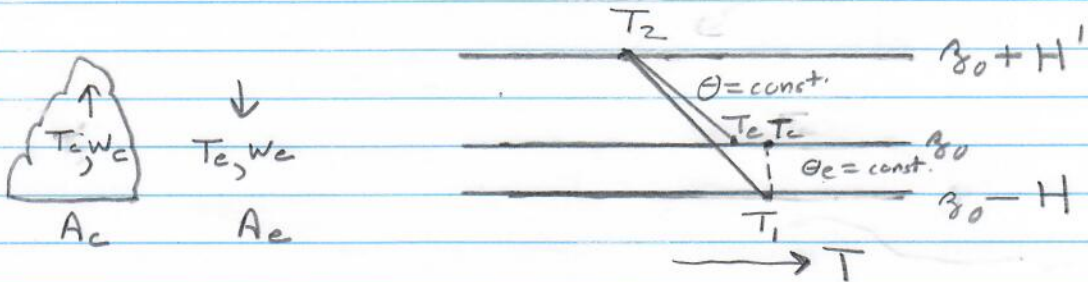


9/26/17

Bjerknes "slice method" $W_c$  — unmixed moist adiabatic ascent $W_e$  — dry adiabatic descent of environmental air $T_c$  — cloud temp. at arbitrary height  $\beta_0$  $T_e$  — <sup>disturbed</sup> environment temp. at arbitrary height  $\beta_0$  $T_2$  — undisturbed temp. at cloud top ( $\beta_0 + H'$ ) $T_1$  — undisturbed temp. at cloud base ( $\beta_0 - H$ )

$$\left\{ \begin{array}{l} T_c(\beta_0) = T_1 - H \Gamma_m \\ T_c(\beta_0) = T_2 + H' \Gamma_d \end{array} \right.$$

$$\text{mass continuity} \Rightarrow W_c A_c = W_e A_e$$

$$\Rightarrow \frac{\Delta \beta_c}{\Delta t} A_c = \frac{\Delta \beta_e}{\Delta t} A_e \Rightarrow$$

$$A_c H = A_e H'$$

$$\left\{ \begin{array}{l} \Delta \beta_c \equiv H \\ \Delta \beta_e \equiv H' \end{array} \right.$$

$$T_1 - T_2 = T_c - T_e + H \Gamma_m + H' \Gamma_d$$

$$\text{but } \frac{T_1 - T_2}{H + H'} = -\frac{\partial T}{\partial z} = \Gamma$$

$$\text{so, } \Gamma = \frac{T_c - T_e}{H + H'} + \frac{H \Gamma_m + H' \Gamma_d}{H + H'}$$

$$\begin{aligned}
\text{so, } \frac{T_c - T_e}{H + H'} &= \Gamma - \left( \frac{H\Gamma_m + H'\Gamma_d}{H + H'} \right) \\
&= \Gamma - \left( \frac{\frac{A_e H' \Gamma_m + H' \Gamma_d}{A_c}}{\frac{A_e H' + H'}{A_c}} \right) \\
&= \Gamma - \left( \frac{A_e \Gamma_m + A_c \Gamma_d}{A_c} \right) \\
&= \Gamma - \left( \frac{A_e + A_c}{A_c} \right) \\
&= \Gamma - \left( \frac{A_e \Gamma_m + A_c \Gamma_d}{A_e + A_c} \right)
\end{aligned}$$

buoyancy  $\sim T_c - T_e > 0 \Rightarrow \Gamma > \left( \frac{A_e \Gamma_m}{A_e + A_c} + \frac{A_c \Gamma_d}{A_e + A_c} \right)$   
 $(H + H' > 0 \quad \Gamma > 0)$

conditional instability  $\Rightarrow \Gamma > \Gamma_m \Rightarrow \Gamma > \Gamma_d \Rightarrow$  want to minimize term on RHS

$$\Gamma < \Gamma_d$$

$\frac{A_c}{A_e}$  must  $\rightarrow 0$  for most favorable condition:

$\Rightarrow$  2nd term  $\rightarrow$  small  $\rightarrow$  small

$\Rightarrow$  1st term  $\rightarrow \Gamma_m$  (2nd  $\Gamma > \Gamma_m$ ) ✓

(If  $\frac{A_e}{A_c} \rightarrow 0$ ,  $\Rightarrow$  2nd term  $\rightarrow \Gamma_d$   
 $\Rightarrow$  1st term  $\rightarrow$  small

$A_e \leftarrow$  large but  $\Gamma \neq \Gamma_d!$