Solutions to Exam I

METR 6223

Fall 2019

(1) $B = g\dfrac{(T-\overline{T})}{\overline{T}}$ $\qquad \overline{T} \equiv T$ of env. $\quad T \equiv T$ of parcel

$$T \equiv T_{cv} = T(1-r_\ell) \qquad r_i = 0 \quad r_v = 0$$

$$B = g\frac{\left[T(1-r_\ell) - \overline{T}\right]}{\overline{T}} \le 0$$

$$T(1-r_\ell) \le \overline{T}$$

$$T - r_\ell T \le \overline{T}$$

$$-r_\ell T \le \overline{T} - T$$

$$r_\ell \ge \frac{T-\overline{T}}{\overline{T}} = \frac{2\,K}{263\,K} = 0.0076 = 7.6\,g\,kg^{-1}_{\text{dry air}}$$

at 500 hPa, $\overline{\rho} = \dfrac{p}{RT} = \dfrac{50\,\cancel{kPa}}{(287\,m^2\,\cancel{s^{-2}}\,\cancel{K^{-1}})(263\,\cancel{K})}\,\dfrac{10^3\,kg\,m\,\cancel{s^{-2}}\,m^{-2}}{\cancel{kPa}}$

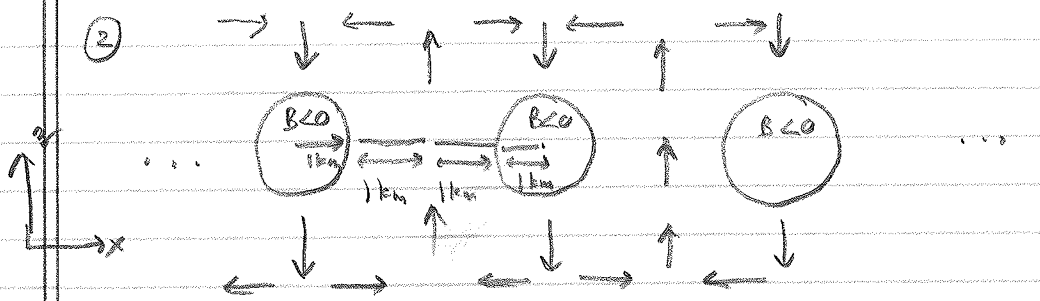
$$= 0.662\,kg\,m^{-3} = \frac{M_d}{V}$$

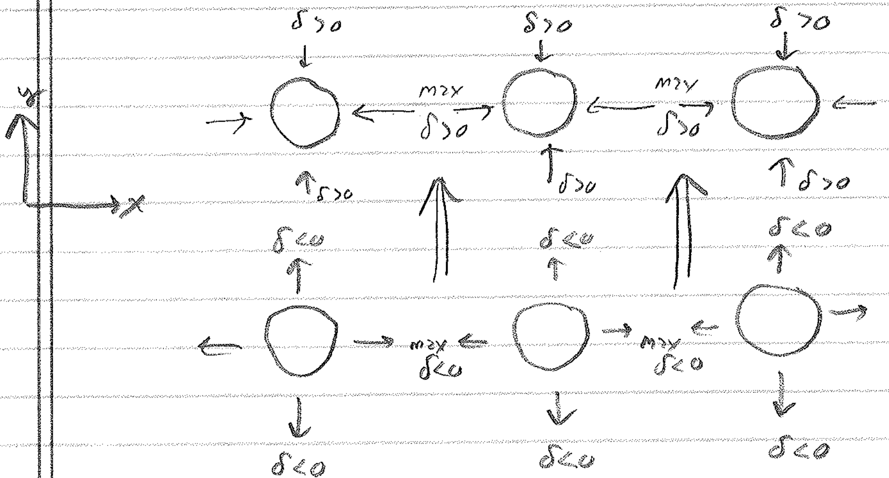
so, in a volume of $1\,m^3$ there is $0.662\,kg$ dry air

$$\frac{7.6\,g\ \text{liquid}}{kg\ \text{dry air}} = \frac{x\,g\ \text{liquid}/m^3}{0.662\,kg\ \text{dry air}/m^3} \Rightarrow x = \frac{7.6\,g\ \text{liquid}}{\cancel{kg}\ \text{dry air}}\left(0.662\,\cancel{kg}/m^3\right) =$$

$$\ge 5.03\,g\,m^{-3}$$

(2)





by continuity, above bubbles $\delta < 0$

by continuity, below bubbles $\delta > 0$

max $w < 0$ at $B < 0$ bubbles, $w < 0$ both above and below

max $w > 0$ in between bubbles where $\delta < 0$ max below & $\delta > 0$ max above

less $w > 0$ to N & S of bubbles where $\delta < 0$ below & $\delta > 0$ aloft but $|\delta|$ less than along E-W line

If parcels shrunken down, $|\delta|$ in between neighboring bubbles will decrease $\Rightarrow$ $|w|$ will decrease everywhere, but right at bubbles

(3) If cyclonic rotation is added, then $Ra_c \sim (\ ) + T_0$

$T_0 \sim \Omega^2 \Rightarrow R_{ac}$ will increase

$R_{ac} \sim$ vertical temp. gradient $\Rightarrow$ onset of stationary overturning will occur for larger temp. differences

If anticyclonic rotation is added, Ω^2 remains the same sign

$\Rightarrow$ same answer as for cyclonic rotation steady $\quad$ 2D $\quad$ at $z=0, w=0$

