

5.13

$$\omega = \frac{\partial p}{\partial t} + \vec{v}_g \cdot \vec{\nabla} p - w \rho g$$

$$\vec{v}_g = \vec{v} - \vec{v}_g, \text{ where } \vec{v}_g = \frac{1}{f\rho} \hat{k} \times \vec{\nabla} p$$

$$\rho = \frac{p}{RT} \equiv \frac{(85 \text{ kPa})(10^3 \text{ kg m s}^{-2} \text{ m}^{-2} / \text{kPa})}{(287 \text{ m}^2 \text{ s}^{-2} \text{ deg}^{-1})(273.15 \text{ deg})} = 1.084 \text{ kg m}^{-3}$$

$$f = 2\left(\frac{2\pi}{86164 \text{ s}}\right) \sin 35^\circ = 8.37 \times 10^{-5} \text{ s}^{-1}$$

$$\vec{v}_g = \frac{(0.4 \text{ kPa} / 2 \times 10^5 \text{ m})(10^3 \text{ kg m s}^{-2} \text{ m}^{-2} / \text{kPa}) \hat{i}}{(8.37 \times 10^{-5} \text{ s}^{-1})(1.084 \text{ kg m}^{-3})} \\ = (22.0 \text{ m s}^{-1}) \hat{i}$$

$$\vec{v} = (17.3 \text{ m s}^{-1}) \hat{i} + (10 \text{ m s}^{-1}) \hat{j}$$

$$\vec{v}_g = (-4.7 \text{ m s}^{-1}) \hat{i} + (10 \text{ m s}^{-1}) \hat{j}$$

$$w = \vec{v} \cdot \vec{\nabla} z_0 = (17.3 \text{ m s}^{-1}) \frac{-10^3 \text{ m}}{400 \times 10^3 \text{ m}} = -4.33 \times 10^{-2} \text{ m s}^{-1}$$

So,

$$\begin{aligned} \omega &\equiv \frac{-1.0 \text{ kPa}}{12 \text{ hrs} \times 60 \text{ min hr}^{-1} \times 60 \text{ s min}^{-1}} \\ &+ (10 \text{ m s}^{-1})(-0.4 \text{ kPa} / 2 \times 10^5 \text{ m}) \\ &- (-4.33 \times 10^{-2} \text{ m s}^{-1})(1.084 \text{ kg m}^{-3})(9.8 \text{ m s}^{-2}) \left( \frac{1 \text{ kPa}}{10^3 \text{ kg m s}^{-2} \text{ m}^{-2}} \right) \\ &= -2.315 \times 10^{-5} \text{ kPa s}^{-1} - 2.0 \times 10^{-5} \text{ kPa s}^{-1} + 4.60 \times 10^{-4} \text{ kPa s}^{-1} \\ &= 4.17 \times 10^{-4} \text{ kPa s}^{-1} = 4.17 \mu\text{b s}^{-1} \end{aligned}$$

Note that even with a large  $|\vec{v}_g|$ ,  $\omega \equiv -\rho g w$ .

The estimate for  $\rho$  is based on  $p \equiv 85 \text{ kPa}$ .

5.16

$$\textcircled{4} \quad \frac{f_0}{\sigma} \left\{ - \frac{\partial \vec{V}_g}{\partial (-P)} \cdot \vec{\nabla}_p S_g - \left[ \vec{V}_g \cdot \vec{\nabla}_p \frac{\partial S_g}{\partial (-P)} \right] + \left[ D_1 \frac{\partial D_2}{\partial (-P)} - D_2 \frac{\partial D_1}{\partial (-P)} \right] \right\}$$

$$- \frac{\partial \vec{V}_g}{\partial (-P)} \cdot \vec{\nabla}_p S_g = \frac{\partial U_g}{\partial P} \frac{\partial S_g}{\partial X} + \frac{\partial V_g}{\partial P} \frac{\partial S_g}{\partial Y}$$

$$= \frac{\partial U_g}{\partial P} \frac{\partial}{\partial X} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) + \frac{\partial V_g}{\partial P} \frac{\partial}{\partial Y} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right)$$

$$- \left[ \vec{V}_g \cdot \vec{\nabla}_p \frac{\partial S_g}{\partial (-P)} \right] = - U_g \frac{\partial}{\partial P} \frac{\partial S_g}{\partial X} - V_g \frac{\partial}{\partial P} \frac{\partial S_g}{\partial Y}$$

