5.14

$$\frac{\partial T}{\partial t} = \frac{\rho g C_H U_o (T_s - T_o)}{|\Delta p|}$$

$$\equiv \frac{100 \text{ kPa}}{(287 \text{ m}^2 \text{ s}^{-2} \text{ deg}^{-1})(278 \text{ deg})} (9.8 \text{ m s}^{-2})(2\times10^{-3})(15 \text{ m s}^{-1})(273.15\text{–}283.15) \text{ deg} / 10 \text{ kPa}$$

$$\left(T \equiv \frac{273.15 + 283.15}{2} \text{ deg}\right)$$

$$= -3.68\times10^{-4} \text{ deg s}^{-1} \times \frac{86400 \text{ s}}{\text{day}} = -31.8 \text{ deg day}^{-1}$$

5.15

First consider the cloud layer (90 to 30 kPa).

$$R = \int_{90 \text{ kPa}}^{30 \text{ kPa}} C \sin\left[\frac{2\pi}{120 \text{ kPa}} (p - 30 \text{ kPa})\right] dp$$

where R is the rainfall rate per unit area (in kg m^{-2} s^{-1}) at cloud base (90 kPa).

Assume all condensate aloft makes it down to cloud base.

$-C \sin\left[\frac{2\pi}{120 \text{ kPa}} (p - 30 \text{ kPa})\right]$ is the condensation rate per unit pressure increment as a function of pressure for 30 kPa $\leq p \leq$ 90 kPa.

Then,

$$R = \frac{-120 \text{ kPa}}{2\pi} C \cos\left[\frac{2\pi}{120 \text{ kPa}} (p\text{–}30 \text{ kPa})\right]_{90 \text{ kPa}}^{30 \text{ kPa}}$$

$$= \frac{-120 \text{ kPa}}{2\pi} C [1 - (-1)]$$

$$= \frac{-120 \text{ kPa}}{\pi} C$$

$$R = \frac{(2 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{10^{-2} \text{ m}}{1 \text{ cm}})(10^3 \text{ kg m}^{-3})}{1 \text{ day} \times \frac{86400 \text{ s}}{1 \text{ day}}}$$

$$= \frac{50.8 \text{ kg m}^{-2} \text{ day}^{-1}}{86400 \text{ s day}^{-1}}$$

$$C = -\frac{\pi R}{120 \text{ kPa}} = -\frac{\pi (50.8 \text{ kg m}^{-2} / 86400 \text{ s})}{(120 \text{ kPa})(10^3 \text{ kg m s}^{-2} \text{ m}^{-2} \text{ kPa}^{-1})}$$

$$= 1.54\times10^{-8} \text{ m}^{-1} \text{ s}$$

So,

$$L\,C\sin\left[\frac{2\pi}{120\text{ kPa}}(p-30\text{ kPa})\right] \equiv \text{rate of latent heat release per unit area per unit pressure increment}$$

$$\frac{dQ}{dt} = L\,C\sin\left[\frac{2\pi}{120\text{ kPa}}(p-30\text{ kPa})\right] g \equiv \text{rate of latent heat release per unit mass}$$

$$\frac{1}{c_p}\frac{dQ}{dt} = \frac{g\,L\,C}{c_p}\sin\left[\frac{2\pi}{120\text{ kPa}}(p-30\text{ kPa})\right] \equiv \text{rate of temperature change}$$

$$\frac{L\,C\,g}{c_p} = \frac{(2.5\times10^{6}\text{ m}^2\text{ s}^{-2})(1.54\times10^{-8}\text{ m}^{-1}\text{ s})(9.8\text{ m s}^{-2})}{1004\text{ m}^2\text{ s}^{-2}\text{ deg}^{-1}}$$

$$= 3.76\times10^{-4}\text{ deg s}^{-1}\times 86400\text{ s day}^{-1}$$

$$= 32.5\text{ deg day}^{-1}$$

Now consider the subcloud layer.

$$\int_{90\text{ kPa}}^{100\text{ kPa}} K\,dp = K\,(10\text{ kPa}) = 0.20\,R$$

where K is the evaporation rate per unit pressure increment for 90 kPa $\leq$ p $\leq$ 100 kPa.

