METR 6413

Advanced Mesoscale Meteorology

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The generation of mesoscale vortices in a well-mixed boundary layer

References:

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1. Formulation of the governing equations

Consider the horizontal equation of motion in vector form as follows:

$$\partial \mathbf{v}_h/\partial t + \mathbf{v} \bullet \nabla_h \mathbf{v} = -f\, \mathbf{k} \times \mathbf{v}_h - C_p\, \theta\, \nabla_h \pi + 1/\rho\, \partial \boldsymbol{\tau}/\partial z, \qquad (1)$$

where $\mathbf{v}_h$ is the horizontal component of the the three-dimensional wind $\mathbf{v}$, $\pi = (p/p_o)^{R/C_p}$, p is the pressure, ρ is the density, f is the Coriolis parameter, and $\boldsymbol{\tau}$ is the turbulent stress. The vertical equation of motion is the hydrostatic equation:

$$-C_p\, \theta\, \partial\pi/\partial z - g = 0. \qquad (2)$$

We will now evaluate (1) for the special condition of a well-mixed boundary layer and for a layer above the well-mixed layer, which we will call the "upper layer." Let the basic state, which is hydrostatic, be designated by variables having a zero subscript; the subscript "ml" refers to the mixed layer and the subscript "ul" refers to the upper layer. Perturbation quantities, which are assumed to be relatively small in comparison to the basic-state variables, are denoted by tilde superscripts. *The crucial assumptions about the mixed layer are that both potential temperature and momentum are independent of height*. Then,

$\theta_{ml}\,(x,y,t) = \theta_O + \theta_{ml}\,(x,y,t)$ (3)

$\theta_{ul}\,(x,y,z,t) = \theta_O + \theta_{ul}\,(x,y,z,t)$ (4)

$\pi(x,y,z,t) = \pi_O(z) + \pi(x,y,z,t)$ (5)

$\rho(x,y,z,t) = \rho_O(z) + \rho(x,y,z,t)$, (6)

where θ_O is constant and the basic state is hydrostatic, so that

$C_p\ \theta_o\ \partial\pi/\partial z = -g.$ (7)

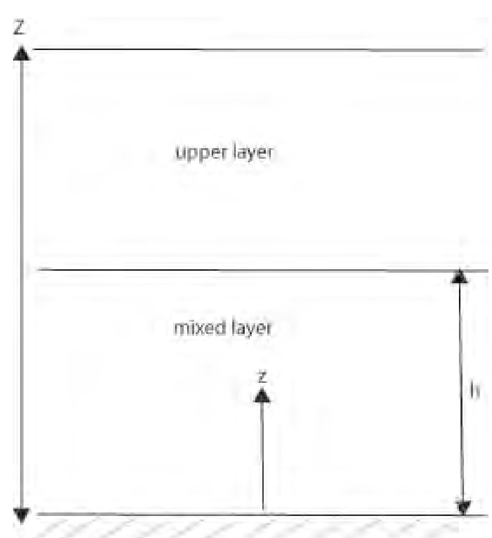
It follows from (7) and (2) that

$C_p\ \partial\pi/\partial z = g/\theta_o\ \theta_{ml}/\theta_o$ (8)

in the mixed layer and

$C_p\ \partial\pi/\partial z = g/\theta_o\ \theta_{ul}/\theta_o$ (9)

in the upper layer. To find π in the mixed layer, we integrate the vertical perturbation-pressure gradient term from a reference level Z above the mixed layer down to a height z within the mixed layer, which has a depth h. We note that θ_{ml} is independent of height in the mixed layer.