$$= - U_g \frac{\partial}{\partial P} \frac{\partial}{\partial X} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) - V_g \frac{\partial}{\partial P} \frac{\partial}{\partial Y} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right)$$

$$D_1 \frac{\partial D_2}{\partial (-P)} = - \left( \frac{\partial U_g}{\partial X} - \frac{\partial V_g}{\partial Y} \right) \frac{\partial}{\partial P} \left( \frac{\partial V_g}{\partial X} + \frac{\partial U_g}{\partial Y} \right)$$

$$= - \frac{\partial U_g}{\partial X} \frac{\partial}{\partial P} \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial X} \frac{\partial}{\partial P} \frac{\partial U_g}{\partial Y} + \frac{\partial V_g}{\partial P} \frac{\partial}{\partial Y} \frac{\partial V_g}{\partial X} + \frac{\partial V_g}{\partial P} \frac{\partial}{\partial Y} \frac{\partial U_g}{\partial X}$$

$$- D_2 \frac{\partial D_1}{\partial (-P)} = - \left( \frac{\partial V_g}{\partial X} + \frac{\partial U_g}{\partial Y} \right) \frac{\partial}{\partial P} \left( \frac{\partial U_g}{\partial X} - \frac{\partial V_g}{\partial Y} \right)$$

$$= - \frac{\partial V_g}{\partial X} \frac{\partial}{\partial P} \frac{\partial U_g}{\partial X} - 2 \frac{\partial V_g}{\partial X} \frac{\partial}{\partial P} \frac{\partial V_g}{\partial Y} + \frac{\partial U_g}{\partial P} \frac{\partial}{\partial Y} \frac{\partial U_g}{\partial X} - \frac{\partial U_g}{\partial Y}$$

$$\frac{\partial U_g}{\partial P} = R \frac{\partial T}{\partial P} \frac{\partial Y}{\partial Y}$$

$$\frac{\partial V_g}{\partial P} = - R \frac{\partial T}{\partial P} \frac{\partial X}{\partial X}$$

$$\boxed{\frac{f_0}{\sigma} \left[ - \frac{\partial \vec{V}_g}{\partial (-P)} \cdot \vec{\nabla}_p S_g \right]} = \frac{R}{\sigma} \left[ R \frac{\partial T}{\partial P} \frac{\partial Y}{\partial X} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) - R \frac{\partial T}{\partial P} \frac{\partial X}{\partial Y} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) \right]$$

$$= \frac{R}{\sigma P} \left[ \frac{\partial T}{\partial Y} \frac{\partial}{\partial X} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) - \frac{\partial T}{\partial X} \frac{\partial}{\partial Y} \left( \frac{\partial V_g}{\partial X} - \frac{\partial U_g}{\partial Y} \right) \right]$$

$$= \frac{R}{\sigma P} \left[ \frac{\partial T}{\partial Y} \frac{\partial^2 V_g}{\partial X^2} - \frac{\partial T}{\partial Y} \frac{\partial^2 U_g}{\partial X^2} - \frac{\partial T}{\partial X} \frac{\partial^2 V_g}{\partial Y^2} + \frac{\partial T}{\partial X} \frac{\partial^2 U_g}{\partial Y^2} \right]$$

$$\frac{\partial U_g}{\partial X} = - \frac{\partial V_g}{\partial Y}$$

$$= \frac{R}{\sigma P} \left[ \frac{\partial T}{\partial Y} \frac{\partial^2 V_g}{\partial X^2} + \frac{\partial T}{\partial Y} \frac{\partial^2 V_g}{\partial Y^2} + \frac{\partial T}{\partial X} \frac{\partial^2 U_g}{\partial X^2} + \frac{\partial T}{\partial X} \frac{\partial^2 U_g}{\partial Y^2} \right] \equiv A$$

$$\boxed{\frac{f_0}{\sigma} \left\{ - \left[ \vec{V}_g \cdot \vec{\nabla}_p \frac{\partial S_g}{\partial (-P)} \right] \right\}} = \frac{f_0}{\sigma} \left[ - U_g \frac{\partial}{\partial X} \frac{\partial}{\partial X} \left( - \frac{R \partial T}{\partial P} \frac{\partial Y}{\partial X} \right) + U_g \frac{\partial}{\partial X} \frac{\partial}{\partial Y} \left( \frac{R \partial T}{\partial P} \frac{\partial Y}{\partial Y} \right) \right.$$

$$\left. - V_g \frac{\partial}{\partial Y} \frac{\partial}{\partial X} \left( - \frac{R \partial T}{\partial P} \frac{\partial X}{\partial X} \right) + V_g \frac{\partial}{\partial Y} \frac{\partial}{\partial Y} \left( \frac{R \partial T}{\partial P} \frac{\partial X}{\partial Y} \right) \right]$$

