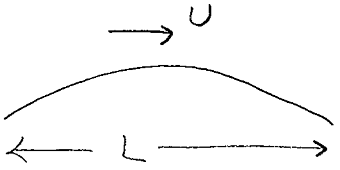
Mountain waves 1986

see Chap. 20 by Dale Durran 472–492 ; 1990 Mountain waves & downslope winds.

ed. P. Ray Mesoscale Meteorology & Forecasting

Atmos. Processes over Complex Terrain AMS 59-82
W. Blumen, ed.

role of mountain width:



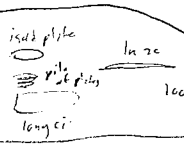
if $\frac{L}{U} > \frac{1}{f}$, rotational effects imp. → large horiz. displacements

$$\text{if } \frac{L}{U} < \frac{1}{f} \Rightarrow L < \frac{U}{f} = \frac{10\ \mathrm{ms^{-1}}}{10^{-4}\ \mathrm{s^{-1}}} = 10^5\ \mathrm{m} = 100\ \mathrm{km}$$

horiz. perturbations disappear, waves in vertical dominate

gravity waves → called mountain waves or lee waves

→ show wave cloud photos (diff. types



lnac look smooth

large-amplitude waves → strong downslope winds
· CAT

idealized, simple case

linear waves forced by sinusoidal mt. ridges

small-amplitude

2-D

$$Ro \gg 1 \quad \left(\frac{U}{fL} \gg 1 \Rightarrow \frac{U}{f} \gg L \Rightarrow L \ll \frac{U}{f} \right)$$

Boussinesq, f=0
no friction

comb. of our earlier analysis of grav. waves [& our earlier analysis of mixed-layer dynamics]

steady

$$\cancel{\frac{\partial U}{\partial t}}^{0} + U\frac{\partial U}{\partial x} + V\cancel{\frac{\partial U}{\partial y}}^{\frac{\partial}{\partial y}=0} + W\frac{\partial U}{\partial z} = -c_p\theta\frac{\partial \pi}{\partial x} \quad \text{where} \quad \pi = \left(\frac{p}{p_0}\right)^{R/c_p}$$

Exner fn

$$\cancel{\frac{\partial W}{\partial t}}^{0} + U\frac{\partial W}{\partial x} + V\frac{\partial W}{\partial y}^{?} + W\frac{\partial W}{\partial z} = -c_p\theta\frac{\partial \pi}{\partial z} - g$$

$$\cancel{\frac{\partial B}{\partial t}}^{0} + \cancel{U\frac{\partial B}{\partial x}} + \cancel{V\frac{\partial B}{\partial y}}^{?} + \cancel{W\frac{\partial B}{\partial z}} = 0$$

(actually, haven't defined B yet!) B is linearized form of θ

$\frac{\partial \theta}{\partial t} + \ldots$

$$U\frac{\partial \theta}{\partial x} + W\frac{\partial \theta}{\partial z} = 0$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}^{?} + \frac{\partial W}{\partial z} = 0$$

$\frac{1}{\rho}\frac{\partial p}{\partial x}$ $c_p T \left(\frac{p_0}{p}\right)^{R/c_p} \frac{R}{c_p}$

11/21/96

Whenever the Boussinesq eqns appear, do <u>not</u> start with adiabatic form of thermo eqn. as $\frac{DB}{Dt} = 0$; it requires linearizing B!

start with thermo. eqn.

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} + w\frac{\partial\theta}{\partial z} = 0$$

$$u = \bar{U} + u'(x,y,z,t)$$

$$w = \quad w'(x,y,z,t \qquad \bar{w} = 0$$

$$\theta = \bar{\theta}(z) + \theta'(x,y,z,t)$$

$$\cancelto{0}{\frac{\partial\bar{\theta}}{\partial t}} + \frac{\partial\theta'}{\partial t} + \bar{U}\cancelto{0}{\frac{\partial\bar{\theta}}{\partial x}} + u'\cancelto{0}{\frac{\partial\bar{\theta}}{\partial x}} + \bar{U}\frac{\partial\theta'}{\partial x} + u'\cancelto{0}{\frac{\partial\theta'}{\partial x}} +$$

$$\bar{V}\cancelto{0}{\frac{\partial\bar{\theta}}{\partial y}} + v'\cancelto{0}{\frac{\partial\bar{\theta}}{\partial y}} + \bar{V}\frac{\partial\theta'}{\partial y} + v'\cancelto{0}{\frac{\partial\theta'}{\partial y}} +$$

$$w'\frac{\partial\bar{\theta}}{\partial z} \qquad + w'\cancelto{0}{\frac{\partial\theta'}{\partial z}} = 0$$

now, $B = g\frac{\theta'}{\bar{\theta}}$ appears in linearized w eqn of motion

so, $\times\ \frac{g}{\bar{\theta}} \rightarrow \quad \frac{\partial B}{\partial t} + \bar{U}\frac{\partial B}{\partial x} + \bar{V}\frac{\partial B}{\partial y} + w'\underbrace{\left[\frac{g}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z}\right]}_{N^2} = 0$

linearize abt basic state — not const or 0 as before

$$u = \overline{U}(z) + u'(x,z)$$
$$w = w'(x,z) \qquad (\overline{w}=0)$$
$$\pi = \overline{\pi}(z) + \pi'(x,z) \qquad \theta = \overline{\theta}(z) + \theta'(x,z)$$

hydrostatic mean state

$$c_p \frac{\partial \overline{\pi}}{\partial z} = -\frac{g}{\overline{\theta}}$$

$$\overline{U}\cancel{\frac{\partial \overline{U}}{\partial x}}^{0} + \overline{U}\frac{\partial u'}{\partial x} + u'\cancel{\frac{\partial \overline{U}}{\partial x}}^{0} + \cancel{u'\frac{\partial u'}{\partial x}} + w'\frac{\partial \overline{U}}{\partial z} + \cancel{w'\frac{\partial u'}{\partial z}} = -c_p\overline{\theta}\frac{\partial \pi'}{\partial x} \cancel{- c_p\theta'\frac{\partial \pi'}{\partial x}}$$

$$\overline{U}\frac{\partial w'}{\partial x} + \cancel{u'\frac{\partial w'}{\partial x}} + \cancel{w'\frac{\partial w'}{\partial z}} = \underbrace{\cancel{-c_p\overline{\theta}\frac{\partial \overline{\pi}}{\partial z}}}_{-g} - c_p\theta'\underbrace{\frac{\partial \overline{\pi}}{\partial z}}_{-\frac{g}{\overline{\theta}c_p}} - c_p\overline{\theta}\frac{\partial \pi'}{\partial z} \cancel{- c_p\theta'\frac{\partial \pi'}{\partial z}}$$

$$+\frac{\cancel{c_p}\theta' g}{\cancel{c_p}\underbrace{\overline{\theta}}_{\sim\theta_0}} = B$$

$$\cancel{\overline{U}\frac{\partial \overline{\theta}}{\partial x}} + \overline{U}\frac{\partial \theta'}{\partial x} + \cancel{u'\frac{\partial \theta'}{\partial x}} + w'\frac{\partial \overline{\theta}}{\partial z} + \cancel{w'\frac{\partial \theta'}{\partial z}} = 0$$

$$\cancel{\overline{U}\frac{\partial \theta'/\overline{\theta}}{\partial x} + \frac{w'}{\overline{\theta}}\frac{\partial \overline{\theta}}{\partial z} + w'\frac{\partial \theta'/\overline{\theta}}{\partial z} = 0}$$

$\times\, g$

$$\cancel{\overline{U}\frac{\partial B}{\partial x}} + \cancel{u'\frac{\partial B}{\partial x}} + w'\underbrace{\frac{\partial B}{\partial z}} = 0$$

$$\underbrace{\frac{g}{\overline{\theta}}\frac{\partial \theta'}{\partial z}}_{N^2} \qquad \overline{\theta} \sim \theta_0$$

$$-\frac{\theta' g}{\overline{\theta}^2}\frac{\partial \overline{\theta}}{\partial z} \qquad \cancel{\left(\frac{\theta'}{\overline{\theta}}\right)\frac{g}{\overline{\theta}}\frac{\partial \overline{\theta}}{\partial z}} \text{ small}$$

$$\cancel{\frac{\partial \theta}{\partial t}} + u\frac{\partial \theta}{\partial x} + \cancel{v\frac{\partial \theta}{\partial y}} + w\frac{\partial \theta}{\partial z} = 0$$

$$\cancel{\overline{U}\frac{\partial \overline{\theta}}{\partial x}} + \overline{U}\frac{\partial \theta'}{\partial x} + w'\frac{\partial \overline{\theta}}{\partial z} = 0$$

$\times \frac{g}{\theta}$

$$B = \frac{g\theta'}{\theta} \quad \left\{ \begin{array}{l} \overline{U}\frac{g}{\theta}\frac{\partial \theta'}{\partial x} + w'\frac{g}{\theta}\frac{\partial \overline{\theta}}{\partial z} = 0 \\ \overline{U}\frac{\partial B}{\partial x} + w' N^2 = 0 \end{array} \right.$$

$$\cancel{\frac{\partial \overline{U}}{\partial x}}^{0} + \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\overline{\theta} = \theta_0$$

let $P = c_p\overline{\theta}\,\pi \underset{-}{=} c_p\theta_0\pi$

so,

$$\begin{cases} \overline{U}\dfrac{\partial u'}{\partial x} + w'\dfrac{d\overline{U}}{dz} + \dfrac{\partial P'}{\partial x} = 0 \\ \overline{U}\dfrac{\partial w'}{\partial x} + \dfrac{\partial P'}{\partial z} = B \\ \overline{U}\dfrac{\partial B}{\partial x} + N^2 w' = 0 \\ \dfrac{\partial u'}{\partial x} + \dfrac{\partial w'}{\partial z} = 0 \end{cases}$$

4 eqns, u', w', P', B

solve for w'

eliminate p'

$\frac{\partial}{\partial z}$ x-eqn of motion
$$\frac{d\bar{U}}{dz}\frac{\partial u'}{\partial x} + \bar{U}\frac{\partial^2 u'}{\partial x\partial z} + \frac{\partial w'}{\partial z}\frac{d\bar{U}}{dz} + w'\frac{d^2\bar{U}}{dz^2} + \frac{\partial^2 p'}{\partial z\partial x} = 0$$

$-$ $\frac{\partial}{\partial x}$ z-eqn of motion
$$\xcancel{\frac{\partial\bar{U}}{\partial x}}^{\,0}\frac{\partial w'}{\partial x} + \bar{U}\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 p'}{\partial x\partial z} = \frac{\partial B}{\partial x}$$

$$\frac{d\bar{U}}{dz}\frac{\partial u'}{\partial x} + \bar{U}\frac{\partial}{\partial x}\left(\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x}\right) + \frac{\partial w'}{\partial z}\frac{d\bar{U}}{dz} + w'\frac{d^2\bar{U}}{dz^2} = -\frac{\partial B}{\partial x}$$

elim. B

$-$ thermo eqn.
$$\frac{N^2 w'}{\bar{U}} = -\frac{\partial B}{\partial x}$$

$$\cancel{\frac{d\bar{U}}{dz}\frac{\partial u'}{\partial x}} + \bar{U}\frac{\partial}{\partial x}\left(\frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x}\right) + \cancel{\frac{\partial w'}{\partial z}\frac{d\bar{U}}{dz}} + w'\frac{d^2\bar{U}}{dz^2} - \frac{N^2 w'}{\bar{U}} = 0$$

elim. u'

cont. eqn. $\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$

cont.
$$\bar{U}\frac{\partial}{\partial z}\left(-\frac{\partial w'}{\partial z}\right) - \bar{U}\frac{\partial^2 w'}{\partial x^2} + w'\frac{d^2\bar{U}}{dz^2} - \frac{N^2}{\bar{U}}w' = 0$$