It follows that

$$C_p\, \theta_o\, \pi(z) = C_p\, \theta_o\, \pi(Z) + {}_{z}\!\int^{h} B_{ul}\, dz + B_{ml}\, (z - h), \qquad (10)$$

where B_{ml} = = g θ_{ml}/θ_o and $B_{ul} = g\, \theta_{ul}/\theta_o$. From (3) and (5), we can see that the pressure-gradient term in (1)

$$- C_p\, \theta\;\; \nabla_h \pi = \;\; - C_p\, \theta_o\;\; \nabla_h \pi \qquad (11)$$

in the mixed layer. So from (11), (10), and (1), we can express the horizontal equation of motion in the mixed layer as follows:

$$\partial \mathbf{v}_h/\partial t + \mathbf{v} \bullet \boldsymbol{\nabla}_{\mathbf{h}}\, \mathbf{v} + f\, \mathbf{k}\ \mathbf{X}\ \mathbf{v}_h = -C_p\, \theta_o\, \pi(Z) + \nabla[\, {}_{h}\!\int^{z} B_{ul}\, dz] + \nabla(hB_{ml}) - z\, \nabla B_{ml}$$
$$+ 1/\rho_{oo}\, \partial \boldsymbol{\tau}/\partial z, \qquad (12)$$

where ρ in the stress term in (1) has been replaced by its value in the basic state at the surface ($\rho_{oo} = \rho_o(z{=}0)$). All the terms in (12) are independent of height in the mixed layer except for the $z\, \nabla B_{ml}$ term. We can therefore express the stress term as follows:

$$1/\rho_{oo}\, \partial \boldsymbol{\tau}/\partial z = z\, \nabla B_{ml} + \mathbf{c}(x,y,t), \qquad (13)$$

where **c** is independent of height. If we integrate (13) from the height of the underlying topography (z_s) to the top of the mixed layer (h), it follows that

$$\mathbf{c}(x,y,t) = - (\nabla\, B_{ml})[1/2\, (h + z_s)] + 1/\rho_{oo}\, [\tau(h) - \tau(z_s)]/(h - z_s), \qquad (14)$$

where τ(h) represents the entrainment of momentum through the top of the mixed layer and $\tau(z_s)$ represents the effects of surface drag. So,

$$1/\rho_{oo}\, \partial \boldsymbol{\tau}/\partial z = [z - 1/2\, (h + z_s)]\, \nabla\, B_{ml} + 1/\rho_{oo}\, [\tau(h) - \tau(z_s)]/(h - z_s). \qquad (15)$$

Substituting (15) into (12), we find that we can express the horizontal equation of motion in the mixed layer as follows:

$$\partial \mathbf{v}_h/\partial t + \mathbf{v} \bullet \boldsymbol{\nabla}_{\mathbf{h}}\, \mathbf{v} + f\, \mathbf{k}\ \mathbf{X}\ \mathbf{v}_h = \{ -C_p\, \theta_o\, \pi(Z) + \nabla[\, {}_{h}\!\int^{z} B_{ul}\, dz] + \nabla(hB_{ml}) - z\, \nabla B_{ml} \}$$
$$+ \{z\, \nabla\, B_{ml} - 1/2\, (h + z_s)\, \nabla\, B_{ml} + 1/\rho_{oo}\, [\tau(h) - \tau(z_s)]/(h - z_s)\} \qquad (16)$$

The first expression (all the terms between the first set of braces) on the right-hand side of (16) is the pressure-gradient term in the mixed layer and the second expression (all the terms between the second set of braces) is the vertical stress divergence term in the mixed layer. Note how the $- z \nabla B_{ml}$ pressure-gradient term cancels the $z \nabla B_{ml}$ vertical-stress divergence term. Since there can be no horizontal component of vorticity in the mixed layer (momentum must be constant with height), there can be no net generation of horizontal vorticity; the terms that are a function of height therefore cancel each other out.

The equation for the vertical component of vorticity in the mixed layer is computed by operating on the horizontal equation of motion (16) with $\mathbf{k} \bullet \nabla \mathbf{X}$ to get the following:

$$\partial\zeta/\partial t = - \mathbf{v} \bullet \nabla_\eta \zeta - (\zeta + f)\delta + \mathbf{k} \bullet \nabla B_{ml} \, \mathbf{X} \, \nabla[1/2\,(h + z_s)] + \mathbf{k} \bullet \nabla \mathbf{X} \, \{1/\rho_{oo} [\tau(h) - \tau(z_s)]/(h - z_s)\}, \qquad (17)$$

where ζ is the vertical component of relative vorticity and δ is the horizontal divergence.