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$$= \frac{R}{\sigma p} \left[ U_g \frac{\partial^2}{\partial x^2} \frac{\partial T}{\partial x} + U_g \frac{\partial^2}{\partial y^2} \frac{\partial T}{\partial x} + V_g \frac{\partial^2}{\partial x^2} \frac{\partial T}{\partial y} + V_g \frac{\partial^2}{\partial y^2} \frac{\partial T}{\partial y} \right]$$

$$\frac{f_0}{\sigma} D_1 \frac{\partial D_2}{\partial (-p)} = \frac{f_0}{\sigma} \left[ - \frac{\partial U_g}{\partial x} \frac{\partial}{\partial x} \left( - \frac{R}{\sigma p} \frac{\partial T}{\partial x} \right) - \frac{\partial U_g}{\partial y} \frac{\partial}{\partial y} \left( \frac{R}{\sigma p} \frac{\partial T}{\partial y} \right) \right]$$

$$+ \frac{\partial V_g}{\partial x} \frac{\partial}{\partial x} \left( - \frac{R}{\sigma p} \frac{\partial T}{\partial x} \right) + \frac{\partial V_g}{\partial y} \frac{\partial}{\partial y} \left( \frac{R}{\sigma p} \frac{\partial T}{\partial y} \right)$$

$$\frac{\partial V_g}{\partial y} = - \frac{\partial U_g}{\partial x}$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial U_g}{\partial x} \frac{\partial^2 T}{\partial x^2} - \frac{\partial U_g}{\partial y} \frac{\partial^2 T}{\partial y^2} - \frac{\partial V_g}{\partial y} \frac{\partial^2 T}{\partial x^2} + \frac{\partial V_g}{\partial y} \frac{\partial^2 T}{\partial y^2} \right]$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial \partial U_g}{\partial x^2} \frac{\partial^2 T}{\partial x^2} + \frac{\partial \partial V_g}{\partial y^2} \frac{\partial^2 T}{\partial y^2} \right] \equiv C$$

$$- \frac{f_0}{\sigma} D_2 \frac{\partial D_1}{\partial (-p)} = \frac{f_0}{\sigma} \left[ - \frac{\partial U_g}{\partial x} \frac{\partial}{\partial x} \left( \frac{R}{\sigma p} \frac{\partial T}{\partial y} \right) - \frac{\partial V_g}{\partial x} \frac{\partial}{\partial y} \left( - \frac{R}{\sigma p} \frac{\partial T}{\partial x} \right) \right]$$

$$+ \frac{\partial V_g}{\partial y} \frac{\partial}{\partial x} \left( \frac{R}{\sigma p} \frac{\partial T}{\partial y} \right) - \frac{\partial U_g}{\partial y} \frac{\partial}{\partial y} \left( - \frac{R}{\sigma p} \frac{\partial T}{\partial x} \right)$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial \partial V_g}{\partial x^2} \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial y} \right) + \frac{\partial \partial V_g}{\partial x^2} \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) + \frac{\partial \partial U_g}{\partial y^2} \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial x} \right) + \frac{\partial \partial U_g}{\partial y^2} \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) \right]$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial \partial V_g}{\partial x^2} \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial \partial U_g}{\partial y^2} \frac{\partial^2 T}{\partial x \partial y} \right] = D$$

$$-\frac{R}{\sigma p} \left[ \nabla_p^2 \left( -\vec{V}_g \cdot \vec{\nabla}_p T \right) \right] = \frac{R}{\sigma p} \left[ \nabla_p^2 \left( U_g \frac{\partial T}{\partial x} + V_g \frac{\partial T}{\partial y} \right) \right]$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial}{\partial x} \left( U_g \frac{\partial T}{\partial x} + V_g \frac{\partial T}{\partial x^2} \right) + \frac{\partial}{\partial x} \left( U_g \frac{\partial T}{\partial y} + V_g \frac{\partial^2 T}{\partial x \partial y} \right) \right]$$

$$+ \frac{\partial}{\partial y} \left( U_g \frac{\partial T}{\partial x} + U_g \frac{\partial^2 T}{\partial x \partial y} + V_g \frac{\partial T}{\partial y} + V_g \frac{\partial^2 T}{\partial y^2} \right)$$

