

Amanda Murphy
 Mesoscale
 Problem Set 1
 6 September 2020

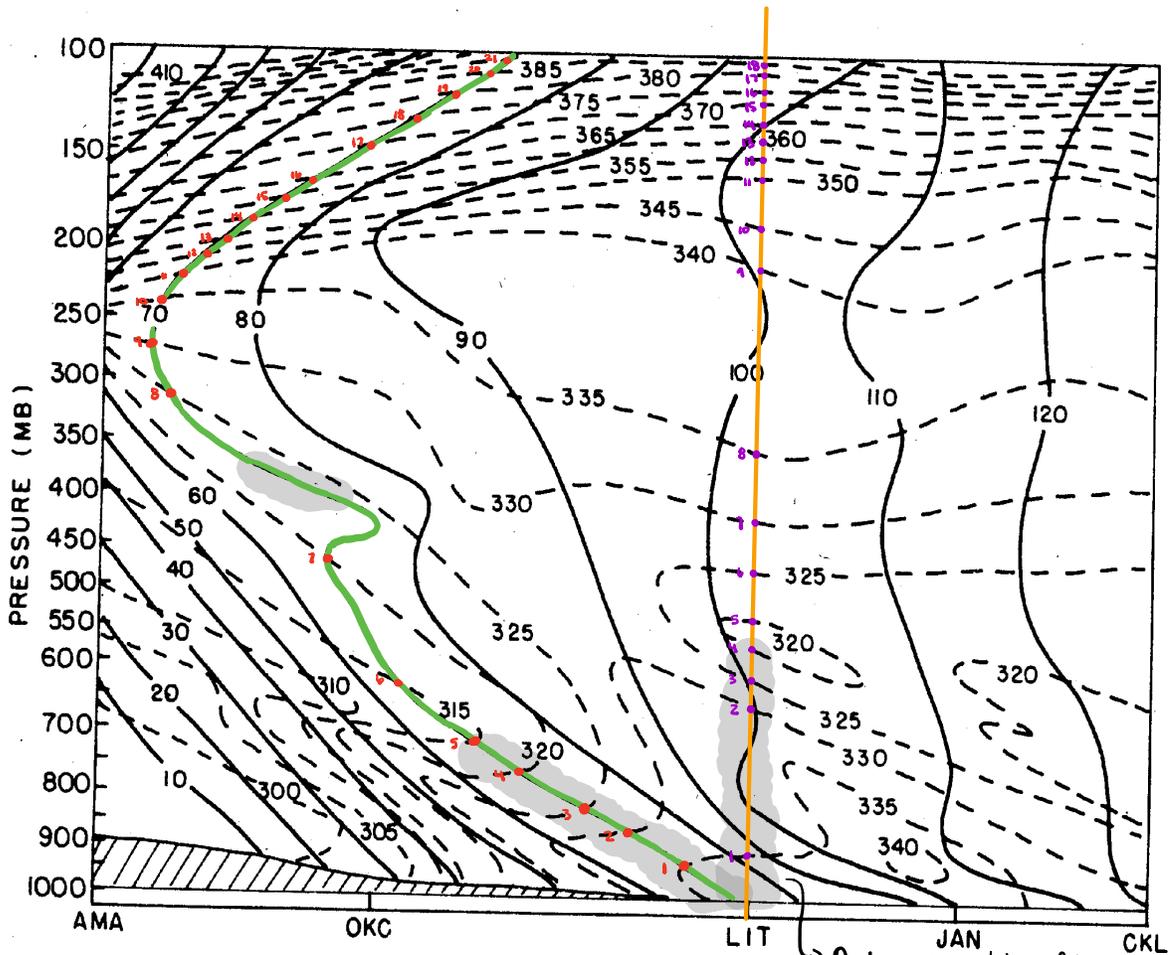
Problem Set #1

Advanced Mesoscale Meteorology
 METR 6413, Sec. 1
 Howie "Cb" Bluestein

Handed out: Thurs., 27 Aug. 2020

Due: Tues, 8 Sept. 2020

1. Consider the vertical cross section of m_g (solid lines; $m\ s^{-1}$) and equivalent potential temperature (dashed lines; K) shown below:



