① $p = \rho R T$

$$\rho_{sample} = \frac{100\ \cancel{kPa} \times \frac{kg\ m s^{-2} m^{-2}}{\cancel{Pa}} \times \frac{10^3\ \cancel{Pa}}{\cancel{kPa}}}{(287\ m^2 s^{-2} K^{-1})(301.15\ K)} = 1.16\ kg\ m^{-3}$$

$$\rho_{env} = \rho_{sample} \times \frac{301.15\ K}{273.15\ K} = (1.16\ kg\ m^{-3})(1.103) = 1.28\ kg\ m^{-3}$$

$$B = -g\,\frac{\rho_{sample} - \rho_{env}}{\rho_{sample}}$$ acceleration due to buoyancy force

$\therefore$ Buoyancy force $= B\,\rho_{sample}$(Volume of sample)

$$= \left[-9.8\ ms^{-2}\ \frac{(1.16\ kg\ m^{-3} - 1.28\ kg\ m^{-3})}{1.16\ kg\ m^{-3}}\right](1.16\ kg\ m^{-3})(10^6\ m^3)$$

$$= 1.18 \times 10^6\ kg\ ms^{-2} \times \frac{nt}{kg\ m s^{-2}} = \boxed{1.18 \times 10^6\ nt}$$

$$B = \frac{\text{Force}}{\text{unit mass}} = \frac{1.18 \times 10^6\ kg\ ms^{-2}}{(1.16\ kg\ m^{-3})(10^6\ m^3)} = \boxed{1.02\ ms^{-2}}$$

② $\frac{Dw}{Dt} = \frac{D^2 z}{Dt^2} = B$

$w = \frac{Dz}{Dt} = Bt + a_1 \quad w(0) = 0 \Rightarrow a_1 = 0$

$z = \frac{Bt^2}{2} + a_2 \quad z(0) = 0 \Rightarrow a_2 = 0$

$t = \sqrt{\frac{2z}{B}} \Rightarrow$ for $z = 1, 2, 3, 4, 5$ km, $t = 44.3, 62.6, 76.7, 88.6, 9$

$\Rightarrow \boxed{w = 45.2, 63.9, 78.2, 90.4, 101\ ms^{-1}}$

③ $\pi = \left(\frac{p}{1000\,mb}\right)^{\kappa}$ $\kappa = \frac{R}{c_p} = \frac{287\,m^2 s^{-2} K^{-1}}{1004\,m^2 s^{-2} K^{-1}} = 0.286$

p(mb)	π
100	0.518
200	0.631
300	0.709
400	0.769
500	0.820
600	0.864
700	0.903
800	0.938
900	0.970
1000	1.0

(4) $B = -\frac{\rho'}{\bar{\rho}} g = \left(\frac{T'}{\bar{T}} - \frac{p'}{\bar{p}}\right) g$

$B = \frac{\theta'}{\bar{\theta}} g$

$T' = +5\,K \qquad \bar{T} = 273.15 \begin{cases} +20 = 293.15 & 850\ hPa \\ -10 = 263.15 & 500\ hPa \\ -35 = 238.15 & 300\ hPa \end{cases}$

$p' = +1\ hPa$

$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p} \qquad \bar{\theta} = \bar{T}\left(\frac{p_0}{\bar{p}}\right)^{R/c_p}$

$\bar{\theta} + \theta' = (\bar{T} + T')\left(\frac{p_0}{\bar{p}+p'}\right)^{R/c_p} = \theta$

$R/c_p = \frac{287}{1004} = 0.286$

$\therefore\ \theta' = \underbrace{(\bar{T}+T')\left(\frac{p_0}{\bar{p}+p'}\right)^{R/c_p}}_{\theta} - \underbrace{\bar{T}\left(\frac{p_0}{\bar{p}}\right)^{R/c_p}}_{\bar{\theta}}$

(a) $B_{850} = (9.8\,ms^{-2})\left[\frac{5\,K}{293.15\,K} - \frac{1\,hPa}{850\,hPa}\right] = 0.156\,ms^{-2}$

$B_{500} = (9.8\,ms^{-2})\left[\frac{5\,K}{263.15\,K} - \frac{1\,hPa}{500\,hPa}\right] = 0.167\,ms^{-2}$

$B_{300} = (9.8\,ms^{-2})\left[\frac{5\,K}{238.15\,K} - \frac{1\,hPa}{300\,hPa}\right] = 0.173\,ms^{-2}$

(b) $\bar{\theta}_{850} = 307.1 \qquad \theta = 312.2 \qquad B_{850} = 9.8\,ms^{-2}\left(\frac{312.2 - 307.1}{307.1}\right) = 0.163\,ms^{-2}$

$\bar{\theta}_{500} = 320.9 \qquad \theta = 326.8 \qquad B_{500} = 9.8\,ms^{-2}\left(\frac{326.8 - 320.9}{320.9}\right) = 0.180\,ms^{-2}$

$\bar{\theta}_{300} = 336.0 \qquad \theta = 342.8 \qquad B_{300} = 9.8\,ms^{-2}\left(\frac{342.8 - 336.0}{336.0}\right) = 0.198\,ms^{-2}$

(c) $\frac{Dw}{Dt} = -c_p \bar{\theta} \frac{\partial \pi'}{\partial z} + B$

$$\frac{Dw}{Dt} \approx 0 \Rightarrow \frac{\partial \pi'}{\partial z} = \frac{B}{c_p \bar{\theta}} = \frac{0.180 \text{ m s}^{-2}}{(1004 \text{ m}^2 \text{s}^{-2} \text{K}^{-1})(320.9 \text{ K})}$$

$$= 5.59 \times 10^{-7}$$

(5) $B = \frac{g}{T}\left[T' + T(0.609 r_v - r_e - r_i)\right] = 0$

so, $T' + T\,0.609 r_v - T r_e - T r_i = 0$

$T_{500\,hPa} = -4°C$ probably no ice $\Rightarrow r_i = 0$
$(-9+5)°C$

$273.15 - 4 = 269.15\,K$

assume $r_v = r_s$ inside cloud

at 500 hPa, $T = -4°C$, $r_v \simeq 6 - 0.25 = 5.75\,g\,kg^{-1}$
from a skew T-log p diagram

$$\frac{T' + T\,0.609 r_v}{T} = r_e = \frac{+5K + 269.15\,K(0.609)(0.00575)}{269.15\,K}$$

$$= 0.0221 = \boxed{22.1\,g\,kg^{-1}}$$