$$= \frac{R}{\sigma p} \left[ \frac{\partial^2 U_g}{\partial x^2} \frac{\partial T}{\partial x} + \frac{\partial U_g}{\partial x} \frac{\partial^2 T}{\partial x^2} + \frac{\partial U_g}{\partial x} \frac{\partial^2 T}{\partial x^2} + U_g \frac{\partial}{\partial x} \frac{\partial^2 T}{\partial x^2} \right]$$

$$+ \frac{\partial^2 V_g}{\partial x^2} \frac{\partial T}{\partial y} + \frac{\partial V_g}{\partial x} \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial V_g}{\partial x} \frac{\partial^2 T}{\partial x \partial y} + V_g \frac{\partial}{\partial y} \frac{\partial^2 T}{\partial x^2}$$

$$+ \frac{\partial^2 U_g}{\partial y^2} \frac{\partial T}{\partial x} + \frac{\partial U_g}{\partial y} \frac{\partial^2 T}{\partial y^2} + \frac{\partial U_g}{\partial y} \frac{\partial^2 T}{\partial x \partial y} + U_g \frac{\partial}{\partial x} \frac{\partial^2 T}{\partial y^2}$$

$$+ \frac{\partial^2 V_g}{\partial y^2} \frac{\partial T}{\partial y} + \frac{\partial V_g}{\partial y} \frac{\partial^2 T}{\partial y^2} + \frac{\partial V_g}{\partial y} \frac{\partial^2 T}{\partial x \partial y} + V_g \frac{\partial}{\partial y} \frac{\partial^2 T}{\partial y^2}$$

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$$= \frac{R}{\sigma p} \left[ \frac{\partial T}{\partial y} \frac{\partial^2 V_g}{\partial x^2} + \frac{\partial T}{\partial y} \frac{\partial^2 V_g}{\partial y^2} + \frac{\partial T}{\partial x} \frac{\partial^2 U_g}{\partial x^2} + \frac{\partial T}{\partial x} \frac{\partial^2 U_g}{\partial y^2} \right]$$

$$+ \frac{R}{\sigma p} \left[ U_g \frac{\partial}{\partial x} \frac{\partial^2 T}{\partial x^2} + U_g \frac{\partial}{\partial x} \frac{\partial^2 T}{\partial y^2} + V_g \frac{\partial^2}{\partial x^2} \frac{\partial T}{\partial y} + V_g \frac{\partial^2}{\partial y^2} \frac{\partial T}{\partial y} \right]$$

$$+ \frac{2R}{\sigma p} \left[ \frac{\partial U_g}{\partial x} \frac{\partial^2 T}{\partial x^2} + \frac{\partial V_g}{\partial y} \frac{\partial^2 T}{\partial y^2} \right]$$

$$+ \frac{2R}{\sigma p} \left[ \frac{\partial V_g}{\partial x} \frac{\partial^2 T}{\partial x \partial y} + \frac{\partial U_g}{\partial y} \frac{\partial^2 T}{\partial x \partial y} \right] = A + B + C + D$$

Solutions to Problem Set #  
METR 5413  
Advanced Synoptic Meteorology  
H. Bluestein

5.18 (a)  $\vec{D} = -\frac{R}{\sigma p} \left( \begin{array}{c} \frac{\partial U_g}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial V_g}{\partial x} \frac{\partial T}{\partial y} \\ \frac{\partial U_g}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial V_g}{\partial y} \frac{\partial T}{\partial y} \end{array} \right)$

$$\frac{\partial U_g}{\partial p} = \frac{\partial \hat{U}}{\partial p} = -\alpha \frac{R}{f_0 p} \frac{\partial T}{\partial y}$$

$$\frac{\partial V_g}{\partial p} = \frac{\partial \hat{V}}{\partial p} \sin \left[ \frac{2\pi}{L} (x - ct) \right] = -\frac{R}{f_0 p} \frac{\partial T}{\partial x} = -\frac{R}{f_0 p} \hat{T} \left( \frac{2\pi}{L} \right) \cos \left[ \frac{2\pi}{L} (x - ct) \right]$$

$$\hat{U}(p=1000) = \hat{U}_o$$

$$\hat{V}(p=1000) = \hat{V}_o$$

$$\hat{U}(p) = \hat{U}_o - \frac{\alpha R}{f_0} \ln \frac{p}{1000} = -\alpha R \frac{\ln \frac{p}{1000}}{f_0}$$

