \right)^{\frac{287}{1004}}}{200 \times 10^3 \text{ m}} = -3.05 \times 10^{-5} \text{ K m}^{-1}$$

$$\frac{\partial \omega}{\partial y} \equiv \frac{-4 \times 10^{-4} \text{ kPa s}^{-1}}{200 \times 10^3 \text{ m}} = -2 \times 10^{-9} \text{ kPa s}^{-1} \text{ m}^{-1}$$

$$\begin{aligned} \frac{\partial \theta}{\partial p} &\equiv \frac{(270.5 \text{ K}) \left(\frac{1000}{700} \right)^{\frac{287}{1004}} - (250.5 \text{ K}) \left(\frac{1000}{500} \right)^{\frac{287}{1004}}}{20 \text{ kPa}} \\ &= \frac{299.5 \text{ K} - 305.4 \text{ K}}{20 \text{ kPa}} = -0.30 \text{ K kPa}^{-1} \end{aligned}$$

$$\begin{aligned} F &= (-3.04 \times 10^{-5} \text{ s}^{-1}) (-3.05 \times 10^{-5} \text{ K m}^{-1}) + (-2.0 \times 10^{-9} \text{ kPa s}^{-1} \text{ m}^{-1}) (-0.30 \text{ K kPa}^{-1}) \\ &= 1.53 \times 10^{-9} \text{ K m}^{-1} \text{ s}^{-1} = 13.2 \text{ K (100 km)}^{-1} \text{ d}^{-1} \end{aligned}$$

(b)

$$\frac{\partial v_g}{\partial y} \equiv -3.04 \times 10^{-5} \text{ s}^{-1}$$

$$\frac{\partial \theta}{\partial y} \equiv -3.05 \times 10^{-5} \text{ K m}^{-1}$$

$$\frac{\partial \omega}{\partial y} \equiv +2.0 \times 10^{-9} \text{ kPa s}^{-1} \text{ m}^{-1}$$

$$\begin{aligned} \frac{\partial \theta}{\partial p} &\equiv \frac{(275.5 \text{ K}) \left(\frac{1000}{700} \right)^{\frac{287}{1004}} - (255.5 \text{ K}) \left(\frac{1000}{500} \right)^{\frac{287}{1004}}}{20 \text{ kPa}} \\ &= \frac{305.1 \text{ K} - 311.5 \text{ K}}{20 \text{ kPa}} = -0.32 \text{ K kPa}^{-1} \end{aligned}$$

$$\begin{aligned} F &= (-3.04 \times 10^{-5} \text{ s}^{-1}) (-3.05 \times 10^{-5} \text{ K m}^{-1}) + (2.0 \times 10^{-9} \text{ kPa s}^{-1} \text{ m}^{-1}) (-0.32 \text{ K kPa}^{-1}) \\ &= 2.87 \times 10^{-10} \text{ K m}^{-1} \text{ s}^{-1} = 2.48 \text{ K (100 km)}^{-1} \text{ d}^{-1} \end{aligned}$$