$\times(-1)$
$$\bar{U}\frac{\partial^2 w'}{\partial z^2} + \bar{U}\frac{\partial^2 w'}{\partial x^2} + \left(\frac{N^2}{\bar{U}} - \frac{d^2\bar{U}}{dz^2}\right)w' = 0$$

$$\frac{\partial^2 w'}{\partial z^2} + \frac{\partial^2 w'}{\partial x^2} + \ell^2 w' = 0 \quad , \quad \ell^2 = \frac{N^2}{\bar{U}^2} - \frac{1}{\bar{U}}\frac{d^2\bar{U}}{dz^2}$$

↑ Scorer parameter

define terrain height $h(x) = h_0 \cos kx$
(above ref. level, e.g. sea level)
↑ amplitude of mtns

simplify: $N, \bar{U}$ const. (typically $\frac{d^2\bar{U}}{dz^2}$ is tiny in comp. w/ $N^2/\bar{U}^2$)

$z = z(x,t)$

$$\frac{Dz}{Dt} = \frac{dz}{dx}\underbrace{\frac{dx}{dt}}_{u}$$

kinematic b.c. at gnd: $w[x, h(x)] = \overbrace{(\bar{U}+u')}^{\text{total } u}\frac{\partial h}{\partial x}$

so, $w'(x,0) \approx \bar{U}\frac{\partial h}{\partial x} = -\bar{U}h_0 k \sin kx$

2nd derivs $\Rightarrow$ Sinusoidal (+ phase)

sin & cos
$\leftarrow$ allows for phase shift

sdns. Separable

$$w'(x,z) = \hat{w}_1(z)\cos kx + \hat{w}_2(z)\sin kx$$

plug into eqn.

$$\frac{\partial w'}{\partial z} = \frac{d\hat{w}_1}{dz}\cos kx + \frac{d\hat{w}_2}{dz}\sin kx$$

$$\frac{\partial^2 w'}{\partial z^2} = \frac{d^2\hat{w}_1}{dz^2}\cos kx + \frac{d^2\hat{w}_2}{dz^2}\sin kx$$

$$\frac{\partial w'}{\partial x} = -\hat{w}_1 k \sin kx + \hat{w}_2 k \cos kx$$

$$\frac{\partial^2 w'}{\partial x^2} = -\hat{w}_1 k^2 \cos kx - \hat{w}_2 k^2 \sin kx = -k^2 w'$$

not needed!

$$l^2 w' = l^2 \hat{w}_1 \cos kx + l^2 \hat{w}_2 \sin kx$$

so,
$$\frac{d^2\hat{w}_1}{dz^2}\cos kx + \frac{d^2\hat{w}_2}{dz^2}\sin kx$$
$$- \hat{w}_1 k^2 \cos kx - \hat{w}_2 k^2 \sin kx$$
$$+ l^2 \hat{w}_1 \cos kx + l^2 \hat{w}_2 \sin kx = 0$$

$$\left\{\begin{array}{l} \dfrac{d^2\hat{w}_1}{dz^2} + (l^2 - k^2)\hat{w}_1 = 0 \\ \dfrac{d^2\hat{w}_2}{dz^2} + (l^2 - k^2)\hat{w}_2 = 0 \end{array}\right.$$

$$l^2 \cong \frac{N^2}{U^2} = \text{const.}$$

$$k^2 = \text{const}$$

$$\Rightarrow l^2 - k^2 \equiv m^2 \quad \text{const}$$

so, $\hat{w}_1 \sim e^{imz}$ etc. $(\hat{w}_2)$

$\mu^2 = -m^2$

$$= \begin{cases} A_1 e^{\mu z} + B_1 e^{-\mu z} & m^2 < 0 \Rightarrow l^2 - k^2 < 0 \Rightarrow l^2 < k^2 \Rightarrow k > l \\ A_1' \cos mz + B_1' \sin mz & m^2 > 0 \Rightarrow k < l \end{cases}$$

horiz. wavenumber

Scorer parameter

etc.

$\Rightarrow$ Vertical structure of wave ($\sim e^{\pm\mu z}$ or sinusoidal) $\sim$ l, k relative to each other

When $k > l$

b.c. – w' doesn't blow up at $z \to \infty$

$$\Rightarrow A_1 = 0$$

$$\boxed{A_2 = 0} \text{ for } \hat{w}_2$$

kinematic lower b.c.

$$\text{at } z = 0, \quad \overbrace{B_1 \cos kx + B_2 \sin kx = -\bar{U} h_0 k \sin kx}$$

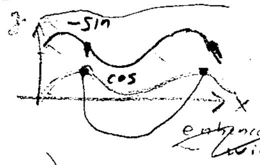
$$\Rightarrow B_1 = 0$$

$$B_2 = -\bar{U} h_0 k$$

$$\text{so, } \boxed{w'(x,z) = -\bar{U} h_0 k e^{-\mu z} \sin kx}$$

evanescent waves

phase shift w.r.t. mts.



~~closely spaced stream fn.~~

~~enhancement of winds above ridges~~

$k < l$

in gen'l
$$w'(x,z) = \left(A_1' \cos mz + B_1' \sin mz\right) \cos kx + \left(A_2' \cos mz + B_2' \sin mz\right) \sin kx$$

$$= A_1' \cos mz \cdot \cos kx + B_1' \sin mz \cdot \cos kx + A_2' \cos mz \cdot \sin kx + B_2' \sin mz \cdot \sin kx$$

can be written as ↙ using trig. identities

$$= \underbrace{C_1 \sin(kx + mz)} + \underbrace{C_2 \sin(kx - mz)} + \underbrace{C_3 \cos(kx + mz)} + C_4 \cos(kx - mz)$$

side / check: $C_1[\sin kx \cos mz + \cos kx \sin mz]$ $C_2[\sin kx \cos mz - \cos kx \sin mz]$ $C_3[\cos kx \cos mz - \sin kx \sin mz]$ $C_4[\cos kx \cos mz + \sin kx \sin mz]$

$$C_1 + C_2 = A_2'$$
$$C_1 - C_2 = B_1'$$
$$C_3 + C_4 = A_1'$$
$$C_4 - C_3 = B_2'$$

l.b.c. – $w'(x,0) = -\bar{U} h_0 k \sin kx$

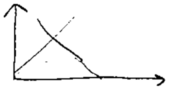
$$C_3 + C_4 \equiv A_1' = 0 \qquad (\cos kx \text{ dependence})$$

$$C_1 + C_2 \equiv A_2' = -\bar{U} h_0 k$$

now, must use upper b.c.'s

<u>not</u> <u>obvious</u>!

lines of constant phase $= kx \pm mz$



tilted lines

C_1 & C_3 $kx + mz$ lines tilt \ $\phi_1 = kx + mz$, $z = \frac{\phi_1}{m} - \frac{k}{m} x$

C_2 & C_4 $kx - mz$ lines tilt / $\phi_2 = kx - mz$, $z = -\frac{\phi_2}{m} + \frac{k}{m} x$

Can be shown

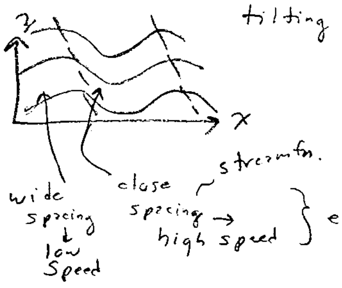
upstream tilting waves transport energy upward & momentum downward
downstream " " " " downward & " upward

mtns. are energy source ⇒ select upstream tilting soln.

$$C_2, C_4 = 0$$
$$\Rightarrow C_3 = 0$$
$$\Rightarrow C_1 = A_2' = -\bar{U} h_0 k$$

$w' \sim \sin kx$

so, $\boxed{w'(x,z) = -\bar{U} h_0 k \sin(kx+mz)}$ for $\bar{U}$ = const.



tilting w/ht

enhancement on lee side like in Colo. Chinook

an alternative argument: for selecting upstream tilting

x-eqn. $\bar{U}\frac{\partial u'}{\partial x} + w'\cancel{\frac{d\bar{U}}{dz}} + \frac{\partial p'}{\partial x} = 0$

cont. $\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$

$$\bar{U}\left(-\frac{\partial w'}{\partial z}\right) + \frac{\partial p'}{\partial x} = 0 \quad \rightarrow \quad \frac{\partial p'}{\partial x} = \bar{U}\frac{\partial w'}{\partial z}$$

plug in w'

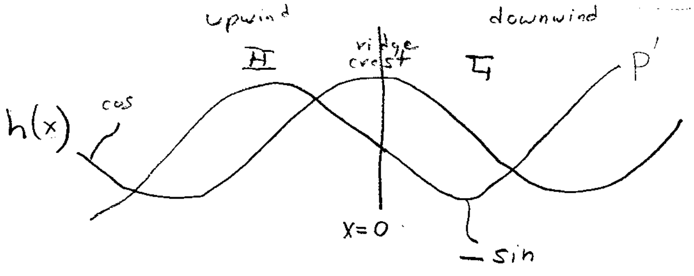
$$\frac{\partial p'}{\partial x} = -\bar{U}^2 h_0 k m \cos(kx+mz)$$

$\Rightarrow$ $p' = -\bar{U}^2 h_0 m \sin(kx+mz)$ for upstream tilting

(for downstream tilting $p' \sim \bar{U}^2 h_0 m \sin(kx-mz)$)

recall, $h(x) = h_0 \cos kx$

$\Rightarrow$ phase shift 90° at $z=0$ between p' ($\sim -\sin kx$) & h ($\sim \cos kx$)



[illegible] Eta model, NGM

← See numerical models!

H windward side

L lee side

→ force in direction of mean flow on topography ⇒ terrain exerts equal & opposite force to decelerate flow, which makes sense

if waves tilted downstream w/ht, p-forces would accel flow in direction it's going! not good!

terrain?

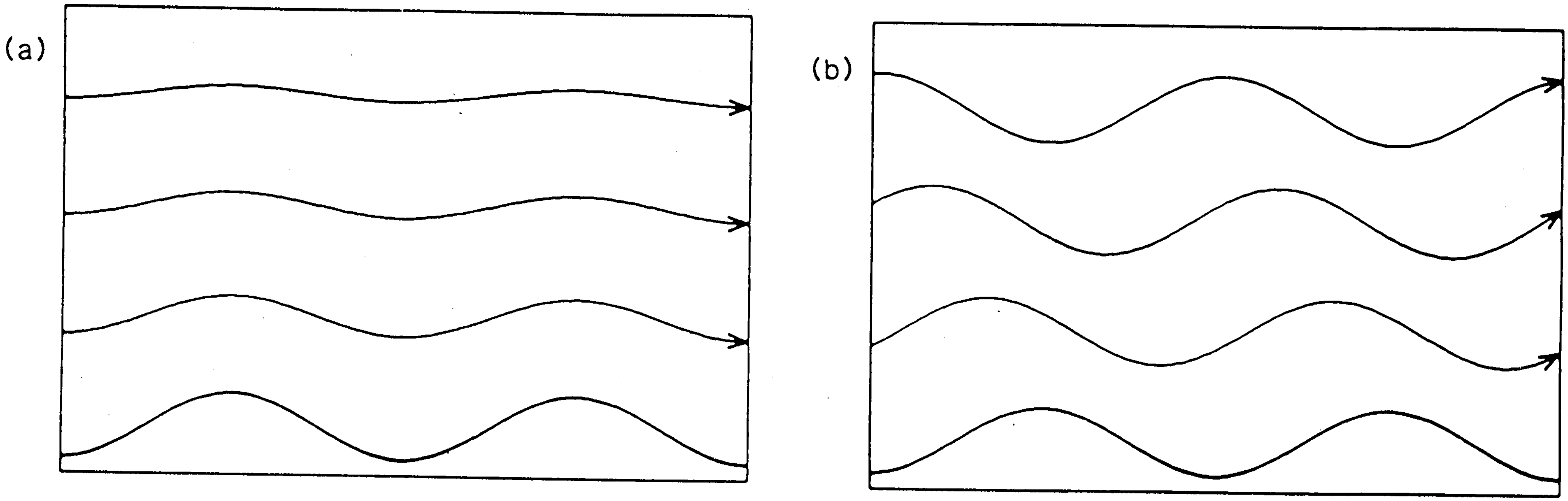


Figure 20.1. Streamlines in the steady airflow over an infinite series of sinusoidal ridges when (a) the wavenumber of the topography exceeds the Scorer parameter (narrow ridges) or (b) the wavenumber of the topography is less than the Scorer parameter (wide ridges).