$$U_g = \hat{U}(p) = \hat{U}_o - \frac{\alpha R}{f_0} \ln \frac{p}{1000}$$

$$\hat{V}(p) - \hat{V}_o = -R \frac{\hat{T} \left( \frac{2\pi}{L} \right)}{f_0} \frac{\cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right]}{\sin \frac{2\pi}{L} (x - ct)} \ln \frac{p}{1000}$$

$$\hat{V}(p) = \hat{V}_o - \frac{R \hat{T} \left( \frac{2\pi}{L} \right)}{f_0} \frac{\cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right]}{\sin \frac{2\pi}{L} (x - ct)} \ln \frac{p}{1000}$$

$$V_g = \left\{ \hat{V}_o - \frac{R \hat{T} \left( \frac{2\pi}{L} \right)}{f_0} \frac{\cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right]}{\sin \frac{2\pi}{L} (x - ct)} \ln \frac{p}{1000} \right\} \sin \frac{2\pi}{L} (x - ct)$$

$$= \left\{ \hat{V}_o \sin \left[ \frac{2\pi}{L} (x - ct) \right] - \frac{R \hat{T} \left( \frac{2\pi}{L} \right)}{f_0} \ln \frac{p}{1000} \cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right] \right\}$$

$$(b) \begin{cases} Q_1 = -\frac{R}{\sigma P} \left[ a \hat{V}_o \frac{2\pi}{L} \cos \left[ \frac{2\pi}{L} (x - ct) \right] - a \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^2 \ln \frac{P}{1000} \sin \left[ \frac{2\pi}{L} (x - ct + \mu) \right] \right] \\ Q_2 = 0 \end{cases}$$

$$(c) \vec{\nabla}_p \cdot \vec{Q} = \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} = -\frac{R}{\sigma P} \left\{ a \hat{V}_o \left( \frac{2\pi}{L} \right)^2 \sin \left[ \frac{2\pi}{L} (x - ct) \right] - a \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^3 \ln \frac{P}{1000} \right. \\ \left. \cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right] \right\}$$

$$(d) \frac{f_o}{\sigma} \frac{\partial}{\partial x} \left[ -\vec{V}_g \cdot \vec{\nabla}_p (S_g + f) \right] \\ S_g = \frac{\partial V_g}{\partial x} - \frac{\partial U_g}{\partial y} = \frac{2\pi}{L} \hat{V}_o \cos \left[ \frac{2\pi}{L} (x - ct) \right] + \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^2 \ln \frac{P}{1000} \\ \sin \left[ \frac{2\pi}{L} (x - ct + \mu) \right]$$

$$-\vec{V}_g \cdot \vec{\nabla}_p S_g = -U_g \frac{\partial S_g}{\partial x} - V_g \frac{\partial S_g}{\partial y} \\ = -\left( \hat{V}_o - \frac{a R}{f_o} \ln \frac{P}{1000} \right) \left[ -\left( \frac{2\pi}{L} \right)^2 \hat{V}_o \sin \left[ \frac{2\pi}{L} (x - ct) \right] \right. \\ \left. + R \frac{\hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^3 \ln \frac{P}{1000} \cos \left[ \frac{2\pi}{L} (x - ct + \mu) \right] \right]$$

$$-\frac{f_o}{\sigma} \frac{\partial}{\partial p} \left[ -U_g \frac{\partial S_g}{\partial x} - V_g \beta \right] = \\ -\frac{f_o}{\sigma} \left[ -\hat{V}_o \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^3 \cos \frac{2\pi}{L} (x - ct + \mu) \frac{1}{P} - \frac{a R}{f_o} \left( \frac{2\pi}{L} \right)^2 \hat{V}_o \sin \frac{2\pi}{L} (x - ct) \frac{1}{P} \right. \\ \left. + \frac{a R \hat{T}}{f_o^2} \left( \frac{2\pi}{L} \right)^3 \cos \frac{2\pi}{L} (x - ct + \mu) \frac{2}{P} \ln \frac{P}{1000} \frac{(1000)}{1000} \right. \\ \left. + \beta \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right) \frac{1000}{P} \frac{1}{1000} \cos \frac{2\pi}{L} (x - ct + \mu) \right]$$

$$-\frac{R}{\sigma P} \nabla_p^2 (-\vec{V}_g \cdot \vec{\nabla}_p T) = -\frac{R}{\sigma P} \nabla_p^2 (-U_g \frac{\partial T}{\partial x} - V_g \frac{\partial T}{\partial y})$$