2. An analysis of how vorticity is generated

The first term on the right-hand side of (17) represents horizontal advection of vorticity, which can only transport already existing vorticity around; it cannot generate new vorticity. The second term, the divergence term, can amplify pre-existing relative vorticity, but cannot generate vorticity from scratch; divergence/convergence acting on Earth's vorticity is too slow to account for the formation of mesoscale vortices. The most significant term is the third term on the right-hand side of (17), the baroclinic term involving the gradient of mixing depth and terrain slope. If the mixed layer depth is constant, then the magnitude of the baroclinic-slope term is proportional to the magnitude of $\nabla B_{ml} \, \mathbf{X} \, \nabla z_s$.

A physical explanation of the baroclinic-slope term is as follows: Consider that part of the vertical stress divergence term in the expression between the second set of braces in (16) which is proportional to the horizontal gradient in mixed-layer buoyancy,

$$[z - 1/2\,(h + z_s)] \nabla B_{ml}.$$

Above (below) $z = 1/2\,(h + z_s)$, the mid-point of the mixed layer, air is accelerated along (in the opposite direction to) the buoyancy gradient in the mixed layer. If the surface elevation is sloped, then vertical vorticity is generated as a result of unequal accelerations at a given height along the surface-elevation gradient as depicted below.

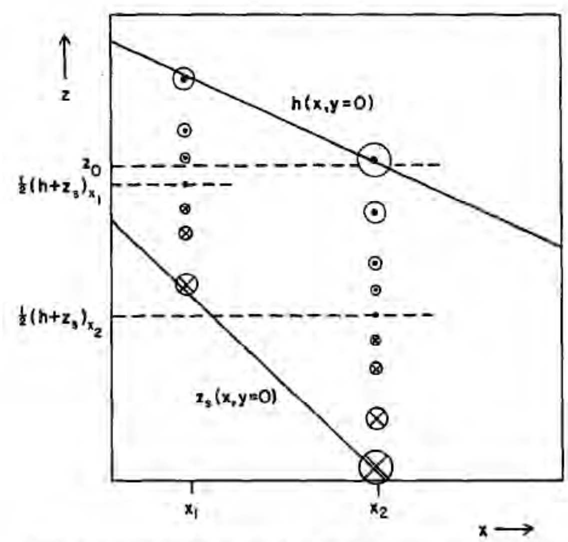


FIG. 2. Profiles of the Reynolds-stress divergence at two adjacent points, x_1 and x_2, in a vertical cross section through the mixed layer along the x-axis. The Reynolds-stress divergence is $[z - \frac{1}{2}(h + z_s)]\partial B_{ml}/\partial y$ in the y-direction, where $\partial B_{ml}/\partial y > 0$. Circles enscribing dots and crosses represent vectors into and out of the page, respectively. The magnitudes of the vectors, represented by the size of the circles, are proportional to the distance above the midpoint of the mixed layer. At any particular $z = z_0$, the flow experiences greater acceleration in the y-direction at x_2 than at x_1 because the midpoint of the mixed layer is lower at x_2 than at x_1. This implies a positive torque around the vertical axis.

(from Dempsey and Rotunno 1988)

3. Mesoscale vortices in nature

The "Denver convergence-vorticity zone" is either a mesoscale cyclone or an elongated convergence zone having cyclonic vorticity at the surface. It is postulated that when the synoptic flow is from the southeast over the Palmer Divide, a region of relatively high terrain that juts out from the Rocky Mountains eastward, between Denver and Colorado Springs, acts as an elevated heat source; potential temperature is advected northward out over terrain that slopes downward to the north. In the figures below, it can be seen that cyclonic vorticity is generated.

