**INSTABILITY**

**Symmetric Stability**

**Howard B. Bluestein**

School of Meteorology, University of Oklahoma

120 David L. Boren Blvd., Rm. 5900

Norman, OK 73072

USA

**Introduction**

 Symmetric stability is a state of the atmosphere in which an inviscid, dry air parcel displaced from its equilibrium position with respect to some axis along/about which the flow has no variations, i.e. along/about an axis of symmetry, experiences a restoring force which makes it oscillate about its original position. For axially symmetric displacements in a ring about an axially symmetric vortex the wave motions are called *inertial* or *centrifugal* waves. Centrifugal oscillations are like buoyancy waves with the horizontal centrifugal (inertial) force playing the role of buoyancy (gravity).

 Similar oscillations can also occur in a statically stable, rotating atmosphere when the thermal wind shear vector is unidirectional and does not vary along the direction it is oriented. Parcels in the form of a tube are displaced in a vertical plane normal to the thermal-wind vector. In this case the axis of symmetry is the axis along which the thermal wind is directed.

 If potential temperature increases with height and the geostrophic absolute vorticity is anticylonic, the atmosphere is *inertially unstable*; if the potential temperature decreases with height and the geostrophic absolute vorticity is cyclonic, the atmosphere is *gravitationally (or statically) unstable*. If the geostrophic absolute vorticity is cyclonic and potential temperature increases with height the atmosphere is both inertially stable and gravitationally stable; however, if infinitesimal displacements in the plane normal to the vertical shear are accompanied by forces that move the air parcel farther away from its equilibrium position, the atmosphere is *symmetrically unstable.* Since the atmosphere is baroclinic, owing to the thermal wind, this instability is a special case of baroclinic instability for a flow in which there is no temperature-gradient component along the axis of symmetry.

 When tubes of moist, unsaturated air are lifted in a symmetrically stable atmosphere to a level at which condensation occurs (and water and ice loading are not significant or are neglected) and thence to a level at which the atmosphere is symmetrically unstable with respect to saturated processes (i.e., when vertical trajectories follow surfaces of constant equivalent or wet-bulb potential temperature instead of surfaces of potential temperature), then the atmosphere is in a state of *conditional symmetric instability (CSI)*. CSI is analogous to *conditional instability* for air parcels lifted vertically. Since CSI involves forces that are both horizontally and vertically directed, the process by which the instability is released is also referred to as *slantwise convection.* When a layer of moist air that is initially symmetrically stable is lifted to saturation and the vertical displacement of air itself creates the conditions for slantwise convection, then the process is referred to as *potential symmetric instability (PSI)*, which is analogous to potential instability for upright convection*.* At saturation, CSI and PSI are equivalent.

 The importance of CSI is that it is thought to be responsible for the formation of some mesoscale bands of precipitation that are oriented along the thermal wind. Since the thermal wind is oriented along the elongated zone of strong temperature gradient associated with fronts and is quasi-two-dimensional, CSI may be triggered in response to slantwise, ageostrophic, frontal circulations initiated by confluence/diffluence acting on a cross-frontal temperature gradient. It is also thought that CSI may be responsible for eyewall rainbands in some tropical cyclones.

**The parcel theory of symmetric instability in an inviscid, dry atmosphere**

 The analysis of symmetric stability is simplified by using a parcel approach analogous to that used in the parcel theory of upright convection. Consider a Cartesian coordinate system in which there is a temperature gradient in the y-p plane and that ∂/∂x of all variables is zero (this choice of an axis of symmetry is arbitrary; sometimes the y axis is chosen to be the axis of symmetry). For simplicity the dynamics are described for the Northern Hemisphere. Consider the quantity

 m = u - fy, [1]

where u is the x-component of the wind and f is the Coriolis parameter. In inviscid flow m, the *absolute momentum* or *pseudo-angular momentum*, is conserved; it is attributed to an infinitesimal tube of air extending through some point (y,p) infinitely off in both the +x and -x directions.