$$-U_g \frac{\partial T}{\partial x} - V_g \frac{\partial T}{\partial y} = -\left( \hat{V}_o - \frac{a R}{f_o} \ln \frac{P}{1000} \right) \frac{\hat{T}}{L} \left( \frac{2\pi}{L} \right) \cos \frac{2\pi}{L} (x - ct + \mu) \\ + a \left[ \hat{V}_o \sin \frac{2\pi}{L} (x - ct) - \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right) \ln \frac{P}{1000} \cos \frac{2\pi}{L} (x - ct + \mu) \right]$$

$$\frac{\partial^2}{\partial x^2} \left( -U_g \frac{\partial T}{\partial x} - V_g \frac{\partial T}{\partial y} \right) = \left( \hat{V}_o - \frac{a R}{f_o} \ln \frac{P}{1000} \right) \frac{\hat{T}}{L} \left( \frac{2\pi}{L} \right)^3 \cos \frac{2\pi}{L} (x - ct + \mu) \\ - a \left[ \hat{V}_o \left( \frac{2\pi}{L} \right)^2 \sin \frac{2\pi}{L} (x - ct) - \frac{R \hat{T}}{f_o} \left( \frac{2\pi}{L} \right)^3 \ln \frac{P}{1000} \cos \frac{2\pi}{L} (x - ct + \mu) \right]$$

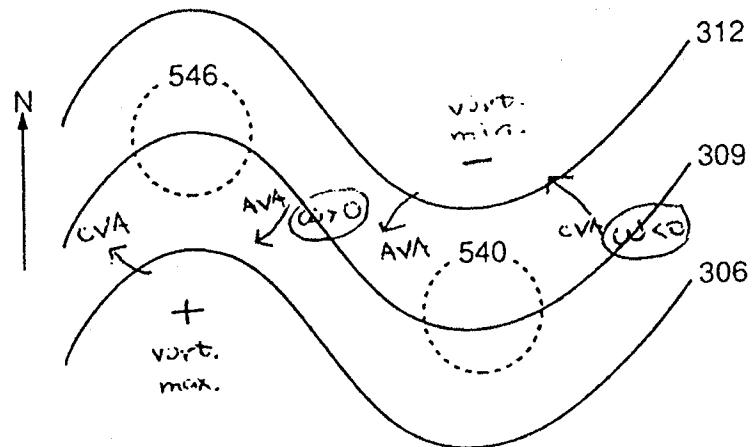
$$\frac{\partial^2}{\partial y^2} \left( -U_g \frac{\partial T}{\partial x} - V_g \frac{\partial T}{\partial y} \right) = 0$$