$$\Rightarrow K = \frac{0.20\,R}{10\text{ kPa}}$$

So,

$-L\,K \equiv$ rate of latent cooling per unit area per unit pressure increment

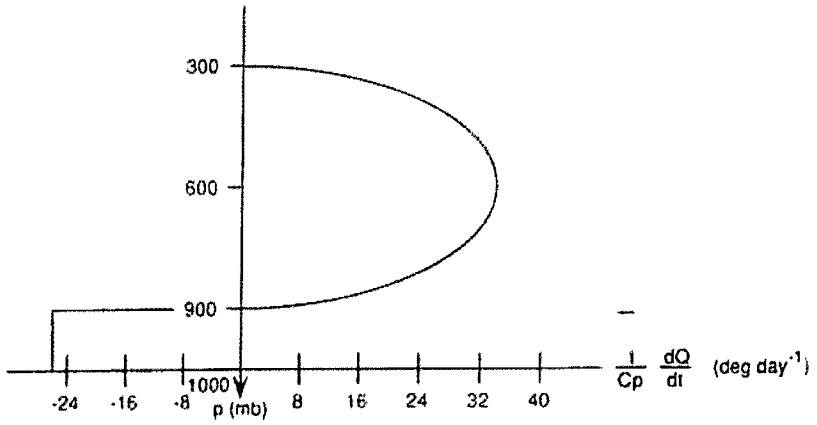
$$\frac{dQ}{dt} = -L\,K\,g \equiv \text{rate of latent cooling per unit mass}$$

$$\frac{1}{c_p}\frac{dQ}{dt} = \frac{-g\,L}{c_p}\frac{0.20\,R}{10\text{ kPa}} \equiv \text{rate of temperature change}$$

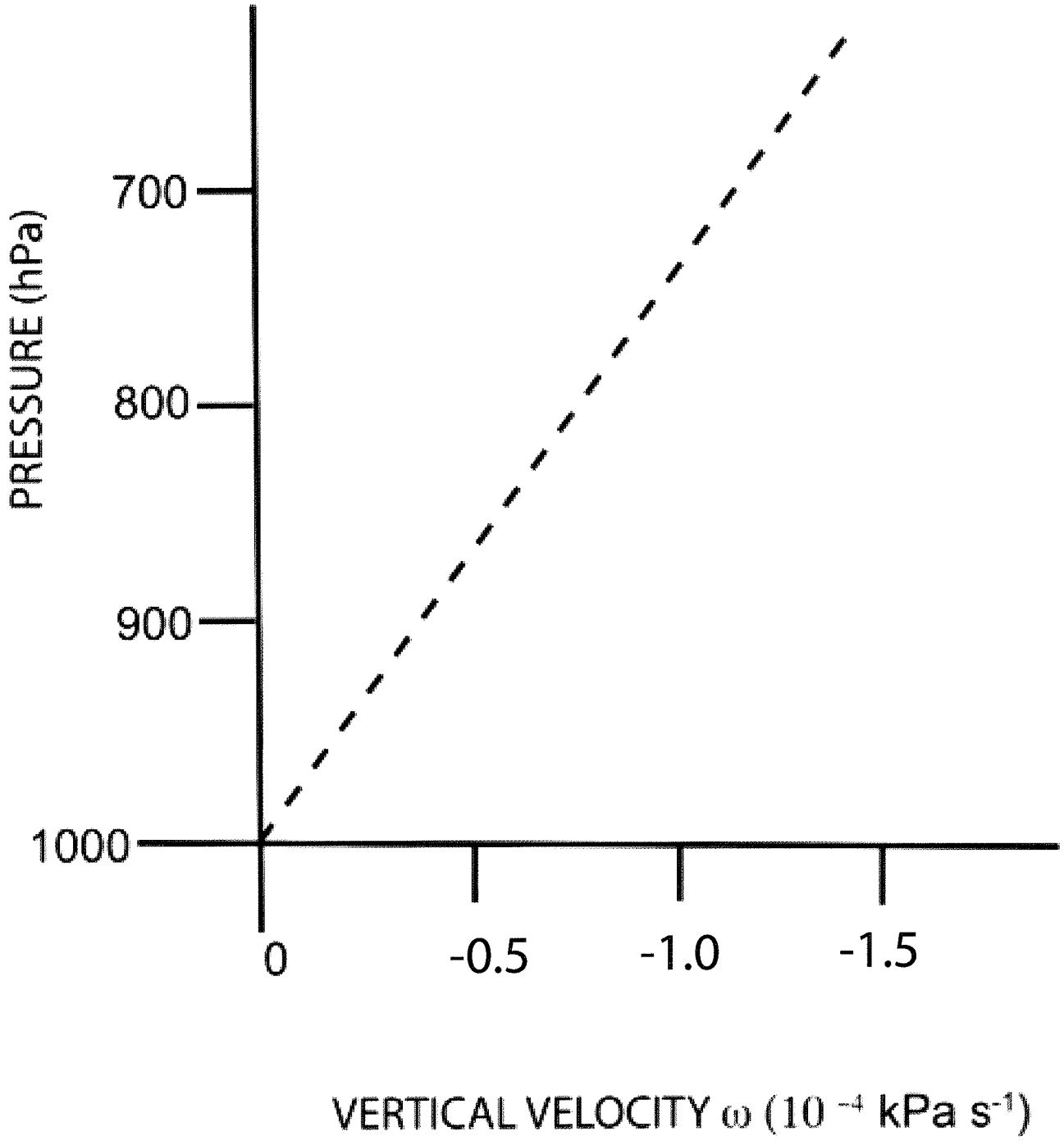
$$= \frac{(9.8\text{ m s}^{-2})(2.5\times10^{6}\text{ m}^2\text{ s}^{-2})(0.20)(50.8\text{ kg m}^{-2})/(86400\text{ s})}{(1004\text{ m}^2\text{ s}^{-2}\text{ deg}^{-1})(10\text{ kPa})(10^{3}\text{ kg m s}^{-2}\text{ m}^{-2})\text{ kPa}^{-1}}$$