 The inviscid momentum equation in the y direction, with height as the vertical coordinate, is

 Dv/Dt = - f u - 1/ ∂p/∂y = - f (m - mg), [2]

where v is the y-component of the wind,  is the density, p is the pressure, D/Dt is the total (material) derivative, and the geostrophic absolute momentum

 mg = ug - f y, [3]

where ug is the geostrophic component of the wind in the x direction. Therefore the net force in the y direction on a tube is proportional to the difference between the m of the tube, which is conserved, and the mg of the environment into which the tube is displaced. It is assumed for simplicity that the tube does not mix with its environment. The original value of m of the tube is just its geostrophic value at its equilibrium point in the y-p plane. Therefore there will be a net force in the y direction on the tube if it moves into an environment where mg is different from that of its equilibrium, starting location. Vertical gradients of mg are associated with thermal wind shear in the x direction (i.e., with temperature gradients in the y direction); gradients of mg in the y direction are associated with geostrophic absolute vorticity.

 The inviscid vertical equation of motion is

 Dw/Dt = g/' 

where w is the vertical velocity, g is the acceleration of gravity,  is the potential temperature of the environment, and ' is the potential temperature of the tube. If the flow is adiabatic and there is no diffusion of heat, ' is conserved following the motion of the tube. It is assumed that the environment is not disturbed by the tube's motion so that there is no vertical perturbation pressure-gradient force. Therefore there will be a net force in the vertical on the tube if it moves into an environment where  is different from that of its equilibrium, starting location.

Whether or not there is a restoring force on the tube that brings it back to its equilibrium point about which it undergoes a stable oscillation (symmetric stability) or whether is continues to move in the direction of its displacement (symmetric instability) depends on how the surfaces of mg and  are oriented and what the direction of displacement is with respect to the surfaces (**Fig. 1**). Symmetric instability is possible (panel b of **Fig. 1**) if the slope of the  surfaces is greater than the slope of the mg surfaces and if the tube is displaced infinitesimally along a plane whose slope is intermediate between that of the surfaces and that of the mg surfaces (i.e., along paths a or c, but not along paths b or d, and if ∂/∂z > 0 and ∂mg/∂y < 0. If ∂/∂z < 0 (panel d of **Fig. 1**) or if ∂mg/∂y > 0 (panel e of **Fig. 1**), then the atmosphere is statically unstable or inertially unstable, respectively, and not symmetrically unstable. Panel a in **Fig. 1** depicts neutral stability and panel c in **Fig. 1** depicts absolute stability.

 The thermal wind relation in terms of potential temperature is to a good approximation

 ∂ug/∂z = -g/f 1/ ∂/∂y. [5]

The slope of a surface of constant  is therefore

 (dz/dy) = (f ∂ug/∂z)/(g/ ∂/∂z) [6]

and the slope of a surface of constant mg is

 (dz/dy)mg = (f - ∂ug/∂y)/(∂ug/∂z). [7]

It follows that the necessary condition for symmetric instability is

 Ri = g/ ∂/∂z / (∂ug/∂z)2 < f / (g + f), [8}

where Ri is the Richardson number for the geostrophic wind and g is the geostrophic vorticity, which for symmetric flow (∂/∂x = 0) is - ∂ug/∂y. In typical synoptic-scale flow in midlatitudes the geostrophic vorticity is an order of magnitude smaller than f; then the necessary condition for symmetric instability is that Ri < 1. In the vicinity of fronts where geostrophic vorticity is much larger, Ri must be smaller.

 Ertel's potential vorticity for an atmosphere in geostrophic and hydrostatic balance is

 Z = (Cp/) f g/ ∂/∂z [ (g + f)/f - 1/Ri]. [9]

It follows from [8] that an equivalent necessary condition for symmetric instability is that Ertel's potential vorticity for the geostrophic wind is negative (anticyclonic in either hemisphere). Since

 (g + f)= (g + f)z - f / Ri, [10]

where (g + f) is the geostrophic absolute vorticity evaluated on an isentropic surface and (g + f)z = f - ∂ug/∂y is the geostrophic absolute vorticity evaluated on a surface of constant height, then negative (anticyclonic in either hemisphere) isentropic geostrophic absolute vorticity is also an equivalent necessary condition for symmetric instability. Thus, symmetric instability is favored on the anticyclonic-shear side of jets and jet streaks or near sharply curved ridges of high pressure.