(3) e

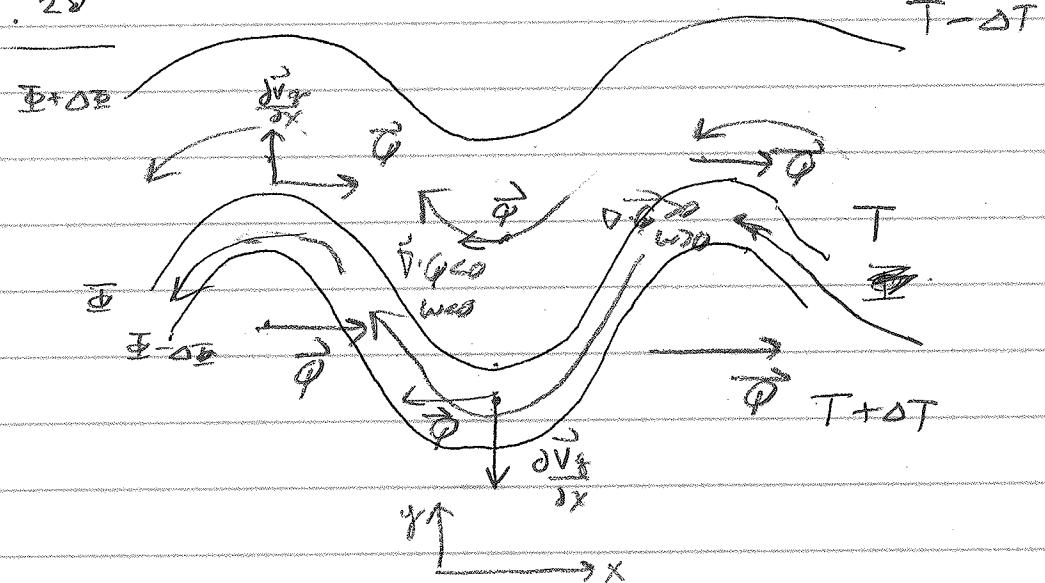
$$\begin{aligned} & \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left[ -\vec{V}_g \cdot \vec{\nabla}_p (\vec{e}_g + \vec{f}) \right] = \\ & + \left( \begin{aligned} & \frac{R \hat{U}_0 \hat{T} (2\pi)^3}{\sigma p} \cos \frac{2\pi}{L} (x - ct + \mu) \\ & + \frac{R \hat{U}_0 \alpha (2\pi)^2}{\sigma p} \sin \frac{2\pi}{L} (x - ct) \checkmark \\ & - \frac{R \hat{U}_0 \alpha \ln \frac{p}{1000}}{\sigma p f_0} (2\pi)^3 \cos \frac{2\pi}{L} (x - ct + \mu) \\ & - \frac{R \hat{U}_0 \beta \hat{T} (2\pi)}{\sigma p} \cos \frac{2\pi}{L} (x - ct + \mu) \\ & - \frac{R}{\sigma p} \nabla_p^2 (-\vec{V}_g \cdot \vec{\nabla}_p T) = - \frac{R \hat{U}_0 \hat{T} (2\pi)^3}{\sigma p} \cos \frac{2\pi}{L} (x - ct + \mu) \\ & + \frac{R \alpha R \hat{T}}{\sigma p f_0} (2\pi)^3 \cos \frac{2\pi}{L} (x - ct + \mu) \ln \frac{p}{1000} \\ & + \frac{R \hat{U}_0 \alpha (2\pi)^2}{\sigma p} \sin \frac{2\pi}{L} (x - ct) \checkmark \\ & - \frac{R \alpha \hat{R} R}{\sigma p f_0} (2\pi)^3 \ln \frac{p}{1000} \cos \frac{2\pi}{L} (x - ct + \mu) \end{aligned} \right) \\ & = 2 \frac{R \hat{U}_0 \alpha (2\pi)^2}{\sigma p} \sin \frac{2\pi}{L} (x - ct) - 2 \frac{R \alpha R \hat{T} (2\pi)^3}{\sigma p f_0} \ln \frac{p}{1000} \cos \frac{2\pi}{L} (x - ct + \mu) \\ & - \frac{R \beta \hat{T} (2\pi)}{\sigma p} \cos \frac{2\pi}{L} (x - ct + \mu) \\ & = -2 \frac{R}{\sigma p} \left[ \alpha \hat{U}_0 (2\pi)^2 \sin \frac{2\pi}{L} (x - ct) - \alpha R \hat{T} (2\pi)^3 \ln \frac{p}{1000} \cos \frac{2\pi}{L} (x - ct) \right. \\ & \quad \left. - \frac{R \beta \hat{T} (2\pi)}{\sigma p} \cos \frac{2\pi}{L} (x - ct + \mu) \right] \\ & = -2 \vec{V}_g \cdot \vec{Q} - \frac{R}{\sigma p} \beta \hat{T} (2\pi) \cos \frac{2\pi}{L} (x - ct + \mu) \end{aligned}$$

5.26

$$\left( \nabla_p^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega \sim -\vec{v}_T \cdot \vec{\nabla}_p (\zeta_g + f)$$