→ physical reason for difference between $k < l$ (tilted) & $k > l$ (evanesc
waves

Skip

$$l^2 = \frac{N^2}{\overline{U}^2}$$ if $\frac{d^2\overline{U}}{dz^2}$ small, i.e. if vertical profile of $\overline{U}$ has little curvature

$$l = \frac{N}{\overline{U}}$$

$$k < l \Rightarrow k < \frac{N}{\overline{U}} \Rightarrow \overline{U}k < N$$

$\left(\frac{2\pi}{N} < \frac{L_x}{\overline{U}} \right)$

period of buoyancy osc.

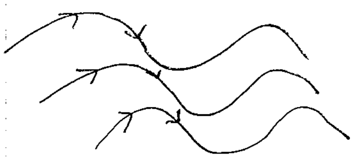
time to move scale of orographic forcing

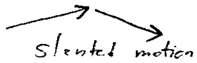
$\frac{2\pi}{L_x}$

"intrinsic freq. of forcing by terrain"

Brunt–Väisälä freq.

↑↓↑ up-down motion





slanted motion

tilt to match N to "intrinsic freq. forced by airflow over terrain"

↑ in vert. prop. gravity waves

if $k > l \Rightarrow \overline{U}k > N$

no slanted motion will work & no buoyancy-driven osc. can occur ⇒ buoyancy acts to dampen oscillation

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→ more realistic cases (not constant N, not constant $\overline{U}$) $\frac{N^2}{\overline{U}^2}$ not const.

not endless series of sinusoidal ridges)

consider first

Smith (1979) Adv. in Geophys 21 87–230 "The influence of mountains on the atmosphere"

Queney et al. (1960) "The airflow over mountains" WMO Tech. Note 34, 135 pp.

isolated mtn. → use Fourier transforms

$$f(x) \longleftrightarrow F(k)$$

$$F\left(\frac{\partial f^n}{\partial x^n}\right) = (ik)^n F(k)$$

so, take eqn. for w', $\frac{\partial^2 w'}{\partial z^2} + \frac{\partial^2 w'}{\partial x^2} + l^2 w' = 0$

$$\overset{\displaystyle w'}{\downarrow}$$
$$f \longleftrightarrow F(W)$$

Fourier transform eqn:

$$\frac{d^2 W}{dz^2} + (ik)^2 W + \ell^2 W = 0$$

$$\frac{d^2 W}{dz^2} + (\ell^2 - k^2) W = 0$$

$s^2 = \ell^2 - k^2$

$'' + s^2(\,) = 0$

$e^{-\sqrt{s^2}}$

get $W(k, z)$ for each k

get inverse transforms W' and add them all up

let (again) N, $\bar{U}$ be const. (i.e. indep. of z)

$$\Rightarrow \ell^2 = \frac{N^2}{\bar{U}^2} - \cancel{\frac{1}{\bar{U}^2}\frac{d^2\bar{U}}{dz^2}} \to 0 \qquad \ell = \frac{N}{\bar{U}} = \text{const}$$

---

not clear

$$\text{so, } W(k, z) = A_1 e^{+\sqrt{\ell^2 - k^2}\, z} + \underbrace{A_2 e^{-\sqrt{\ell^2 - k^2}\, z}}_{\text{complex}}$$

use b.c.'s: W doesn't blow up as $z \to \infty$

free slip

$w'(x, z=0) \approx \bar{U}\frac{dh}{dx}$ ⟷ $W(k, 0) = (ik) H_{(k,z)} \bar{U}$

H ← h

(k,z) Fourier transform of terrain profile

$e^{\sqrt{-1}\sqrt{-a^2}} = e^{\sqrt{a^2}}$

→ $A_1 = 0$

$ik\bar{U}H(k,z) = A_2$

$e^{\sqrt{a^2}} = e^{i\sqrt{-a^2}}$

$$\Rightarrow W(k, z) = ik\bar{U}H(k,z)\, e^{\overset{\sqrt{\ell^2-k^2}\,z}{\cancel{-\sqrt{(k^2-\ell^2)}\,z}}}$$

$$= ik\bar{U}\underbrace{H(k,z)}_{\text{weighting fn}}\, e^{\cancel{\sqrt{(\ell^2-k^2)}\,z}}$$

$e^{-i\sqrt{k^2-\ell^2}} = \cos(-\sqrt{\;\;}) + i\sin(\;\;)$

$k > 0$

~~if $\ell < k \to k > \ell$~~

$= \sqrt{-(k^2-\ell^2)}\, z = -i\sqrt{k^2-\ell^2}\, z$

$= \sqrt{\ell^2 - k^2}\, z = i$?

$-\sqrt{\ell^2-k^2}$

$i\sqrt{\ell^2-k^2}$

$i^2 = -1$

$i = \sqrt{-1}$

if mtn. vy narrow, dominant weighting ?

$k > \ell \Rightarrow$ evanescent waves

$$\frac{2\pi}{L_x} > \frac{N}{\bar{U}} \Rightarrow L_x < \frac{2\pi \bar{U}}{N}$$

distance traveled in buoyancy time scale by flow

if mtn. wide, $k < \ell \Rightarrow$ vert prop waves

$$\text{for } h(x) = \frac{h_0 a^2}{a^2 + x^2}$$

bell-shaped ridge $\Rightarrow a^{-1}$ characteristic of scales forced by mtn. wavenumbers

↑ "Witches of Agnesi" profile

$\begin{cases} a \text{ large} \Rightarrow \text{broad mtn.} \\ a \text{ small} \Rightarrow \text{narrow mtn} \end{cases} = \dfrac{h_0}{1 + \left(\frac{x}{a}\right)^2}$

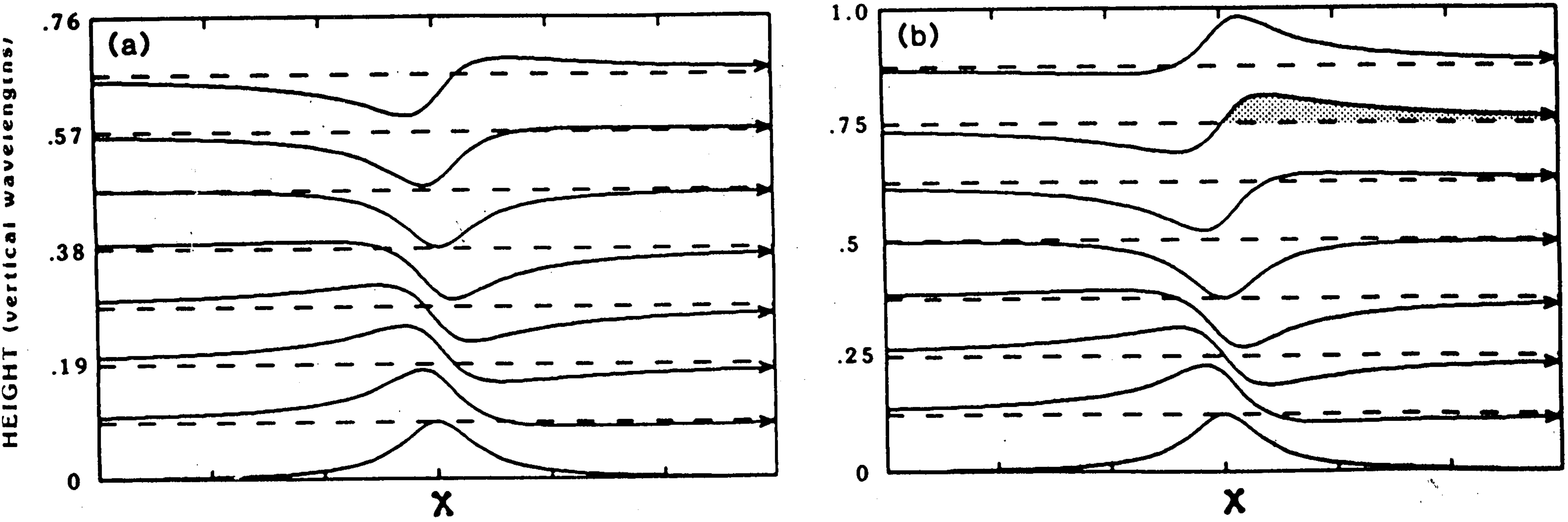


Figure 20.12. Streamlines in the steady airflow over an isolated bell-shaped mountain when the vertical wavelength in (a) exceeds the wavelength in (b) because of a reduction in the stability due to cloudiness over the windward slopes. Probable location of wave-induced cirrus is shaded.

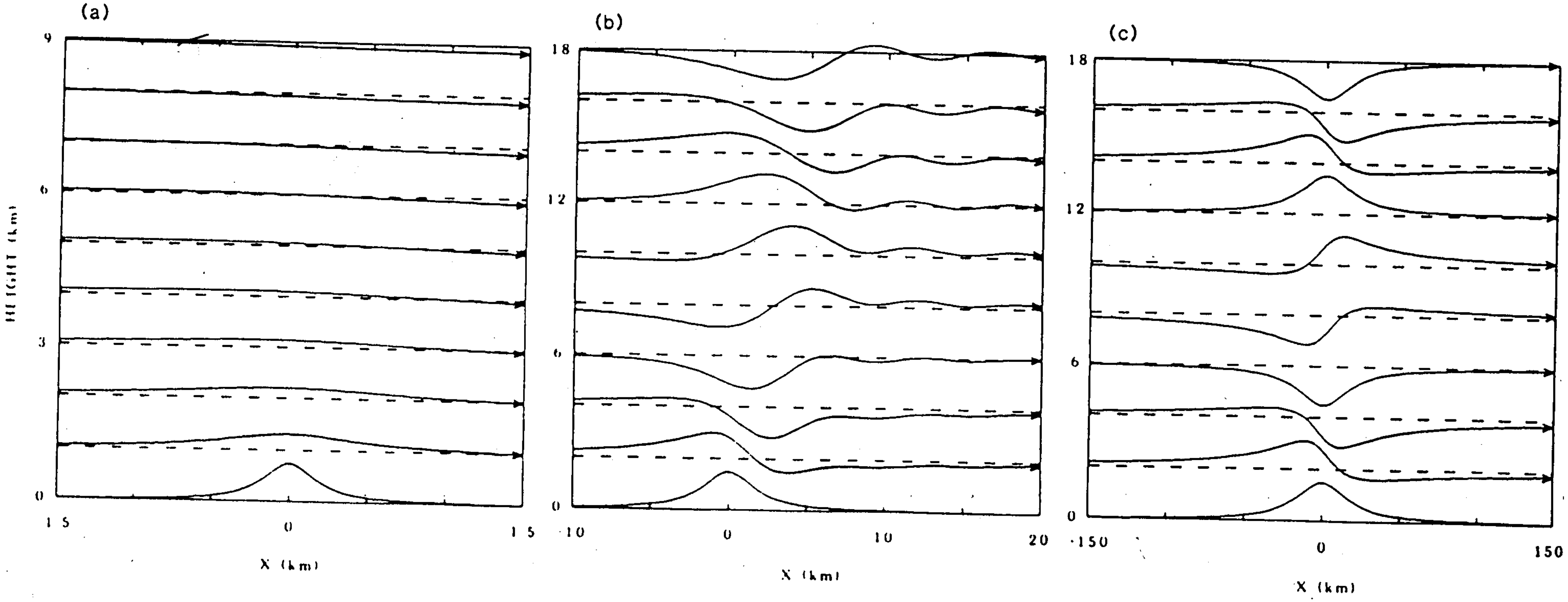


Figure 20.2. Streamlines in the steady airflow over an isolated bell-shaped ridge. (a) $a^{-1} >> \ell$, narrow ridge; (b) $a^{-1} = \ell$, width of the ridge comparable with the Scorer parameter; (c) $a^{-1} << \ell$, wide ridge, but not so wide that rotational effects become significant.

