

Wahl (1974)

①

Rotunno 1983 JAS

Linear theory

$$\frac{\partial U'}{\partial t} = fV' - \frac{\partial \Pi'}{\partial x} \quad \text{simple!}$$

$$\frac{\partial V'}{\partial t} = -fu' \quad \text{linearized} \quad \bar{U} = 0 \quad \text{no fric.}$$

$$\frac{\partial W'}{\partial t} = B - \frac{\partial \Pi'}{\partial y} \quad \bar{V} = 0 \quad \bar{W} = 0$$

$$\frac{\partial B}{\partial t} + N^2 w' = Q \quad \frac{\partial}{\partial y} N^2 \text{ const}$$

$$\frac{\partial U'}{\partial x} + \frac{\partial W'}{\partial y} = 0 \quad \Rightarrow \quad U' = \frac{\partial \psi}{\partial y}, \quad W' = -\frac{\partial \psi}{\partial x}$$

elim. Π'

$$\frac{\partial}{\partial y} \left(\frac{\partial U}{\partial t} \right) = f \frac{\partial V'}{\partial y} - \frac{\partial^2 \Pi'}{\partial y^2}$$

$$\frac{\partial}{\partial x} \left(\frac{\partial W}{\partial t} \right) = \frac{\partial B}{\partial x} - \frac{\partial^2 \Pi'}{\partial x^2}$$

form var.

eqn.

$$\frac{\partial}{\partial t} \left(\frac{\partial U'}{\partial y} - \frac{\partial W'}{\partial x} \right) = f \frac{\partial V'}{\partial y} - \frac{\partial B}{\partial x}$$

elim. B:

$$\frac{\partial}{\partial x} \left(\frac{\partial B}{\partial t} \right) + N^2 \frac{\partial W'}{\partial x} = \frac{\partial Q}{\partial x}$$

$$\frac{\partial^2}{\partial t^2} \left(\frac{\partial U'}{\partial y} - \frac{\partial W'}{\partial x} \right) - f \frac{\partial^2 V'}{\partial y^2} + \frac{\partial^2 B}{\partial t^2} = 0$$

$$N^2 \frac{\partial W'}{\partial x} - \frac{\partial^2}{\partial t^2} \left(\frac{\partial U'}{\partial y} - \frac{\partial W'}{\partial x} \right) + f \frac{\partial^2 V'}{\partial t^2} = -\frac{\partial Q'}{\partial x}$$

$$\underbrace{-\frac{\partial^4}{\partial x^4}}_{\frac{\partial \psi}{\partial y}} \underbrace{\frac{\partial^4}{\partial y^4}}_{-\frac{\partial \psi}{\partial x}} \underbrace{+ f \frac{\partial^2}{\partial t^2}}_{\frac{\partial V'}{\partial t}} = -\frac{\partial Q'}{\partial x}$$

$$+ f \frac{\partial}{\partial y} \left(-fu' \right) \underbrace{-\frac{\partial^4}{\partial y^4}}$$

(2)

$$N^2 \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2}{\partial t^2} \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2}{\partial t^2} \frac{\partial^2 \psi}{\partial y^2} + f^2 \frac{\partial^2 \psi}{\partial y^2} = - \frac{\partial \phi}{\partial x}$$

so,

$$\left(\frac{\partial^2}{\partial t^2} + N^2 \right) \frac{\partial^2 \psi}{\partial x^2} + \left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial^2 \psi}{\partial y^2} = - \frac{\partial \phi}{\partial x}$$

assume $\phi \sim e^{-i\omega t}$

so, $\psi \sim e^{-i\omega t}$

$$\underbrace{[(-i\omega)^2 + N^2]}_{-\omega^2} \frac{\partial^2 \psi}{\partial x^2} + \underbrace{[(-i\omega)^2 + f^2]}_{-\omega^2} \frac{\partial^2 \psi}{\partial y^2} = - \frac{\partial \phi}{\partial x}$$

$$(N^2 - \omega^2) \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial y^2} = - \frac{\partial \phi}{\partial x}$$

$$\begin{cases} N^2 \approx 10^{-2} \text{ s}^{-1} \\ \omega = \frac{2\pi}{86400 \text{ s}} = 7.27 \times 10^{-5} \text{ s}^{-1} \end{cases} \Rightarrow N^2 \gg \omega^2$$

so, $N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial y^2} = - \frac{\partial \phi}{\partial x}$

when $f = \omega \Rightarrow 2\sqrt{2} \sin \phi = \sqrt{2} \equiv \omega$

so, $\sin \phi = \frac{1}{2} \Rightarrow \phi = 30^\circ$

when $f^2 - \omega^2 > 0 \Rightarrow f > \omega$, i.e., when $\phi > 30^\circ$

$\frac{N^2}{L^2} \sim \frac{f^2 - \omega^2}{h^2}$

vert. scale

horiz. scale

③

$$\Rightarrow \frac{h}{L} \sim \frac{\sqrt{f^2 - w^2}}{N}$$

dominant mode
aspect ratio

when N small (low static stability)

$$\frac{h}{L} \cdot \text{large}$$

when N large (high static stability)

$$\frac{h}{L} \cdot \text{small}$$

$$L \sim \frac{Nh}{\sqrt{f^2 - w^2}}$$

gets larger when $f \rightarrow w$

Something weird must happen

$$z + 30^\circ \quad f = w \text{ singular}$$

$\text{no } \frac{\partial^4}{\partial z^4} \text{ term!}$

friction must be included; also N not a const.
nonlinear effects?

nonlinear effects \rightarrow landward advection of sea-breeze

nature of eqn \rightarrow $\frac{\partial}{\partial z} \left[\frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} \right) - N^2 (f^2 - w^2) \right]$ (or any solenoidal circulation)

sea paper (Roothaa)
get eqn $\sim \nabla^2 u$
is better... \rightarrow fundamental diff. in behavior depending on latitude!

when $f > w (> 30^\circ)$, no waves are excited (elliptic eq.)

\rightarrow circ. in phase w/φ (hyperbolic eq.)

$f < w (< 30^\circ)$, inertial grav. waves, but Coriolis force \rightarrow circ. $\sim 180^\circ$ out of phase w/φ no longer dominant \rightarrow don't observe diff. \Rightarrow friction $-K\nabla^2 u$

\rightarrow behavior of sea breeze like a density current?

\rightarrow effects mitigated by inclusion of (Rayleigh) friction!

Phew!