Make a plot of a sounding (temperature and dew point vs. pressure on a skew T - log p diagram) along the $m_g = 70\ m\ s^{-1}$ surface, which passes near the ground level at Little Rock, AR (LIT). Is the sounding along this m_g surface conditionally unstable anywhere? (In other words, is the sounding conditionally symmetrically unstable?) Compare it to the regular sounding at LIT. Feel free to interpolate equivalent potential temperature by eye to get temperature and dewpoint. Assume that the air is saturated everywhere below 700 mb, and neglect moisture above 700 hPa.

1) Points along $m_g = 70 \text{ ms}^{-1}$ surface:

Point	Pressure (hPa)	θ (K)	T ($^{\circ}\text{C}$)	T_d ($^{\circ}\text{C}$)
1)	930	335	?	?
2)	870	330	?	?
3)	825	325	?	?
4)	765	320	?	?
5)	720	315	?	?
6)	635	315	4	$-\infty$
7)	470	320	-15	$-\infty$
8)	315	325	-39	$-\infty$
9)	275	330	-45	$-\infty$
10)	240	335	-50	$-\infty$
11)	225	340	-51	$-\infty$
12)	210	345	-52	$-\infty$
13)	200	350	-52	$-\infty$
14)	190	355	-52	$-\infty$
15)	175	360	-54	$-\infty$
16)	165	365	-55	$-\infty$
17)	145	370	-60	$-\infty$
18)	130	375	-64	$-\infty$
19)	120	380	-66	$-\infty$
20)	110	385	-68	$-\infty$
21)	100	390	-71	$-\infty$

From image

Calculated

$$\theta = T \left(\frac{p_0}{p} \right)^K \quad (1)$$

$$K = \frac{R_d}{c_p} \approx 0.286$$

$$p_0 = 1000 \text{ hPa}$$

$$\therefore T = \theta / \left[\left(\frac{1000}{p} \right)^K \right] \quad (2)$$

$$\text{where } [\theta] = \text{K}$$

$$[p] = \text{hPa}$$

$$[T] = \text{K}$$

$$\text{Convert to C: } T_c = T_k - 273$$

Check units on (2):

$$K = \text{K} \left(\frac{\text{hPa}}{\text{hPa}} \right)^{\left(\frac{\text{J K}^{-1} \text{kg}^{-1}}{\text{J K}^{-1} \text{kg}^{-1}} \right)} = \text{K} \checkmark$$

Can calculate T from θ_e for points where $p < 700 \text{ mb}$, because there $\theta_e = \theta$

For points where $p > 700 \text{ mb}$, must determine T from θ_e graphically.

- Plot θ_e value where $p = 1000 \text{ mb}$
- Follow dry adiabat up to $p = 700 \text{ mb}$ (where there is no moisture)
- Follow moist adiabat down to prescribed pressure level

} pink lines

Can also do a sanity check on T values where $p < 700 \text{ mb}$:

- Plot θ_e value where $p = 1000 \text{ mb}$
- Follow dry adiabat up to prescribed pressure level