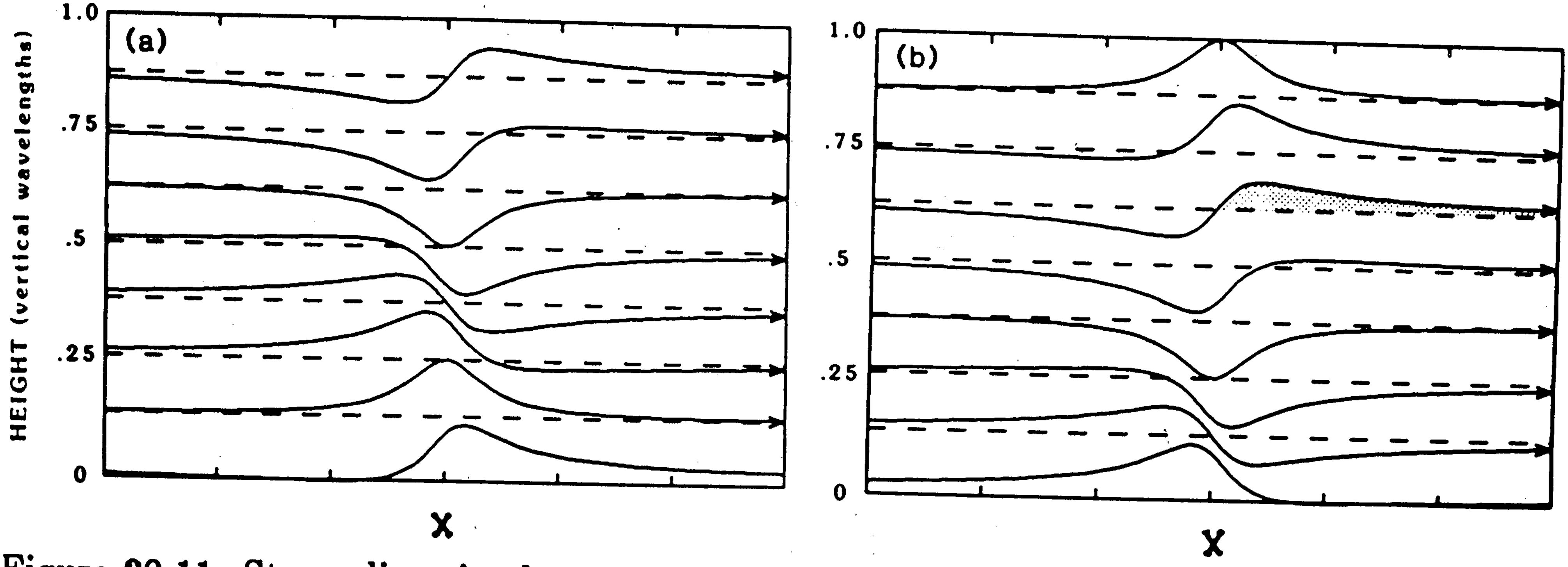
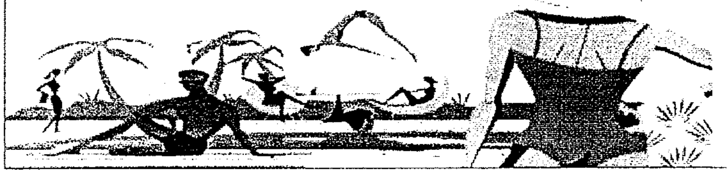


Figure 20.11. Streamlines in the steady airflow over an isolated asymmetric ridge with (a) a steep windward slope; (b) a steep leeward slope. Probable location of wave-induced cirrus is shaded.



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Maria Agnesi

18th cent.

(May 16, 1718 - January 9, 1799)
mathematician, philosopher, philanthropist
Maria Gaetana Agnesi, Maria Gaëtana Agnesi

- wrote first mathematics book by a woman that still survives
- first woman appointed as a mathematics professor at a university

Maria Agnesi's father was Pietro Agnesi, a wealthy nobleman and a professor of mathematics at the University of Bologna. It was normal in that time for the daughters of noble families to be taught in convents, and to receive instruction in religion, household management and dressmaking. A few Italian families educated daughters in more academic subjects; a few attended lectures at the university or even lectured there.

Pietro Agnesi recognized the talents and intelligence of his daughter Maria. Treated as a child prodigy, she was given tutors to learn five languages (Greek, Hebrew, Latin, French and Spanish) and also philosophy and science.

The father invited groups of his colleagues to gatherings at their home, and had Maria Agnesi present speeches to the assembled men. By age 13, Maria could debate in the language of the French and Spanish guests, or she could debate in Latin, the language of the educated. She didn't like this performing, but she could not persuade her father to let her out of the task until she was twenty years old.

In that year, 1738, Maria Agnesi assembled almost 200 of the speeches she had presented to her father's gatherings, and published them in Latin as *Propositiones philosphicae* -- in English, *Philosophical Propositions*. But the topics went beyond philosophy as we think of the topic today, and included scientific topics like celstial mechanics, Isaac Newton's gravitation theory, and elasticity.

Pietro Agnesi married twice more after Maria's mother died, so that Maria Agnesi ended up the eldest of 21 children. In addition to her performances and lessons, her responsibility was to teach her siblings. This task kept her from her own goal of entering a convent.

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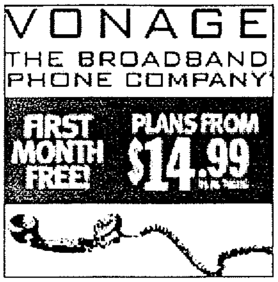

Also in 1783, wanting to do the best job of communicating up-to-date mathematics to her younger brothers, Maria Agnesi began to write a mathematics textbook, which absorbed her for ten years.

The *Instituzioni Analitiche* was published in 1748 in two volumes, over one thousand pages. The first volume covered arithmetic, algebra, trigonometry, analytic geometry and calculus. The second volume covered infinite series and differential equations. No one before had published a text on calculus that included the methods of calculus of both Isaac Newton and Gottfried Liebnitz.

Maria Agnesi brought together ideas from many contemporary mathematical thinkers -- made easier by her ability to read in many languages -- and integrated many of the ideas in a novel way that impressed the mathematicians and other scholars of her day.

As recognition of her achievement, in 1750 she was appointed to the chair of mathematics and natural philosophy at the University of Bologna by an act of Pope Benedict XIV. She was also recognized by the Hapsburg Empress Maria Theresa of Austria.

Did Maria Agnesi ever accept the Pope's appointment? Was it a real appointment or an honorary one? So far, the historical record does not answer those questions.

Maria Agnesi's name lives on in the name that English mathematician John Colson gave to a mathematical problem -- finding the equation for a certain bell-shaped curve. Colson confused the word in Italian for "curve" for a somewhat similar word for "witch," and so today this problem and equation still carries the name "witch of Agnesi."

Maria Agnesi's father was seriously ill by 1750 and died in 1752. His death released Maria from her responsibility to educate her siblings, and she used her wealth and her time to help those less fortunate. She established in 1759 a home for the poor. In 1771 she headed up a home for the poor and ill. By 1783 she was made director of a home for the elderly, where she lived among those she served. She had given away everything she owned by the time she died in 1799, and Maria Agnesi was buried in a pauper's grave.

Also on this site

- Top 10 Early Female Mathematicians
- Women and Mathematics - Women Mathematicians
- Women and University Education
- European Women's History
- Women and Christianity

Print Bibliography

- Smith, Sanderson. *Agnesi to Zeno: Over 100 Vignettes from the History of Math.* 1996.
- Tilche, Giovanni. Maria Gaetana Agnesi.
- book

Maria Agnesi on the web

- Maria Gaetana Agnesi - from the 1911 *Encyclopedia Britannica*

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- Maria Gaetana Agnesi - from a collection of biographies of people important to the development of the calculus
- Maria Gaëtana Agnesi - from a collection of biographies of mathematicians
- Maria Gaetana Agnesi - biography, includes a quotation from the Pope's appointment of her to a chair at the University of Bologna
- Maria Gaetana Agnesi - biography from the Catholic Encyclopedia, 1907
- Life History - Maria Agnesi - requires the (free) Adobe Acrobat Reader to view this illustrated profile of Maria Agnesi
- Timeline of Agnesi - chronology puts Maria Agnesi into the context of her time, comparing her timeline with that of other mathematical contemporaries

- **The Witch of Agnesi**
  - The Living Witch of Agnesi - an animated drawing of the mathematical curve
  - Witch of Agnesi - an illustration of the curve, with mathematical explanation
  - Witch of Agnesi - illustration and equation

About Maria Agnesi

- Categories: mathematician
- Places: Milan, Italy, Hapsburg Empire
- Period: 18th century
- Religious Associations: Roman Catholic

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⑨

$a^{-1} \gg l$ a small ⇒ narrow mtn

See Fig. $\begin{cases}\text{pattern symm. w.r.t. ridge crest} \\ \text{decay w/ ht}\end{cases}$

$a^{-1} \ll l$ a large ⇒ broad mtn.

See Fig. $\begin{cases}\text{waves prop vertically} \\ \text{tilt upstream}\end{cases}$

$$W(k,z) = ik\bar{U}\,H(k,z)\,e^{ilz}$$

$k \ll l$

hydrostatic limit $\longrightarrow W(k,z) \to 0$

$\frac{2\pi}{L_x} \ll l$

$L_x \gg \frac{2\pi}{l}$

no terms in pert. eq. in z

i.e. $\begin{cases}\bar{U}\frac{\partial w'}{\partial x} = 0 \\ \frac{\partial p'}{\partial z} = 0\end{cases}$

$B = 0$

skip

$a^{-1} = l$ (vert prop. nonhydrostatic waves

$k < l$

→ now, add structure to $l^2(\bar{U}(z), N(z))$

trapped waves (resonant lee waves)

lower trap on lee side

Show Fig

— not same as "ducted"

Scorer (1949) l^2 decreases w/ ht

no deriv. here!

$$l^2 = \frac{N^2}{\bar{U}^2} - \frac{1}{\bar{U}}\frac{d^2\bar{U}}{dz^2}$$

⇒ N decreases with height

or

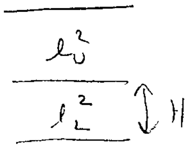
$\bar{U}$ increases with ht

or

$\boxed{\frac{d^2\bar{U}}{dz^2}\text{ (curvature) increases w/ ht}}$

$\frac{\cancel{s^{-2}}}{m^2\cancel{s^{-2}}}$ $\frac{1}{\cancel{m s^{-1}}}\,\frac{\cancel{m s^{-1}}}{m^2}$

2 layer



Wavenumber of trapped waves:

resonance condition

for solns to exist

$$\sqrt{l_L^2 - k^2}\,\cot\left[\sqrt{l_L^2 - k^2}\,H\right] = -\sqrt{(k^2 - l_U^2)} \Rightarrow l_L^2 - l_U^2 > \frac{\pi^2}{4H^2}$$

could be multiple solns.
longest wavelength usually dominates

$$l_L > k > l_U$$

i.e. in lower layer $k < l_L$ prop. (but no tilt!) reflected downward
in upper layer $k > l_U$ evanescent

possible wave motions
- vertically propagating
- evanescent
- resonant & trapped

} depends on $\overline{U}(z), N(z)$

shape of terrain → strength of forcing at each wavelength

usually $0.008 < N < 0.02 s^{-1}$
$10 < \overline{U} < 40 ms^{-1}$

$l^2 = \frac{N^2}{\overline{U}^2} - \frac{1}{\overline{U}}\frac{d^2\overline{U}}{dz^2}$ max l $k > \frac{N_{max}}{U_{min}} = \frac{0.02 s^{-1}}{10 ms^{-1}} = 2\times10^{-3} m^{-1} \Rightarrow \frac{2\pi}{L_x} = 2\times10^{-3} m^{-1}$

