Solutions to Problem Set #4
METR 5413
Advanced Synoptic Meteorology
H. Bluestein

5.20

(a)

$$\theta = T\left(\frac{p_o}{p}\right)^{\kappa}, \text{ where } \kappa = \frac{R}{c_p} = \frac{287}{1004} = 0.286$$

$$\theta_{850} = (284\ \text{K})\left(\frac{100\ \text{kPa}}{85\ \text{kPa}}\right)^{0.286} = 297.5\ \text{K}$$

$$\theta_{500} = (255\ \text{K})\left(\frac{100\ \text{kPa}}{50\ \text{kPa}}\right)^{0.286} = 310.9\ \text{K}$$

$$-\frac{\partial\theta}{\partial p} = -\frac{297.5\ \text{K} - 310.9\ \text{K}}{85\ \text{kPa} - 50\ \text{kPa}} = 0.383\ \text{K kPa}^{-1}$$

$$\begin{aligned}\theta_{700} &= \theta_{500} + (70\ \text{kPa} - 50\ \text{kPa})\frac{\partial\theta}{\partial p}\\ &= 310.9\ \text{K} + (20\ \text{kPa})(-0.383\ \text{K kPa}^{-1})\\ &= 303.2\ \text{K}\end{aligned}$$

$$T_{700} = (303.2\ \text{K})\left(\frac{70\ \text{kPa}}{100\ \text{kPa}}\right)^{0.286} = 273.8\ \text{K}$$

$$\begin{aligned}\sigma &= -\frac{RT}{p\theta}\frac{\partial\theta}{\partial p}\\ &= \frac{(287\ \text{m}^2\ \text{s}^{-2}\ \text{K}^{-1})(273.8\ \text{K})(0.383\ \text{K kPa}^{-1})}{(70\ \text{kPa})(303.2\ \text{K})}\\ &= 1.42\ \text{m}^2\ \text{s}^{-2}\ \text{kPa}^{-2}\end{aligned}$$

(b)

$$\sigma_e = \sigma - \frac{0.622\ R\ L_v}{p^2\ c_p}\left[\frac{de_s}{dT}\left(\frac{dT}{dp}\right)_m - \frac{e_s}{p}\right]$$

$$e_s = 5.9728\times10^{22}\ e^{-6788.026/T}\ T^{-5.0065}\ \text{kPa}$$

$$\frac{de_s}{dT} = e_s\left[\frac{6788.026}{T^2} - \frac{5.0065}{T}\right]\ \text{kPa K}^{-1}$$

$$\left(\frac{dT}{dp}\right)_m = \frac{RT}{pc_p}\left[\frac{1 + \frac{L_v r_s}{RT}}{1 + \left(\frac{0.622\ L_v}{c_p T}\right)\left(\frac{L_v r_s}{RT}\right)}\right]$$

$e_s(273.8\text{ K}) = 0.64\text{ kPa}$

$\frac{de_s}{dT}(273.8\text{ K}) = 0.0462\text{ kPa K}^{-1}$

$r_s = 0.622\frac{e_s}{p} = 0.622\frac{0.64\text{ kPa}}{70\text{ kPa}} = 5.69\times10^{-3}$

$\left(\frac{dT}{dp}\right)_m = 0.654\text{ K kPa}^{-1}$

$$\sigma_e = \sigma - \frac{(0.622)(287\text{ m}^2\text{ s}^{-2}\text{ K}^{-1})(2.5\times10^6\text{ m}^2\text{ s}^{-2})}{(70\text{ kPa})^2(1004\text{ m}^2\text{ s}^{-2}\text{ K}^{-1})}\left[(0.0462\text{ kPa K}^{-1})(0.654\text{ K kPa}^{-1}) - \frac{0.64\text{ kPa}}{70\text{ kPa}}\right]$$
$$= 1.42\text{ m}^2\text{ s}^{-2}\text{ kPa}^{-2} - 1.91\text{ m}^2\text{ s}^{-2}\text{ kPa}^{-2}$$
$$= -0.49\text{ m}^2\text{ s}^{-2}\text{ kPa}^{-2}$$

Therefore, there is conditional instability, and the operator in the Q-G ω equation is not elliptic. ("Moist convective adjustment" would occur in numerical models.)