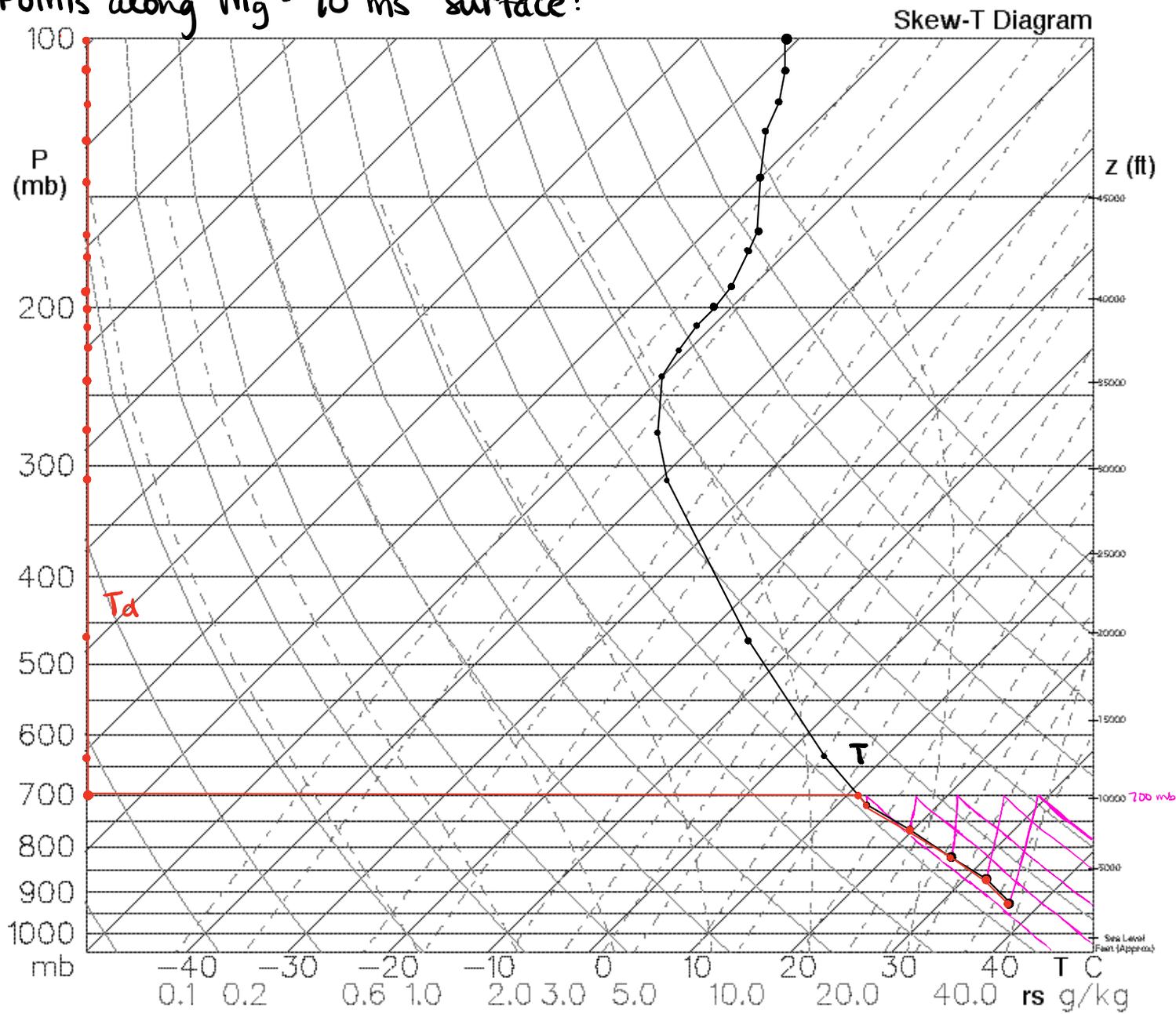
Plot: next page.

Is it conditionally (symmetrically) unstable anywhere?