$L_x = \frac{2\pi}{2\times10^{-3} m^{-1}} \sim 3 km$

i.e. $\boxed{L_x \lesssim 3 km}$

→ evananescent

upper

$-\frac{1}{\overline{U}}\frac{d^2\overline{U}}{dz^2}$ "

lower

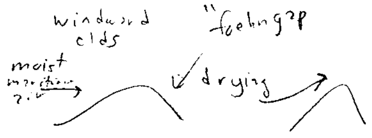
min l $k < \frac{N_{min}}{U_{max}} = \frac{0.008 s^{-1}}{40 ms^{-1}} = 2\times10^{-4} m^{-1} \Rightarrow \boxed{L_x \gtrsim \frac{2\pi}{2\times10^{-4} m^{-1}} \sim 30 km}$

→ propagate vertically
only one crest
long wavelength (could also be forcing is for long waves) can prop. even when l^2 small

$L_x \sim$
trapped waves → 5 → 25 km

multiple crests downstream
{ infr strong winds at ridge-top level
{ $\frac{d\overline{U}}{dz} >> 0$, strong N^2 in lower trop

→ foehn (cap) cloud over crest
distribution of clds
middle & low level
diff. in RH



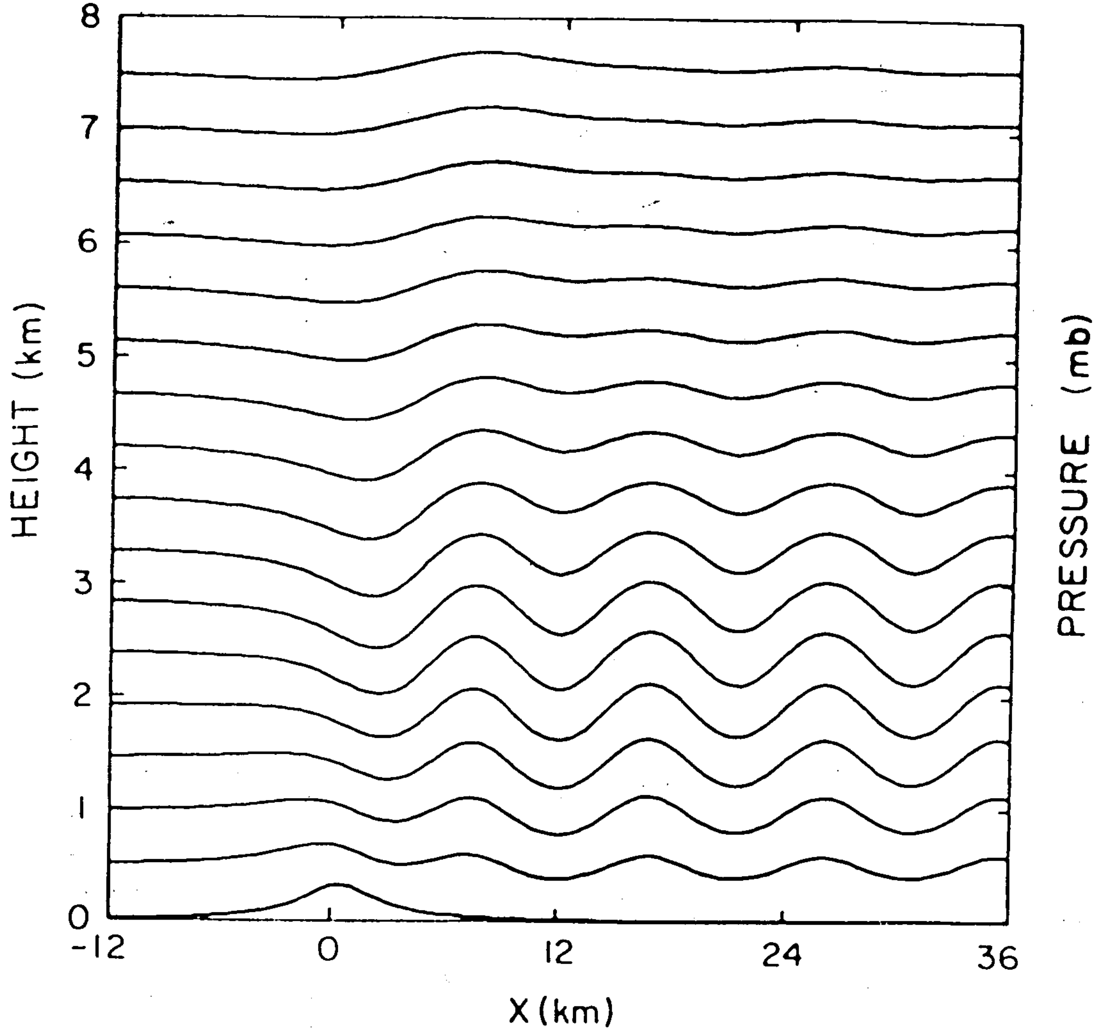
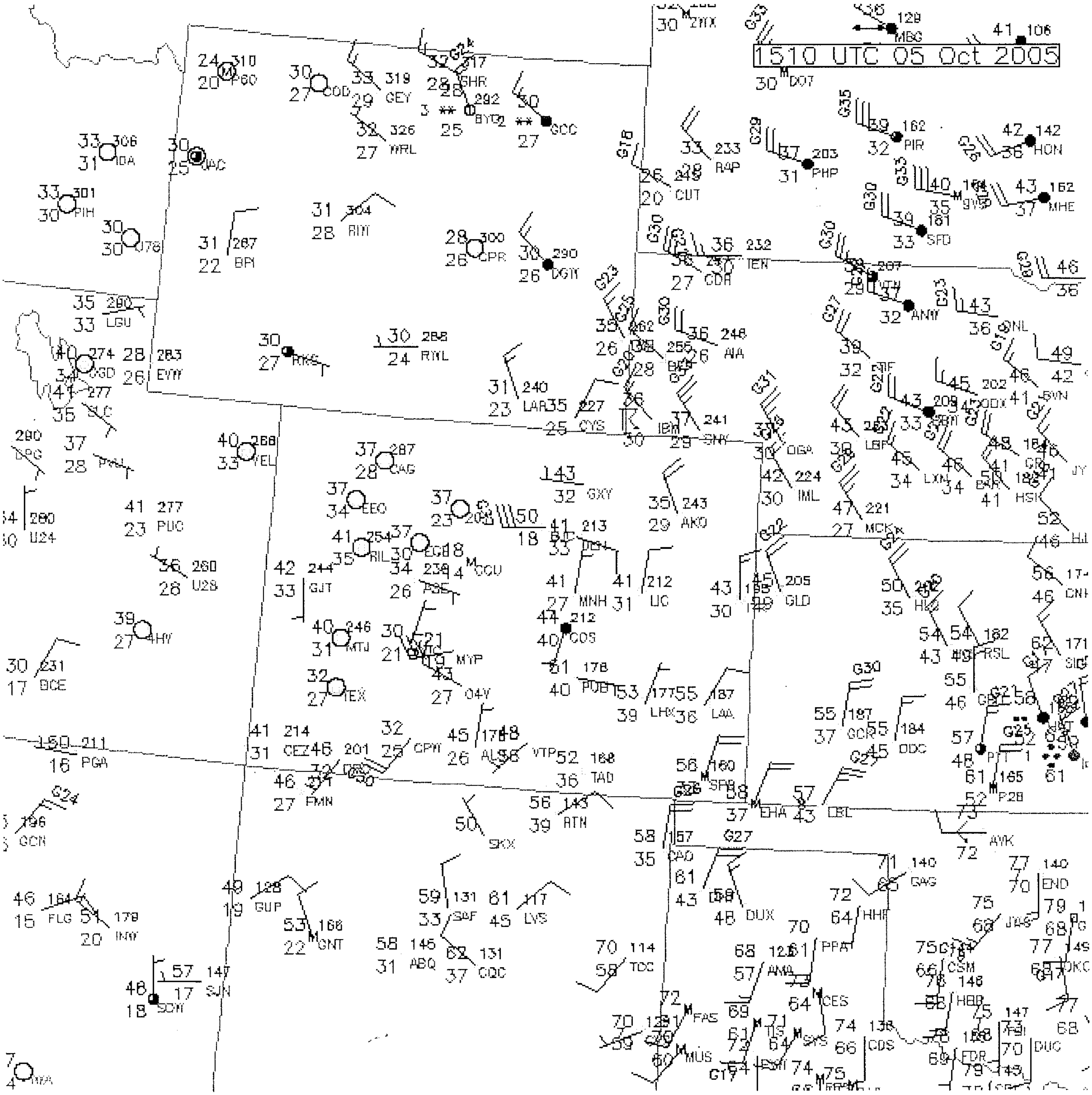


Figure 20.3. Streamlines in the steady airflow over an isolated bell-shaped ridge when the vertical variation of the Scorer parameter has the two-layer structure shown in Fig. 20.4.