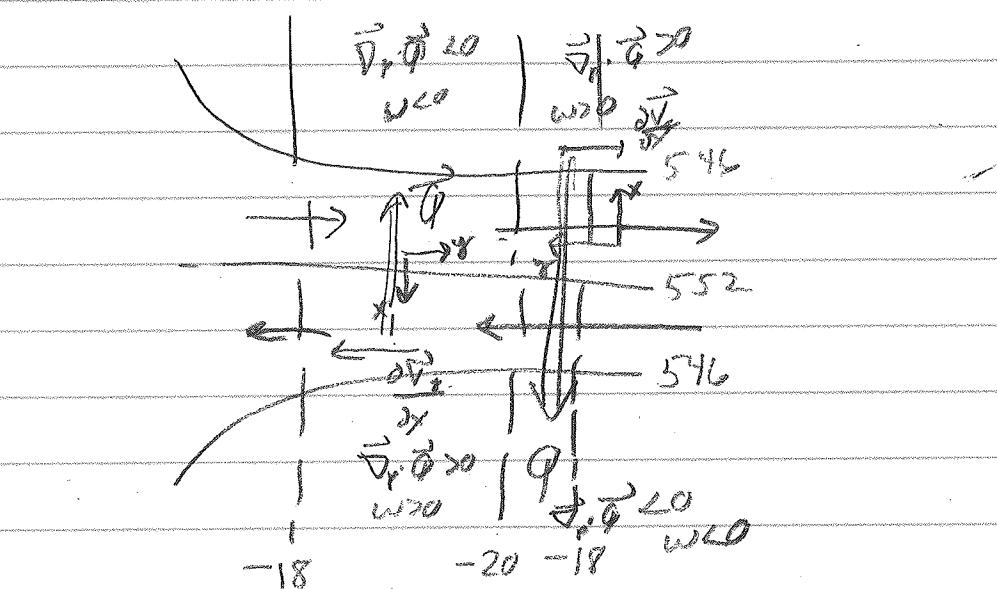


5, 28



$$-\frac{R}{6p} \beta \frac{\partial T}{\partial x} = 0 \quad \text{because } \frac{\partial T}{\partial x} = 0$$

5,37



$$cVA \geq w^{60} \quad pVA \geq w^{70}$$

25 विद्या

$$-\vec{\nabla}_T \cdot \vec{\nabla}_S S$$

~~GRVA = 2000 AVRA = 3000~~

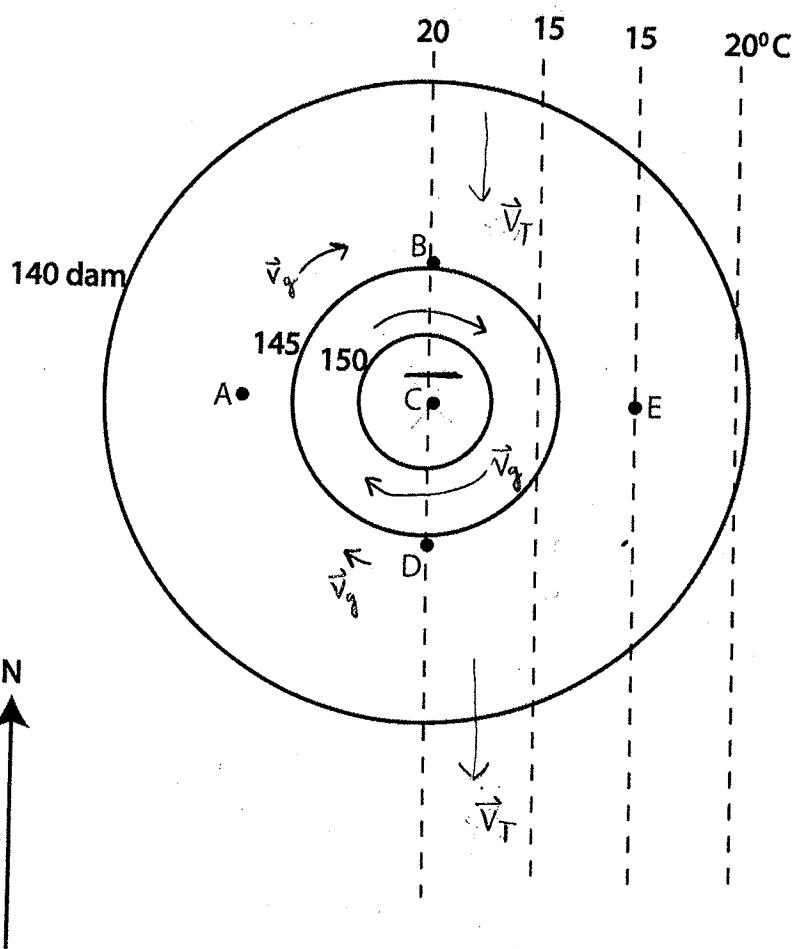
$$\textcircled{1} \quad w \sim -\vec{v}_T \cdot \vec{\nabla}_p S_g \quad B \downarrow \vec{v}_T \quad -\vec{v}_T \cdot \vec{\nabla}_p S_g > 0 \Rightarrow w < 0$$

10 points

C vort. min

$$D \downarrow \vec{v}_T \quad -\vec{v}_T \cdot \vec{\nabla}_p S_g < 0 \Rightarrow w > 0$$

(2)



$$-\vec{v}_T \cdot \vec{\nabla}_p S_g > 0$$

N of C-B  $\Rightarrow w < 0$

$$-\vec{v}_T \cdot \vec{\nabla}_p S_g < 0 \quad S \text{ of } C-D \Rightarrow w > 0$$

$M_{in}$   
in  
 $S_g$   
at