- CSI where slope of θ sfc $>$ slope of m_g sfc on original figure
- From points 1-5, where $\frac{\partial \theta}{\partial z} > 0$ and $\frac{\partial m_g}{\partial z} < 0$.
- Small region between points 7-8, where $\frac{\partial \theta}{\partial z} > \frac{\partial m_g}{\partial z}$ but both < 0 .
- All else $\frac{\partial \theta}{\partial z} < \frac{\partial m_g}{\partial z}$

Answer: Surface to 720 mb and 410-370 mb

Points along $M_g = 70 \text{ ms}^{-1}$ surface:



1) Points at LIT:

Point	Pressure (hPa)	θ (K)	T ($^{\circ}$ C)	T _d ($^{\circ}$ C)
1)	915	335	?	?
2)	665	330	21	$-\infty$
3)	620	325	10	$-\infty$
4)	575	320	0	$-\infty$
5)	540	320	-5	$-\infty$
6)	475	325	-10	$-\infty$
7)	425	330	-15	$-\infty$
8)	360	335	-23	$-\infty$
9)	220	340	-52	$-\infty$
10)	190	345	-58	$-\infty$
11)	165	350	-64	$-\infty$
12)	150	355	-67	$-\infty$
13)	145	360	-66	$-\infty$
14)	135	365	-67	$-\infty$
15)	125	370	-69	$-\infty$
16)	120	375	-69	$-\infty$
17)	110	380	-71	$-\infty$
18)	105	385	-71	$-\infty$

From image

Calculated

$$\theta = T \left(\frac{p_0}{p} \right)^{\kappa} \quad (1)$$

$$\kappa = \frac{R_d}{c_p} \approx 0.286$$

$$p_0 = 1000 \text{ hPa}$$

$$\therefore T = \theta / \left[\left(\frac{1000}{p} \right)^{\kappa} \right] \quad (2)$$

where $[\theta] = \text{K}$

$[p] = \text{hPa}$

$[T] = \text{K}$

Convert to C: $T_c = T_k - 273$

Check units on (2):

$$K = \text{K} \left(\frac{\text{hPa}}{\text{hPa}} \right)^{\kappa} \left(\frac{\text{J K}^{-1} \text{kg}^{-1}}{\text{J K}^{-1} \text{kg}^{-1}} \right) = \text{K} \checkmark$$

Use same thinking to calculate T (rewritten below):

Can calculate T from θ_e for points where $p < 700 \text{ mb}$, because there $\theta_e = \theta$

For points where $p > 700 \text{ mb}$, must determine T from θ_e graphically.

- Plot θ_e value where $p = 1000 \text{ mb}$
- Follow dry adiabat up to $p = 700 \text{ mb}$ (where there is no moisture) } pink lines
- Follow moist adiabat down to prescribed pressure level