1510 UTC 05 Oct 2005
P60
COD
GEY
SHR
BYG
GCC
IDA
JAC
WRL
PIH
RIW
BPI
CPR
DGW
U78
LGU
RKS
RWL
EVW
OGD
SLC
LAR
CYS
BPG
PVU
VEL
CAG
EEO
RIL
EGE
GJT
ASE
MTJ
MYP
COS
TEX
PUB
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ALS
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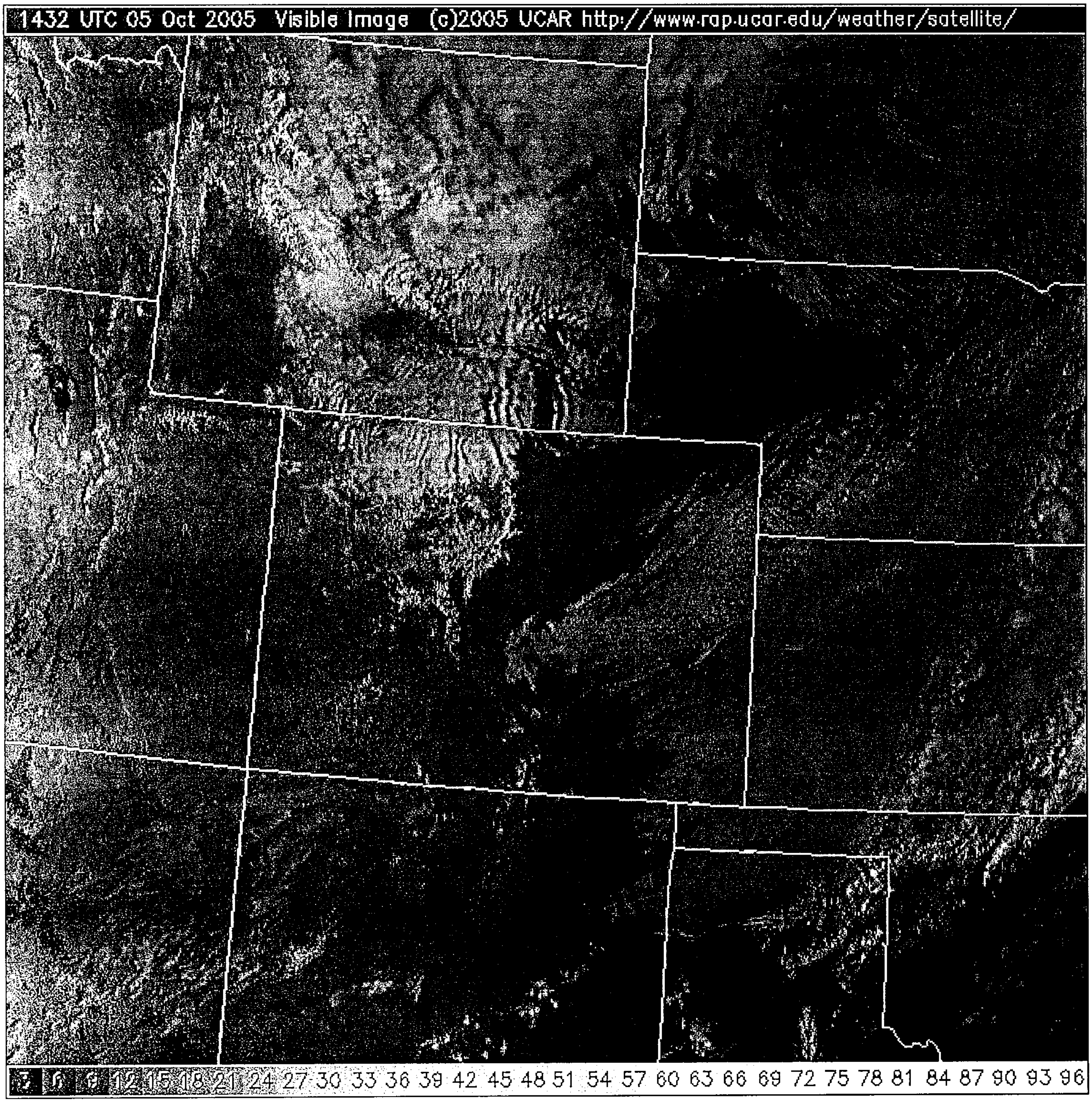
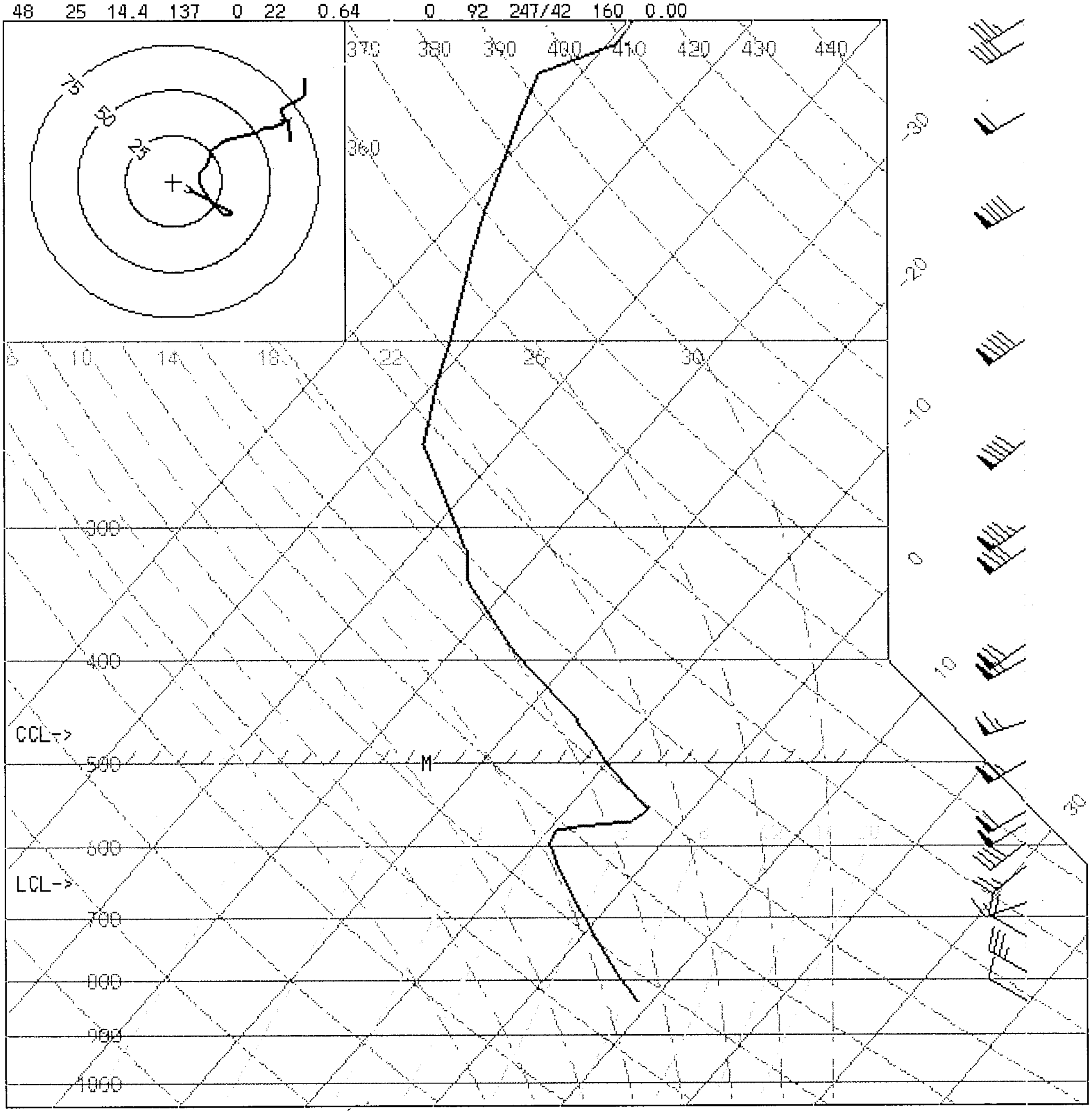
ONL
BBW
ODX
GRI
EAR
HSI
LXN
PHP
PIR
HON
MHE
SFD
9V9
MBG
DOZ
2WX

Satellite image(s) [DEN, vis]

generated from http://www.rap.ucar.edu/weather/satellite/ at Wed, 5 Oct 2005 15:12:19 GMT

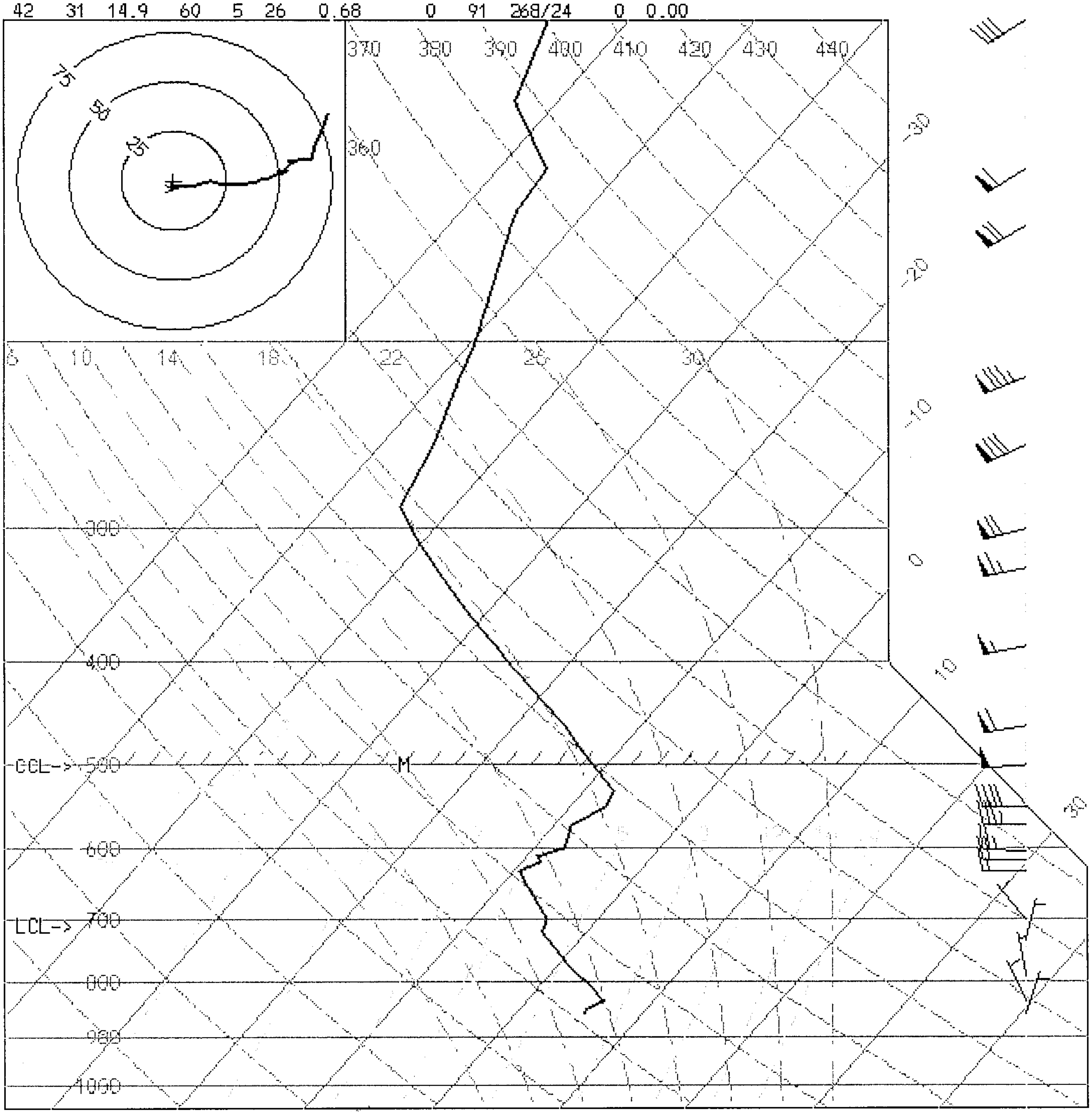


T(F)	Td	LI	SWT	K	TT	Pw(cm)	CAPE	Tc	CELL	SREH	VGP
48	25	14.4	137	0	22	0.64	0	92	247/42	160	0.00



SKEW-T/LOG-P VALID 1200 UTC 10/05/2005 KDNR Lat = 39.75 , Lon = -104.87

T(F)	Td	LI	SWT	K	TT	Pw(cm)	CAPE	Tc	CELL	SREH	VGP
42	31	14.9	60	5	26	0.68	0	91	268/24	0	0.00



SKEW-T/LOG-P VALID 1200 UTC 10/05/2005 KGJT Lat = 39.12 , Lon = -108.52

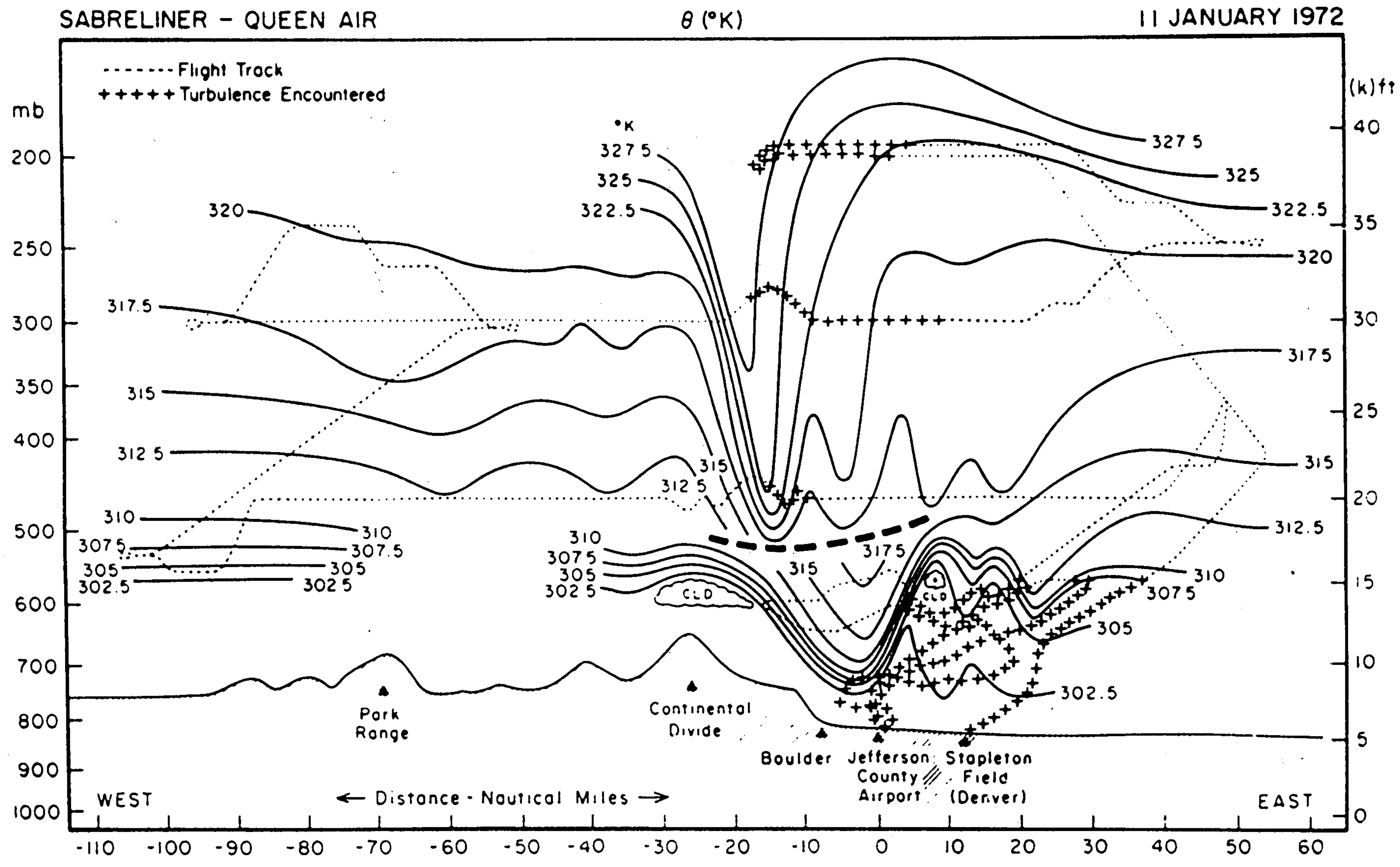


Figure 20.13. Cross section of the potential temperature field observed in a very strong mountain wave over Boulder, Colo., on 11 January 1972. The heavy dashed lines separate observations taken at different times; the dotted lines show the aircraft flight tracks; the crosses indicate regions of turbulence. (From Lilly and Zipser, 1972.)