$$= -2.87\times10^{-4}\text{ deg s}^{-1}$$

$$= -24.8\text{ deg day}^{-1}$$



(1)

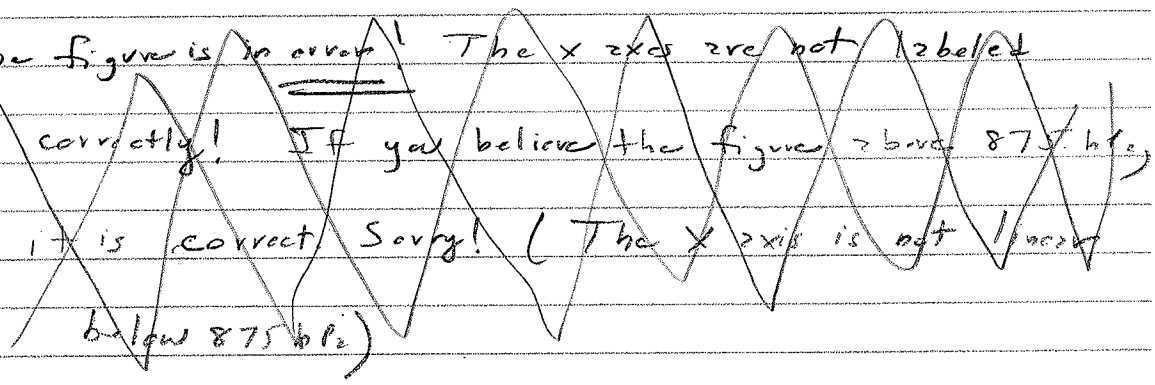

PRESSURE (hPa)
700
800
900
1000
0
-0.5
-1.0
-1.5
VERTICAL VELOCITY ω (10 -4 kPa s-1)

(1) $\frac{D S_g}{Dt} = -f_0 \delta = f_0 \frac{\partial w}{\partial p}$

$f_0 = 2\Omega \sin 35° = 2\left(\frac{2\pi}{86164\,s}\right) \sin 35° = 8.35 \times 10^{-5}\,s^{-1}$

$\left.\frac{\partial w}{\partial p}\right|_{850\,hPa} \simeq \frac{0.5 \times 10^{-4}\,hPa\,s^{-1}}{12.5\,hPa} = -\delta$

The figure is in error! The x axes are not labeled correctly! If you believe the figure above 875 hPa, it is correct. Sorry! (The x axis is not linear below 875 hPa)



so, $\frac{D S_g}{Dt} \simeq \frac{(8.35 \times 10^{-5}\,s^{-1})(0.5 \times 10^{-4}\,hPa\,s^{-1})}{12.5\,hPa} = 3.34 \times 10^{-10}\,s^{-1}$