5.21

parabolic: $\omega = ap^2 + bp + c, \quad \frac{\partial\omega}{\partial p} = 2ap + b$

$\omega(p{=}100\text{ kPa}) = 0 = a(100\text{ kPa})^2 + b(100\text{ kPa}) + c$

$\omega(p{=}60\text{ kPa}) = -10\ \mu\text{b s}^{-1} = a(60\text{ kPa})^2 + b(60\text{ kPa}) + c$

$\frac{\partial\omega}{\partial p}(p{=}60\text{ kPa}) = 0 = 2a(60\text{ kPa}) + b$

solution for a, b, and c:

$a = 6.25\times10^{-7}\text{ kPa}^{-1}\text{ s}^{-1}, \quad b = -7.5\times10^{-5}\text{ s}^{-1}, \quad c = 1.25\times10^{-3}\text{ kPa s}^{-1}$

$$\delta(p{=}\ 800\text{mb}) = -\frac{\partial\omega}{\partial p}(p{=}800\text{ mb})$$
$$= -[2(6.25\times10^{-7}\text{ kPa}^{-1}\text{ s}^{-1})(80\text{ kPa}) - 7.5\times10^{-5}\text{ s}^{-1}]$$
$$= -2.5\times10^{-5}\text{ s}^{-1}$$

$$\frac{\partial}{\partial t}\left(-\frac{\partial\theta}{\partial p}\right) = -\delta\frac{\partial\theta}{\partial p}$$

$$\frac{d\left[-\frac{\partial\theta}{\partial p}\right]}{-\frac{\partial\theta}{\partial p}} = \delta\, dt$$

$$\ln\left(-\frac{\partial\theta}{\partial p}\right)\Big|_0^t = \delta t$$
$$= \ln\left(\frac{0.001\text{ K kPa}^{-1}}{0.383\text{ K kPa}^{-1}}\right)$$
$$= -5.95$$

therefore,

$$t = \frac{-5.95}{\delta(p{=}800\text{ mb})} = \frac{-5.95}{-2.5\times10^{-5}\text{ s}^{-1}} = 2.38\times10^5\text{s} = 2.76\text{ days}$$

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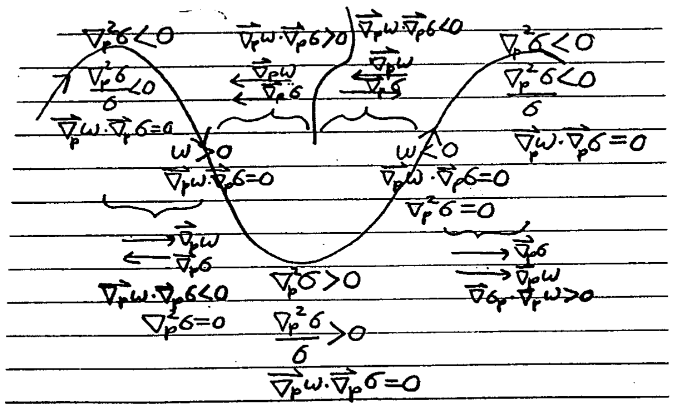
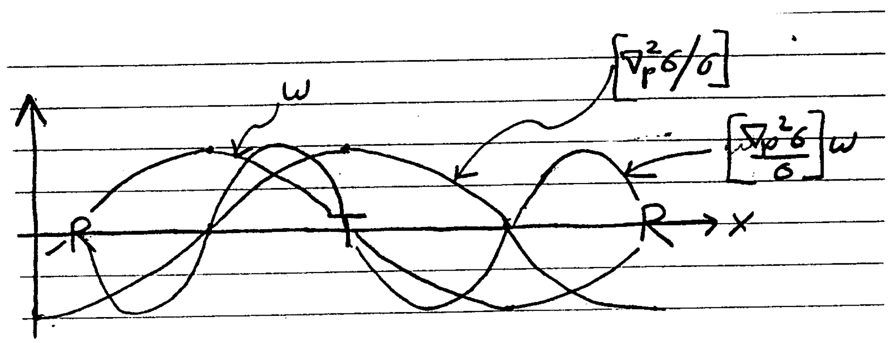
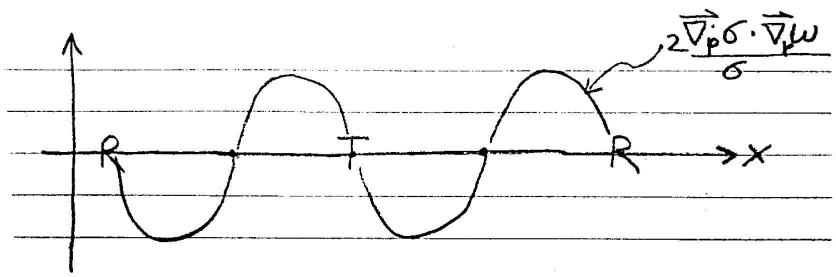


The effect of horizontal variations in σ on vertical motion is contained in the terms

$$\frac{\nabla_p^2\sigma}{\sigma}\omega \quad \text{and} \quad \frac{2}{\sigma}\nabla_p\sigma \cdot \nabla_p\omega.$$

The following diagrams show the general forms of these quantities.





Therefore, effects of horizontal variations in σ are *least* important along trough and ridge axes, and exactly in between trough and ridge axes. Effects of horizontal variations in σ are *most* important 1/8 wavelength downstream and upstream from trough and ridge axes.