 It can also be shown that the necessary condition for symmetric stability is equivalent to the ellipticity condition for the Sawyer-Eliassen equation, which describes the vertical circulation about a front forced by geostrophic confluence/diffluence and differential diabatic heating and whose dynamics are governed by the geostrophic-momentum approximation. Since the Sawyer-Eliassen equation is a second-order, constant-coefficient, partial differential equation, the condition of ellipticity is necessary for it to have unique solutions. Thus, balanced frontal circulations are possible only if the atmosphere is symmetrically stable. However, if friction is included in the equations of motion, it turns out that the ellipticity condition can be met even when Ertel’s potential vorticity is negative.

**The parcel theory of slantwise convection in an inviscid, moist atmosphere**

 The analysis of symmetric instability in a moist atmosphere is complicated by latent heat release, evaporation and melting-related cooling, and by water and ice loading. The governing momentum equation remains [2]. The governing vertical equation of motion, on the other hand, is different from [4] since it must account for latent heat release, and if there is condensate, for water and ice loading also. Surfaces of constant entropy that account for latent heat release and for condensate loading replace potential temperature in [4]. If both the environment and the tube are unsaturated and there is no condensate, then virtual potential temperature (v) may be used in place of potential temperature. If the tube is saturated and the environment is unsaturated, and if condensate is ignored, then potential temperature of the tube may be replaced by equivalent virtual potential temperature (ev); if both the tube and the environment are saturated, and if condensate is ignored, then the potential temperature of both the tube and environment may be replaced by equivalent virtual potential temperature. [Wet-bulb virtual potential temperature (wv) may be used instead of equivalent virtual potential temperature.]

 For the purpose of illustration consider an atmosphere that is unsaturated and has no condensate, but is moist. Suppose that the distribution of v, ev, and mg is as shown in **Fig. 2**. Since the slope of the surfaces of constant v are not steeper than the surfaces of constant mg at low levels, the atmosphere there is symmetrically stable or even neutral with respect to unsaturated displacements. However, there are regions aloft where the slope of surfaces of constant ev is steeper than the surfaces of mg. In these regions, if condensate loading is ignored, the atmosphere is symmetrically unstable with respect to infinitesimal saturated displacements. Since the stability depends upon whether or not a tube is saturated or unsaturated, the symmetric instability condition is *conditional*.

 Suppose an unsaturated tube at low levels is lifted a *finite* distance along a surface of constant v (e.g., by the ascending branch of a frontal circulation or more slowly as a result of quasigeostrophic forcing) until it reaches its lifting condensation level (LCL) and that condensate is ignored: If lifted any further, it follows a surface of constant ev. Thus far the tube is neutrally buoyant. Owing to the inclusion of the effects of latent heat, the surfaces of constant ev have different slopes than that of the surfaces of constant v. The m of the tube is greater than that in its environment everywhere to the right of the original mg surface; therefore according to [2] the tube is symmetrically stable because it feels a restoring force that has a component to the left; if the tube were not forced any further, it would become negatively buoyant and move back down and to the left towards its original equilibrium position. The tube is symmetrically stable even though it is saturated and the slope of the surfaces of constant ev is greater than the slope of the surfaces of constant mg because the tube has undergone a finite displacement rather than an infinitesimal displacement.

 If the tube is lifted further, however, so that eventually it crosses to the left of the original mg surface, and it is displaced upward and to the right at a slope intermediate between that of the ev and mg surfaces, then according to [4] and [2] it would continue to accelerate upward and to the right if it were not forced any more. The level at which it would first realize symmetric instability is called the *level of free slantwise convection (LFSC)*, in analogy with the level of free convection (LFC) for upright convection.