Plot: next page

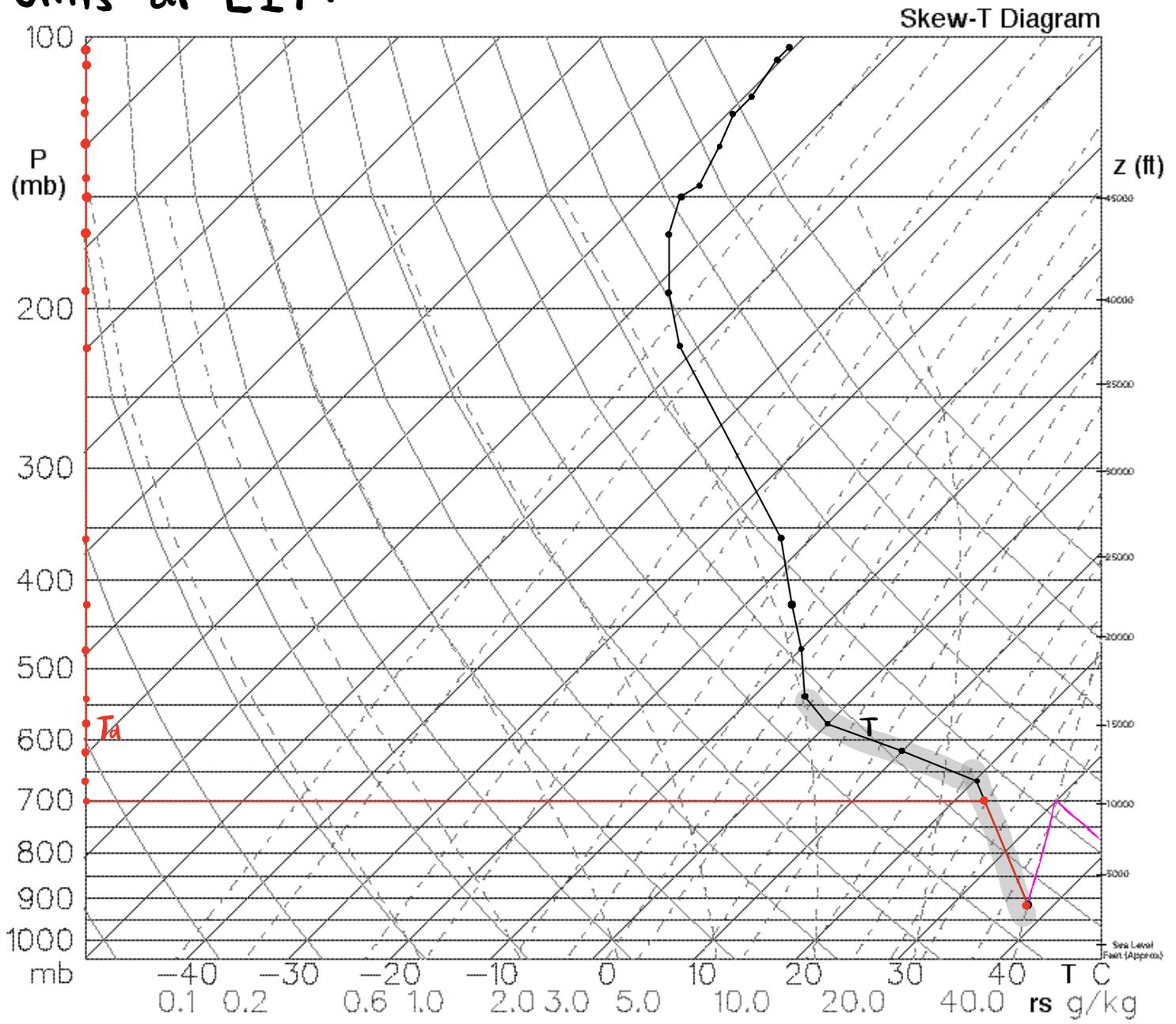
Is it conditionally unstable anywhere?

- Using the plot with θ_e and m_g contours:
 - Have $\frac{\partial \theta_e}{\partial z} < 0$ from surface to $\sim 550 \text{ mb}$. But...
- Using the sounding I drew:
 - Sounding has lapse rates $> \Gamma_d$ from $\sim 700 - 550 \text{ mb}$. Absolutely unstable there.
 - Because $T = T_d$ at the surface, can get parcels to rise freely from sfc to 700 mb ($\Gamma_{\text{parcel}} < \Gamma_{\text{envt}}$). Unstable there as well!

Comparing the possibility for CSI to what I see on the LIT sounding.

- Both show potential for surface-based convection
- LIT sounding is unstable from sfc-550 mb, whereas, along m_g surface, only CSI from sfc-720 mb.

Points at LIT:



mb, and neglect moisture above 700 hPa.

2. Consider a material tube (a symmetric air parcel extending infinitely off in the $\pm x$ direction) that is displaced upward and to the north (+y direction) slightly (just 100 km to the north) of its original position so that it becomes embedded in an environment of $u_g = 15 \text{ m s}^{-1}$ at 36° N .