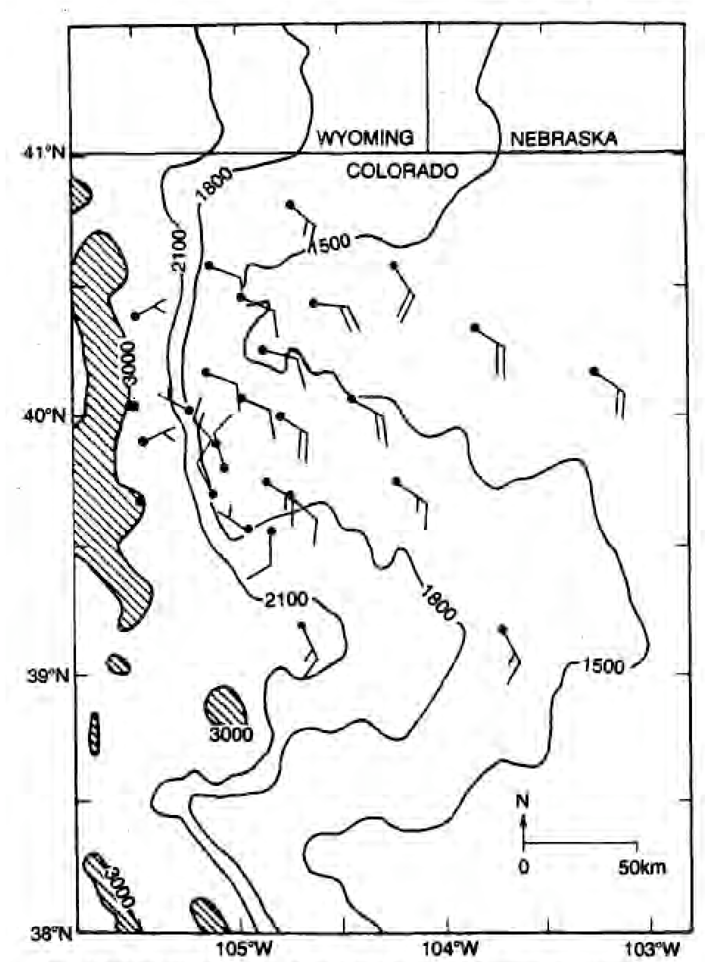


FIG. 1. Surface wind observations of the Denver Cyclone, an example of a topographically forced, mesoscale vortex, in northeastern Colorado at 0000 UTC 16 May 1987. The Front Range of the Rocky Mountains runs north–south along the left side of the figure, while the Palmer Divide extends eastward from the Front Range into the Great Plains in the lower half of the figure. The prevailing surface wind is from the southeast.

(from Dempsey and Rotunno 1988)

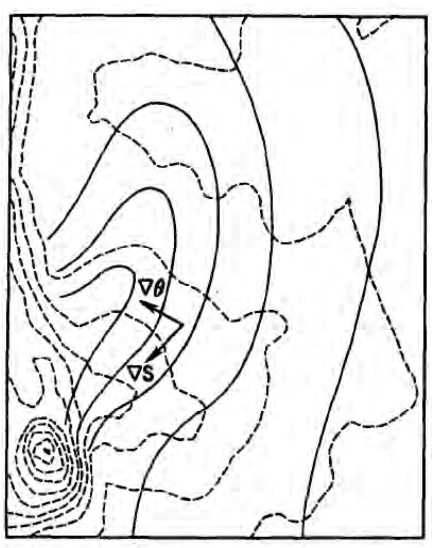


FIG. 18. Schematic of the terrain height (s) and potential temperature (θ) contours on the lee slope of the Palmer Ridge. The baroclinic-slope term generates vorticity where contours of θ intersect contours of S.

(from Wilczak and Glendening 1988; $S \equiv z_s$))

Similar mesoscale vortices may occur to the north by the Cheyenne Ridge and to the south by the Raton Ridge. The mesoscale vortices and regions of enhanced cyclonic vorticity are important for air pollution considerations and in providing a source of pre-existing boundary-layer vorticity that may be stretched to tornadic strength underneath rapidly growing cumulus congestus clouds, especially during the late spring and summer.

4. A word of caution

Despite how neat the theory looks, we must remember that while we often find deep mixed layers that are well mixed with respect to potential temperature, we do not usually see mixed layers in which the momentum is well mixed; there is usually vertical shear. Since our derivation of the vorticity equation (17) depended on the assumption that momentum is well mixed, we must be cautious in applying the theory to the real world.