 Eventually the tube will reach a level at which it crosses back to the right side of the original mg surface where the slope of the ev surfaces is now less than that of the mg surfaces. Above this level, the *level of neutral buoyancy (LNB)* for slantwise convection, the atmosphere is symmetrically stable. The LNB tends to be near the tropopause where  surfaces are more horizontally oriented owing to the strong static stability [6] in the lower stratosphere and where mg surfaces are more vertically oriented owing to the lack of horizontal temperature gradient and thermal-wind shear [7] at the tropopause.

 The amount of kinetic energy it takes to lift a tube to its LFS is called the *slantwise convective inhibition (SCIN)*. The potential energy in the environment that is converted into kinetic energy of the tube while it is symmetrically unstable is called the *slantwise convective available potential energy.* SCIN and SCAPE are the analogs to the convective inhibition (CIN) and convective available potential energy (CAPE) in the parcel theory of upright convection. The SCAPE is equivalent to CAPE computed along a surface of constant mg. In the geostrophic coordinate system used in semigeostrophic theory, mg surfaces are parallel to the geostrophic coordinate, which is directed opposite in direction to the horizontal temperature gradient. Thus SCAPE is CAPE computed in geostrophic coordinates. In the limit of vanishing baroclinicity, mg surfaces become vertically oriented (see [7] when ∂ug/∂z  and SCAPE is identical to CAPE.

 If the atmosphere is saturated, then an equivalent necessary condition for CSI is that Ertel's potential vorticity for saturated moist processes is negative (anticyclonic in either hemisphere). If the effects of condensate are accounted for, the criteria for CSI are more complicated. The loading depends on what phase of water substance is present; condensate that falls out does not follow air parcel motion and may evaporate into unsaturated air.

 In order for precipitation bands to form as a result of slantwise moist convection, there must be an adequate supply of water vapor, strong enough lift to release CSI, and the necessary conditions for CSI must be satisfied.

 The slantwise ascent of symmetrically unstable tubes must be compensated for by slantwise-descending air. The regions of slantwise-descending air modify the environment so as to make the tubes less symmetrically unstable, just as compensating subsidence around buoyant clouds (according to the *slice method* in the theory of upright convection) warms the environment and lessens the buoyancy in the clouds (i.e., the CAPE is diminished). The narrower the slantwise ascending branch and the wider the slantwise descending branch, the less is the slantwise acceleration (i.e. the less is the SCAPE). The most unstable configuration is one of thin, relatively rapidly slantwise-ascending layers of saturated air sandwiched in between thick, less-rapidly slantwise-descending layers of unsaturated air that are being cooled evaporatively from precipitation falling out from above. If the most-unstable configuration is the one most likely to occur, then CSI precipitation bands should be relatively narrow and widely spaced.

 The horizontal scale of CSI precipitation bands estimated from the horizontal extent of a sloping mg surface [7] is Ug/f, where Ug is the change in geostrophic wind in the layer of CSI. For typical values of Ug in midlatitudes, the horizontal scale of CSI precipitation bands is on the order of 100 km, which is mesoscale.

**Observations of conditional symmetric instability and precipitation bands**

 Slantwise convection may be triggered suddenly after a gradual buildup in SCAPE by synoptic-scale or mesoscale processes, or it may be in statistical equilibrium so that the SCAPE is nearly zero and constant. Since the latter is the frequently observed case, it is not easy to use SCAPE as a forecast tool because its absence does not preclude the possibility that CSI is in fact being released; the existing observational network cannot resolve the space and time scales of the production of CSI and its release when the atmosphere is in statistical equilibrium.

 Bands of precipitation ahead of warm fronts may be due to CSI. The bands are important in that large amounts of precipitation may accumulate in mesoscale regions, which makes the forecasting of floods and heavy snow difficult; synoptic-scale dynamics, on the other hand, can only explain how lower amounts of precipitation accumulate over broader regions. The slanted convection in the eyewall of some rapidly intensifying tropical cyclones might also be a result of CSI. In this case, the temperature gradient is directed toward the center of the tropical cyclone.

 While lighting activity is common in when conditional instability is released, it is thought it can also occur when CSI is released, even though updrafts in slantwise convection are usually much weaker. Lightning is commonly observed in the trailing precipitation region of mesoscale convective systems and sometimes in wintertime convection and in the eyewall of deepening tropical cyclones, where the likelihood of conditional instability is small, but there can be CSI.