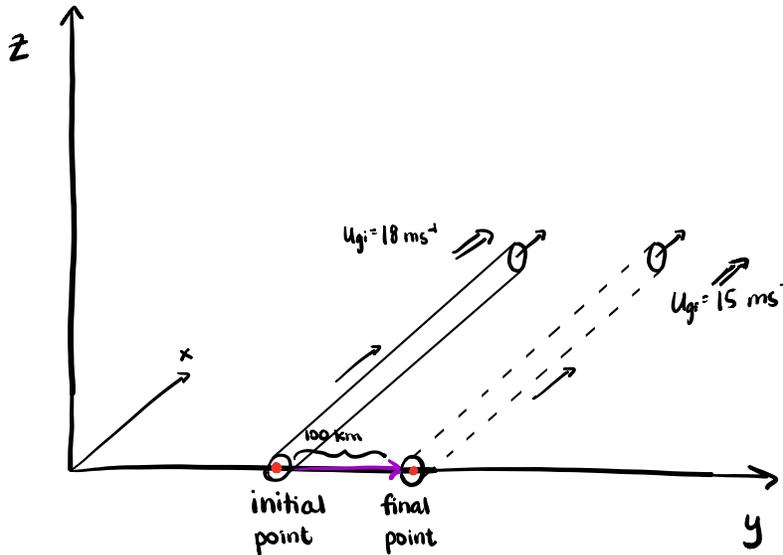
(a) If it originally was moving in the +x direction at 18 m s^{-1} and let go, what is its acceleration in the y direction? Neglect friction and assume that the movement of the tube does not disturb the environmental pressure field.

$\frac{dv}{dt} = ?$

(b) If $\partial m_g / \partial y$ is constant and < 0 in the environment, what is the period of oscillation of the tube in the y direction?

↳ stable!

↳ $T = 2\pi t$



(a) Objective: $\frac{dv}{dt} = ?$

Given:

$$u_{gi} = 18 \text{ ms}^{-1} \text{ and } u_{gf} = 15 \text{ ms}^{-1}$$

$$\varphi = 36^\circ \text{ for final tube position}$$

Equations:

$$m_g = u_g - f_y \rightarrow \frac{\partial m_g}{\partial y} = \frac{\partial u_g}{\partial y} - f$$

$$f = 2 \Omega \sin \varphi \text{ where: } \Omega = 7.2921 \times 10^{-5} \text{ s}^{-1}$$

$$\varphi = 36^\circ$$

$$\frac{dv}{dt} = f(m - m_g) \text{ where: } m = m \text{ of tube}$$

$$m_g = m \text{ of environment}$$

$$m_2 = m_1 + \frac{\partial m}{\partial y} \Delta y \text{ where: } m_2 = m \text{ of final environment}$$

$$m_1 = m \text{ of initial environment}$$

Solve:

$$\frac{\partial u_g}{\partial y} \sim \frac{\Delta u_g}{\Delta y} = \frac{(15-18) \text{ ms}^{-1}}{(100000) \text{ m}} = -3 \times 10^{-5} \text{ s}^{-1}$$

$$f = 2(7.2921 \times 10^{-5}) \sin(36^\circ) = 8.572 \times 10^{-5} \text{ s}^{-1}$$

Determine difference between starting (m_1) and ending (m_2) m :

$$m_2 = m_1 + \frac{\partial m_g}{\partial y} \Delta y \quad \text{Sub } m \text{ for } \frac{\partial m_g}{\partial y}$$

$$m_2 - m_1 = \left(\frac{\partial u_g}{\partial y} - f \right) \Delta y \quad \text{Plug } m \text{ values}$$

$$= (-3 \times 10^{-5} - 8.572 \times 10^{-5})(10^5) \quad \text{Units: } (\text{s}^{-1})(\text{m}) = \text{ms}^{-1}$$

$$m_2 - m_1 = -11.572 \text{ ms}^{-1}$$

$$m_2: m \text{ of ending environment} = m_g$$

$$m_1: m \text{ of starting environment} = m \text{ of parcel} = m$$

$$m_g - m = -11.572 \text{ ms}^{-1} \quad \text{Plug into } \frac{dv}{dt} \text{ equation}$$

$$\frac{dv}{dt} = -f(m - m_g) = f(m_g - m) \quad \text{Plug in numbers}$$

$$= (8.572 \times 10^{-5})(-11.572) \quad \text{Units: } (\text{s}^{-1})(\text{ms}^{-1}) = \text{ms}^{-2} \checkmark$$

$$\boxed{\frac{dv}{dt} = -9.920 \times 10^{-4} \text{ ms}^{-2}} \quad \text{Acceleration in the } -y$$