(4) $\cancel{\frac{\partial u}{\partial t}}^{0} + U\frac{\partial U}{\partial x} + \cancel{v\frac{\partial U}{\partial y}} + \cancel{w\frac{\partial U}{\partial z}} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}U^2 + \frac{p'}{\rho}\right) = 0 \Rightarrow \frac{1}{2}U^2 + \frac{p'}{\rho} = \text{const}$$

so, $\frac{1}{2}(13\,ms^{-1})^2 + 0 = \text{const.}$ $\quad$ $p'=0$ ahead of density current

$0 + \frac{p'}{\rho} = \text{const}$ $\quad$ $U=0$ at leading edge of density current

so, $\frac{p'}{\rho} = \frac{1}{2}(13\,ms^{-1})^2 = 84.5\ m^2 s^{-2}$

$$\bar{\rho} = \frac{p}{RT} = \frac{975\ \cancel{hPa}}{(287\ m^2 s^{-2} K^{-1})(303\ K)} \times \frac{kPa}{10\ hPa} \times \frac{10^3\ kg\ m\ s^{-2}\ m^{-2}}{kPa}$$

$$= 1.12\ kg\ m^{-3}$$

so, $p' = (84.5\ m^2 s^{-2})(1.12\ kg\ m^{-3}) = 94.7\ kg\ m^{-1} s^{-2}$

$$\times \frac{10\ hPa}{1\ kPa}\ \frac{kPa}{10^3\ kg\ m\ s^{-2}\ m^{-2}}$$

$$= +0.947\ hPa$$

(5) $\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2}$

$$\frac{U}{\tau} \sim \nu\frac{U}{H^2} \Rightarrow \tau \sim \frac{H^2}{\nu}$$

(6) $Ra = \dfrac{\alpha H^4}{\nu K}$, where $\alpha \sim$ vertical temp. gradient (temp. difference between bottom and top plates)

H = spacing between top and bottom plates

ν = kinematic coefficient of viscosity

K = coefficient of thermal diffusivity

$\sim$ ratio of rate of transport of heat (buoyancy) by convection

to rate of transport of heat (buoyancy) by (molecular) conduction

when $B \sim K \dfrac{\partial^2 T}{\partial z^2}$

critical Rayleigh number is the smallest Ra for which stationary overturning will occur

(7) p' has units of $\underbrace{kg\, \underbrace{ms^{-2}}_{accel}}_{force\ reqd}\, \underbrace{m^{-2}}_{per\ unit\ area} \rightarrow kg\, m^{-1} s^{-2}$

$p' \sim F^a \rho^b z^c$ $\quad$ F has units of $(ms^{-2})(ms^{-1})m^2 \rightarrow m^4 s^{-3}$

$$kg\, m^{-1} s^{-2} \sim (m^4 s^{-3})^a (kg\, m^{-3})^b (m)^c$$

kg: $1 = b$

m: $-1 = 4a - 3b + c$

s: $-2 = -3a \Rightarrow a = 2/3$

$$\Rightarrow -1 = \underbrace{4\left(\tfrac{2}{3}\right)}_{8/3} - 3(1) + c \Rightarrow c = -1 + 3 - 8/3 = \frac{6}{3} - \frac{8}{3} = -\frac{2}{3}$$

$$\therefore p' \sim F^{2/3} \rho z^{-2/3}$$

(8) fully compressible eq. of continuity — all wind speeds

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\vec{\nabla}\cdot\vec{V}$$

anelastic eq. of continuity subsonic winds, deep

$$W\frac{\partial \ln \rho}{\partial z} = -\vec{\nabla}\cdot\vec{V}$$

Boussinesq eq. of continuity subsonic winds, shallow

$$\vec{\nabla}\cdot\vec{V} = 0$$

(9) $\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + B$

$w\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z}w + Bw$

$\frac{D}{Dt}\left(\frac{1}{2}w^2\right) = -\frac{1}{\rho}\frac{\partial p'}{\partial z}\frac{Dz}{Dt} + B\frac{Dz}{Dt}$

$d\left(\frac{1}{2}w^2\right) = \frac{-dp'}{\rho} + B\,dz$

$\int_0^{H(500\,hPa)} d\left(\frac{1}{2}w^2\right) = -\frac{1}{\rho}\int_{975\,hPa}^{500\,hPa} dp' + \int_0^{z(500\,hPa)} B\,dz$

$w(z=0)=0$

$\rho = \frac{p}{RT} = \frac{975\,hPa}{(287\,m^2s^{-2}\cancel{K^{-1}})(303\cancel{K})} = 0.0112\,hPa\,m^{-2}s^2$

$\frac{1}{2}(20\,ms^{-1})^2 = -\frac{\Delta p}{0.0112\,hPa\,m^{-2}s^{2}} + CAPE$

$-\Delta p = \underbrace{(200\,\cancel{m^2}\cancel{s^{-2}})(0.0112\,hPa\,\cancel{m^{-2}}\cancel{s^2})}_{2.24\,hPa} - \underbrace{1000\,J\,kg^{-1}}\times(0.0112\,hPa\,m^{-2}s^2)$

$\underbrace{-(1000\,\cancel{kg}\,\cancel{m^2}\cancel{s^{-2}}\,\cancel{kg^{-1}})(0.0112\,hPa\,\cancel{m^{-2}}\cancel{s^2})}_{11.2\,hPa}$

$\boxed{\Delta p = 8.96\,hPa}$