 When Ertel's potential vorticity for moist processes is very small, i.e., when the atmosphere is nearly neutral with respect to CSI, the atmosphere's response to frontogentical forcing is enhanced. Since frontal secondary circulations and the secondary circulations in tropical cyclones themselves can produce precipitation bands it is therefore not always easy to distinguish between bands forced by the secondary circulations and the bands forced by CSI.

 The state of the atmosphere sometimes evolves so that the necessary conditions for gravitational, inertial, and symmetric instability/CSI appear and disappear. When small-scale gravitational instability weakens and is followed by mesoscale, symmetric instability, there is said to be “upscale development.” There is some evidence that the trailing, “stratiform” precipitation region in some mesoscale convective systems may represent upscale development, as air ascends at the leading convective line in an environment of conditional instability, and then leans over into a trailing region of CSI.

When mesoscale frontal ascent in a symmetrically unstable atmosphere leads to latent heat release and is followed by small-scale, elevated gravitational instability, there is said to be “downscale development.” In any event, it has been argued that there is a continuum between the gravitational and symmetric instabilities, so that a distinction between the two is somewhat arbitrary.

*Key words:* **Conditional Symmetric Instability, Slantwise Convection, Slantwise Convective Available Potential Energy, M surface, Absolute Momentum, Pseudo-angular Momentum, Potential Symmetric Instability, Level of Free Slantwise Convection, Level of Neutral Buoyancy, Slantwise Convective Inhibition**

*Cross references:* **Centrifugal Instability, Inertial Instability, Gravitational Instability, (Mesoscale) Cloud Bands, Frontal Circulations, Frontogenesis, Geostrophic-Momentum Approximation, Thermal Wind, Geostrophic Wind, Potential Temperature, Equivalent Potential Temperature, Wet-bulb Potential Temperature, Ertel's Potential Vorticity, Sawyer-Eliassen Equation, Virtual Potential Temperature, Quasigeostrophic Theory, Lifting Condensation Level, Convective Inhibition, Convective Available Potential Energy, Geostophic Coordinates, Semi-geostrophic theory, Slice Method, Mesoscale.**

**Further Reading**

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**Figure Captions**

**Figure 1** Stability criteria for symmetric flow (∂/∂x = 0) in terms of the slope of the mg (solid lines) and potential-temperature  (dashed lines) surfaces. Displacements in the directions *a, b, c,* and *d* are referred to in the text. (a) neutral stability for displacements along the  and mg surfaces; otherwise stable for infinitesimal displacements; (b) symmetric instability: unstable for infinitesimal slantwise displacements intermediate in slope between that of  surfaces and mg surfaces; otherwise stable; (c) absolute stability: stable for any infinitesimal slantwise displacement; (d) gravitational instability: unstable for any infinitesimal slantwise displacement not along a  surface; (e) inertial instability: unstable for any infinitesimal slantwise displacement not along an mg surface. (adapted from Bluestein, HB (1993). *Synoptic-Dynamic Meteorology in Midlatitudes, Vol. II: Observations and Theory of Weather Systems.* New York, Oxford University Press.)

**Figure 2** Idealized example of a vertical cross section in the Northern Hemisphere, normal to the thermal-wind shear vector, showing surfaces of constant mg (solid lines), and constant v,ev, andwv. In this example v,ev, andwv increase with height (gravitational and conditional stability), mg decreases with increasing y (inertial stability), and v decreases with y (baroclinic atmosphere, with colder air at larger values of y). Lifting condensation level (LCL); level of free slantwise convection (LFS); level of neutral buoyancy (LNB). Below the LCL the slope of the v surface is less than that of the mg surface. Note that the slope of ev and wv surfaces is greater than the slope of v surfaces because the lapse rate of a v surface is greater than that of a ev orwv surface, and v,ev, andwv decrease with y. (adapted from Bluestein, HB (1993). *Synoptic-Dynamic Meteorology in Midlatitudes, Vol. II: Observations and Theory of Weather Systems.* New York, Oxford University Press.)