From: "Pallutto, Marcia J." <marcia@ou.edu>
To: "Bluestein, Howard B." <hblue@ou.edu>
CC: "Henry, Judith S." <jhenry@ou.edu>, "Jones, Celia G." <cjones@ou.edu>,
"Campbell, Nancy S." <ncampbell@ou.edu>
Date: Tue, 7 Oct 2008 11:34:50 -0500
Subject: Room reservation
Thread-Topic: Room reservation
Thread-Index: Ackomp6USWtHf7+xRLKgBTX1UNvm7A==
Accept-Language: en-US
acceptlanguage: en-US
X-yoursite-MailScanner-Information: Please contact the ISP for more information
X-yoursite-MailScanner: Not scanned: please contact your Internet E-Mail Service Provider for details
X-yoursite-MailScanner-SpamCheck: not spam, SpamAssassin (score=0.101,
required 6, HTML_FONT_BIG 0.10, HTML_MESSAGE 0.00)

Howie,
I reserved NWC 5820 for you on Wednesday, Oct. 15th from 3-4 PM.

* *

Marcia J. Pallutto
Administrative Assistant I
OU School of Meteorology
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BOOMER SOONER!

Showed wave
clouds — photos
+
sat images
tried to relate to
theory

↑ high-level clds.

difference in terrain profiles and/or moisture effects

skip

Show Figs { steep windward, shallow lee
shallow windward, steep lee

↓ **show figs** deep cloudiness —
local stability reduced in sat'd air

see Mercer et al 2008 WaF

→ downslope windstorms (~~large~~ high - amplitude mtn waves)

linear thy — low-amplitude waves ⟹ need nonlinear theory

Show **Lilly & Zipser fig.**

theories

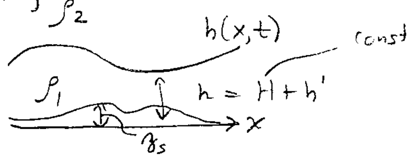
1. hydraulic jump
Long 1953

nonlinear
(?) Unrealistic upward boundary — vertically propagating waves not properly represented
wind!

first: linear theory

2 layers

See Holton



→ recall earlier analyses of Kelvin waves

no f
no fric.

u-eq. of motion

steady; $\frac{\partial}{\partial y}=0$; lower layer; hydrostatic; sym. no forcing

$$\cancel{\frac{\partial u}{\partial t}} + u\frac{\partial u}{\partial x} + v\cancel{\frac{\partial u}{\partial y}} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\Delta\rho}{\rho_1}\frac{\partial h'}{\partial x} - \cancel{\frac{1}{\rho_1}\left(\frac{\partial p}{\partial x}\right)_{z=H+h}}$$

$$u = \overline{u} + u' \qquad \overline{w}=0$$

const

$$\boxed{\overline{u}\frac{\partial u'}{\partial x} + g\frac{\Delta\rho}{\rho_1}\frac{\partial h'}{\partial x} = 0}$$

barotropic
($\overline{u}$ not fnct of z)

is $\frac{\partial u}{\partial x}$ indep of z? yes! what abt $\frac{\partial u'}{\partial x}$?

cont.

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \frac{\partial w}{\partial z} = 0$$

$$\int_{z_s}^{h}\frac{\partial w}{\partial z}dz = \int_{z_s}^{h} -\frac{\partial u}{\partial x}dz$$

$$w(h) - \underbrace{w(z_s)}_{u\frac{\partial z_s}{\partial x}} = -\frac{\partial u}{\partial x}(h - z_s)$$

$$\text{but } w(h) = \frac{Dh}{Dt} = \cancel{\frac{\partial h}{\partial t}}^{0} + u\frac{\partial h}{\partial x}$$

steady

$$\Longrightarrow \quad u\frac{\partial z_s}{\partial x} - \frac{\partial u}{\partial x}(h - z_s) = u\frac{\partial h}{\partial x}$$

$$u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} - z_s\frac{\partial u}{\partial x} - u\frac{\partial z_s}{\partial x} = 0$$

nonlinearize)

$+\bar{U}\frac{\partial z_s}{\partial x}$

small ampl. mtns!

$-z_s\frac{\partial U'}{\partial x} \to 0$ $\quad -U'\cancel{\frac{\partial z_s}{\partial x}}$ small

$$\bar{U}\frac{\partial h'}{\partial x} + \cancel{h'\frac{\partial U'}{\partial x}} + H\frac{\partial U'}{\partial x} - \bar{U}\frac{\partial z_s}{\partial x} = 0$$

$$\boxed{\bar{U}\frac{\partial}{\partial x}(h'-z_s) + H\frac{\partial U'}{\partial x} = 0}$$

$$\frac{\partial U'}{\partial x} = -\frac{\bar{U}}{H}\frac{\partial}{\partial x}(h'-z_s)$$

plug into eq. of motion

$$\bar{U}\left(-\frac{\bar{U}}{H}\frac{\partial}{\partial x}(h'-z_s)\right) + g\frac{\Delta\rho}{\rho_1}\frac{\partial h'}{\partial x} = 0$$

×H

$$\frac{\partial h'}{\partial x}\underbrace{g\frac{\Delta\rho}{\rho_1}H}_{c^2} - \bar{U}^2\frac{\partial h'}{\partial x} = -\bar{U}^2\frac{\partial z_s}{\partial x}$$

c shallow-water wave speed

$$\frac{\partial h'}{\partial x}\left(c^2-\bar{U}^2\right) = -\bar{U}^2\frac{\partial z_s}{\partial x}$$

integrate w.r.t. x

$$\therefore \boxed{h' = \frac{-\bar{U}^2}{c^2-\bar{U}^2}z_s = -\frac{z_s\,\bar{U}^2/c^2}{1-\frac{\bar{U}^2}{c^2}}}$$

$\frac{\partial}{\partial x}$

$$\frac{\partial U'}{\partial x} = -\frac{\bar{U}}{H}\frac{\partial h'}{\partial x} + \frac{\bar{U}}{H}\frac{\partial z_s}{\partial x}$$

plug in h'

$$= -\frac{\bar{U}}{H}\left(\frac{-\bar{U}^2\frac{\partial z_s}{\partial x}}{c^2-\bar{U}^2}\right) + \frac{\bar{U}}{H}\frac{\partial z_s}{\partial x} = \frac{\partial z_s}{\partial x}\frac{\bar{U}}{H}\left(\frac{\bar{U}^2}{c^2-\bar{U}^2}+1\right)$$

$$= \frac{\partial z}{\partial x}\frac{\bar{U}}{H}\left(\frac{\cancel{\bar{U}^2}+c^2-\cancel{\bar{U}^2}}{c^2-\bar{U}^2}\right)$$

$$= \frac{\partial z}{\partial x}\frac{\bar{U}}{H}\left(\frac{1}{1-\bar{U}^2/c^2}\right)$$

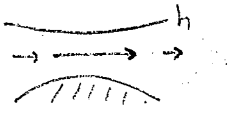
integrate w.r.t. x

$$\therefore \boxed{U' = \frac{z_s}{H}\frac{\bar{U}}{1-\bar{U}^2/c^2}}$$

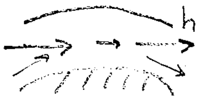
$$F_r = \frac{\bar{U}^2}{c^2}$$

c^2 — $\underbrace{g\frac{\Delta\rho}{\rho_1}}_{N}H$

for $F_r < 1$, $\bar{U} < c$ subcritical flow $h'<0$, $U'>0$



$F_r > 1$, $\bar{U} > c$ supercritical flow $h'>0$, $U'<0$



flow air too fast

⇒ gravity waves cannot play role in establishing steady-state adj. bet h' & U' when $F_r \to 1$, $h' \,\&\, U' \to \pm\infty \Rightarrow$ linear soln not valid

consider nonlinear steady-state eqns.

u-eqn. not linearized

$$U\frac{\partial U}{\partial x} + g\left(\frac{\Delta\rho}{\rho_1}\right)\frac{\partial h}{\partial x} = 0$$

let $\Delta\rho=\rho_1$ for mathematical simplicity $\to \Delta\rho = \rho_1-\rho_2$ small eg air above water

cont non-linearized

$$U\frac{\partial h}{\partial x}+(h-z_s)\frac{\partial U}{\partial x} - U\frac{\partial z_s}{\partial x}=0$$

$$U\frac{\partial h}{\partial x}+h\frac{\partial U}{\partial x}-z_s\frac{\partial U}{\partial x}-U\frac{\partial z_s}{\partial x}=0$$

$$\frac{\partial}{\partial x}\left[U(h-z_s\right]=0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}U^2\right)+\frac{\partial}{\partial x}(gh)=0$$

$$\frac{1}{2}U^2+gh=\text{const.}$$

{ h decr. if U incr.
" incr. if " decr.

$$U(h-z_s)=\text{const.}$$

$\times U \to U^2\frac{\partial U}{\partial x}+Ug\frac{\partial h}{\partial x}=0$

cont. eq. $\frac{\partial}{\partial x}[U(h-z_s)]=0=U\frac{\partial h}{\partial x}+h\frac{\partial U}{\partial x}-U\frac{\partial z_s}{\partial x}-z_s\frac{\partial U}{\partial x}$

$$\frac{\partial h}{\partial x}=-\frac{h}{U}\frac{\partial U}{\partial x}+\frac{\partial z_s}{\partial x}+\frac{z_s}{U}\frac{\partial U}{\partial x}$$

subst. $\frac{\partial h}{\partial x}$ into eqn. motion

so, $U^2\frac{\partial U}{\partial x}+Ug\left(-\frac{h}{U}\frac{\partial U}{\partial x}+\frac{\partial z_s}{\partial x}+\frac{z_s}{U}\frac{\partial U}{\partial x}\right)=0$

$$\frac{\partial U}{\partial x}\left(U^2\underbrace{-gh+gz_s}_{g(z_s-h)}\right)=-Ug\frac{\partial z_s}{\partial x}$$

$$\frac{\partial U}{\partial x}\left[U^2-g(h-z_s)\right]=-Ug\frac{\partial z_s}{\partial x}$$

$$\frac{\partial U}{\partial x}\left[\underbrace{\frac{g(h-z_s)}{c^2}}_{1}-\underbrace{\boxed{\frac{U^2}{c^2}}}_{Fr}\right]=\frac{Ug}{c^2}\frac{\partial z_s}{\partial x}$$

(remember $\frac{\Delta\rho}{\rho_1}=1$ here!)