Sanity check:

$$m_2 = u_{gf} - f_y \text{ and } m_1 = u_{gi} - f_y; \quad \text{where } f_{y_f} \approx f_{y_i}$$

$$m_2 - m_1 = u_{gf} - u_{gi} < 0 \quad \text{and } \frac{\partial m_g}{\partial y} < 0$$

Moved tube from high m environment to low m environment

• Parcel $m >$ new environment's m

• Leads to acceleration in $-y$ \checkmark

(b) Objective: What is the period of the tube's oscillation around the final point, in the y-direction?

Given:

$$\frac{\partial m}{\partial y} < 0 \rightarrow \text{Inertially stable}$$

$$\frac{\partial m}{\partial y} \sim \frac{\Delta m}{\Delta y} = \frac{m_2 - m_1}{\Delta y} \text{ where } m_2, m_1, \text{ and } \Delta y \text{ as defined in (a)}$$

Equations:

$$\text{Period} = T = 2\pi t$$

$$a = \frac{dv}{dt} \text{ where } a = \text{acceleration of tube}$$

$$\frac{dv}{dt} \sim \frac{v_f - v_i}{t} \text{ where: } v_i = \text{speed of tube when it initially arrives at the second point}$$

$$v_f = \text{speed of tube when it oscillates back to the second point}$$

$$v_f = -v_i \text{ when the tube is oscillating about a point}$$

Solve:

$$\frac{dv}{dt} \approx \frac{v_f - v_i}{t}$$

Isolate t, plug in $v_i = -v_f$

$$t = \frac{2v_f}{\frac{dv}{dt}}$$

Plug RHS into expression for period

$$T = 2\pi \left(\frac{2v_f}{\frac{dv}{dt}} \right) \text{ Plug in } \frac{dv}{dt}, \text{ solve, leaving } v_f \text{ as a variable}$$

$$T = 2\pi \left(\frac{2v_f}{-1.348 \times 10^{-5}} \right)$$

$$= v_f \left(\frac{4\pi}{-1.348 \times 10^{-5}} \right)$$

$$\text{Units: } (\text{ms}^{-1})(\text{m}^{-1}\text{s}^2) = \text{s} \checkmark$$

$$T = -9.322 \times 10^{-5} v_f \text{ or } T = 9.322 \times 10^{-5} v_i$$

Sanity check:

- Initial tube motion is to the +y ($v_i > 0$).
- Oscillates around initial point, so v_f in -y ($v_f < 0$)
- T must be > 0
- $T = (\text{term} < 0)(\text{term} < 0) > 0 \checkmark$

Now, searching for a $T \propto \frac{dv}{dt}$ relation because an answer in terms of v_i seems incomplete:

- Equation for oscillation around a point: $T = 2\pi \sqrt{L/g}$ where
 - L = distance tube is displaced. $L = 100 \text{ km} = 10^5 \text{ m}$
 - g = gravitational acceleration. Here it's side-to-side, not up and down, so $g = \frac{dv}{dt}$. Use abs value
 - Source: www.pstcc.edu/departments/natural-behavioral-sciences/Web%20Physics/Experiment%2004web.htm
- Relation to initial v_i is replaced by relation to L

Solve x2:

$$T = 2\pi \sqrt{\frac{L}{\frac{dv}{dt}}}$$

Plug in numbers

$$= 2\pi \sqrt{\frac{10^5}{9.920 \times 10^{-4}}}$$

$$\text{Units: } \sqrt{\frac{\text{m}}{\text{ms}^{-2}}} = \text{s}$$

$$\text{Know } T = 2\pi t = \text{s} \checkmark$$

$$T = 6.31 \times 10^4 \text{ s} \sim 0.73 \text{ days}$$