$c^2 \equiv g(h-z_s)$ "local" thickness of fluid

like $c^2=gH$, but acct for topography

$$(1-F_r)\frac{\partial U}{\partial x}=\frac{Ug}{c^2}\frac{\partial z_s}{\partial x}$$

on upslope side of ridge, $\frac{\partial z_s}{\partial x} > 0$

if $F_r < 1 \Rightarrow \frac{\partial U}{\partial x} > 0$ subcritical on windward side → flow will accelerate

if $F_r > 1 \Rightarrow \frac{\partial U}{\partial x} < 0$ supercritical flow on windward side → flow will decelerate

Consider $F_r < 1$ on windward side

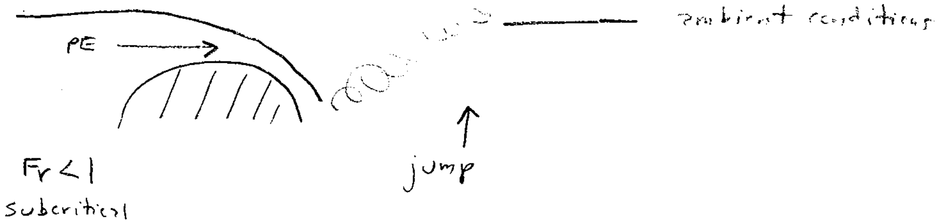
$$\left.\begin{array}{ll} U & \text{increases up mtn.} \\ c = \sqrt{g\underbrace{(h - z_s)}_{\text{decreases}}} & \text{up mtn.} \end{array}\right\} \Longrightarrow F_r = \frac{U^2}{c^2} \text{ increases}$$

if $F_r = 1$ at crest, then since $\frac{\partial U}{\partial x} = \frac{\frac{Ug}{c^2}\overset{>0}{\frac{\partial z_s}{\partial x}}}{1 - F_r \sim 0} \longrightarrow$ vy large ⇒ U becomes vy large

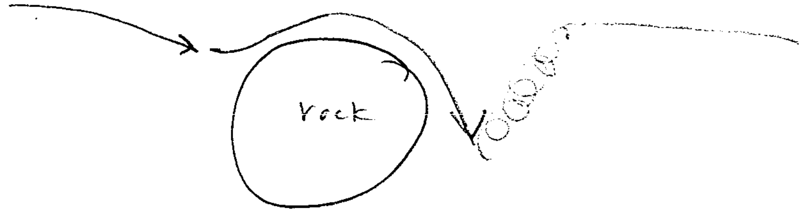
$\Longrightarrow F_r > 1$ becomes supercritical & continues to accelerate on lee side ~ $\frac{\frac{\partial z_s}{\partial x} < 0}{1 - F_r < 0}$

in a turbulent "hydraulic jump"

PE → KE whole way!



like rock in stream



Reed Timmer

What happens if $Fr \overset{>}{\to} 1$ well before mt. crest?

$$\frac{\partial U}{\partial x} = \frac{\dfrac{Ug}{c^2}\overset{>0}{\dfrac{\partial z_s}{\partial x}}}{\underbrace{1-Fr}_{<0}} < 0$$

but then Fr will decrease.

blocking?

when $F_r \to 1$, $\frac{\partial V}{\partial y} << 0$

if F_r becomes < 1 before air reaches ridge

then $\frac{\partial V}{\partial y}$ becomes > 0 & wind increases downstream

if $F_r < 1$ at ridge, then on lee side

$V < 0$, $\frac{\partial z_s}{\partial y} > 0$

$1 - F_r > 0$

$\Rightarrow \frac{\partial V}{\partial y} < 0 \Rightarrow$ wind decreases downstream

if $F_r > 1$ at ridge, then on lee side

$V < 0$, $\frac{\partial z_s}{\partial y} > 0$

$1 - F_r < 0$

$\Rightarrow \frac{\partial V}{\partial y} > 0 \Rightarrow$ wind increases downstream

psbl windstorm!

2. reflection of upward-propagating waves

Klemp & Lilly (1975)

linear waves

multilayer atm. — $N, \frac{\partial \bar{U}}{\partial z}$ const in each layer

includes even smallest values of l

if $\frac{\partial l^2}{\partial z} \neq 0$, part of energy reflected downward

superposes upward & downward moving waves

in 3-layer atm. over sinusoidal topography

optimum superposition

each of lower layers 1/4 wavel. deep

if upper ≡ stratosphere

stratosphere

~dry adiabatic

rel. stable

model <u>linear</u> : extrapolate results to nonlinear regime?

3. self-induced critical layer

Clark & Peltier

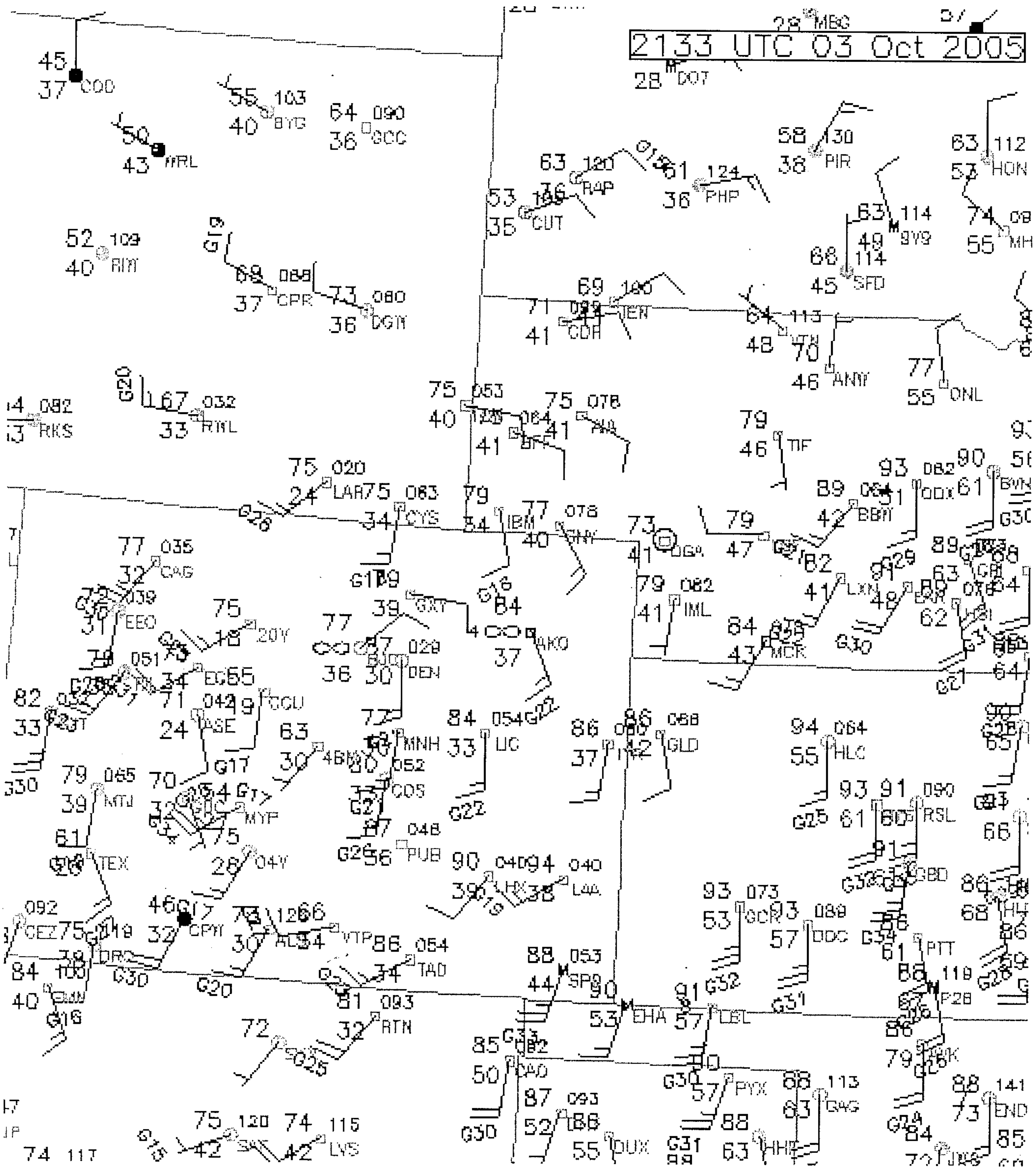
nonlinear, nonhydrostatic model

wave breaks → strong mixing

local reversal of horiz. wind

upward propagating wave trapped below by self-induced critical layer

large amplitude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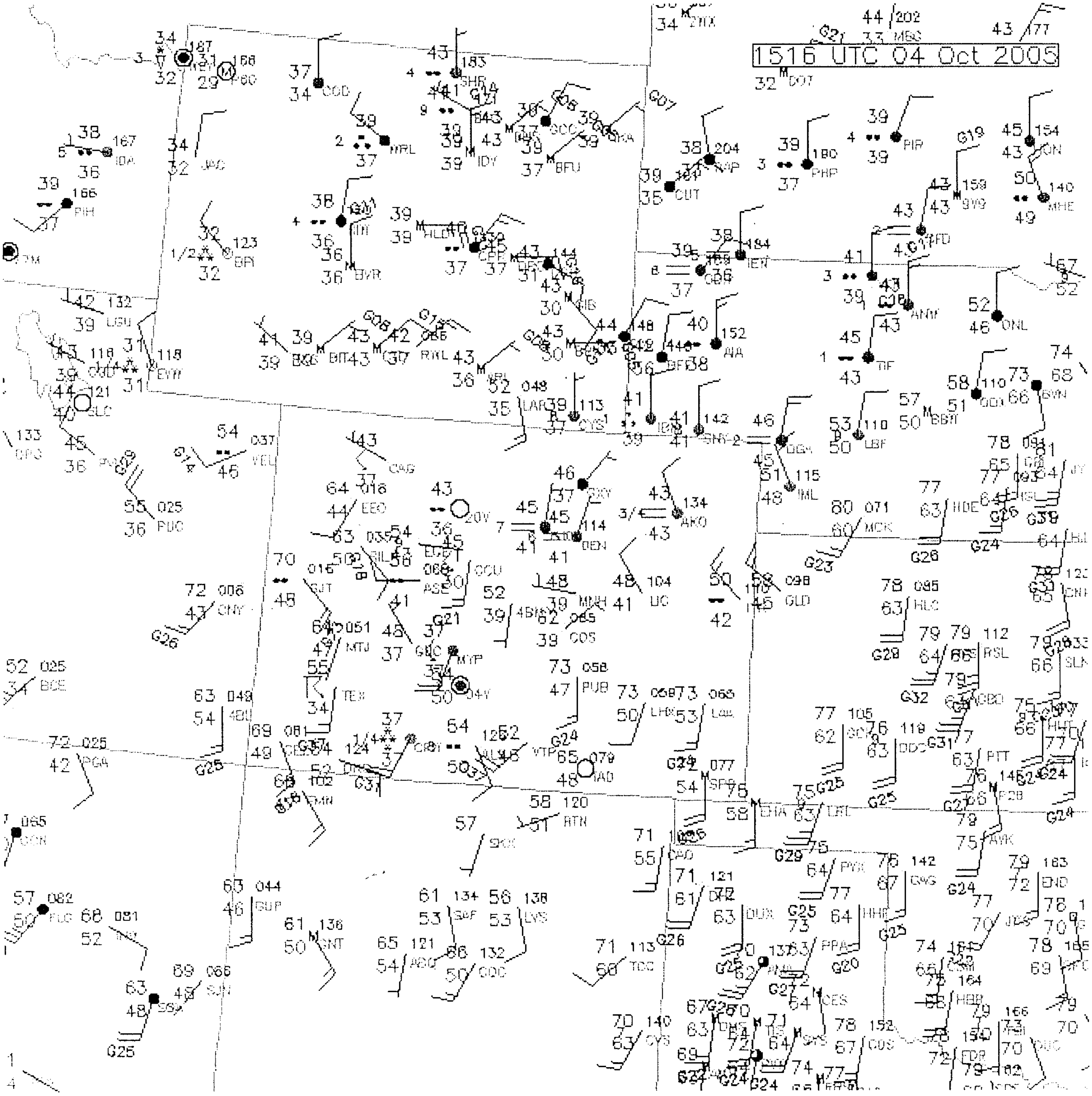
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