THE UNIVERSITY OF OKLAHOMA SCHOOL OF METEOROLOGY

Atmospheric Forces and Force Balances

METR 2021

Introduction

In this lab you will be introduced to the forces governing atmospheric motions as well as some of the common force balances. A common theme throughout all of meteorology is balance. With the non-uniform heating from the sun across Earth's surface comes imbalance in temperature, which creates a localized pressure difference. The atmosphere does not like the imbalance (in temperature or pressure or anything), so the reaction is to ameliorate the temperature/pressure difference. This is where we get weather from, an attempt to regain balance in the atmosphere. Luckily, the atmosphere is never able to achieve the balance (because it is always being pulled out of balance) otherwise we wouldn't have anything to forecast!

Forces

There are a number of forces that govern atmospheric flows. The most important is the pressure gradient force (PGF). The PGF is the only force that can actually cause the wind to blow. All of the other forces act upon the already moving air in the atmosphere. Mathematically we can write a pressure gradient (PG) as,

$$PG = \frac{\Delta p}{\Delta n}$$

where Δp is a change in pressure and Δn is a change in distance. This is not quite our pressure gradient force. The PG has units of N m^{-2}, which is a force per unit area. To get it into our pressure gradient force that we use in meteorology we need to divide by density, ρ,

$$PGF = -\frac{1}{\rho}\frac{\Delta p}{\Delta n}.$$

By convention, we place the negative sign in front of the PGF so that the force always points toward low pressure where the units on PGF of m s^{-2}. **Air will *ALWAYS* move from higher pressure to lower pressure.**

The fact that Earth rotates once every day on its axis results in a force acting on the wind. This force is called the Coriolis Force (CF), which is named after the scientist who quantified the relationship. In order for the CF to impact atmospheric motions, the flow must be going over large time and/or spatial scales. For example, the CF will NOT affect water draining out of your sink or bathtub because it will drain too quickly and is not draining over a large enough spatial scale. We can quantify the Coriolis parameter, f, as follows,

$$f = 2\Omega \sin\phi$$

where ϕ is the latitude and Ω is the Coriolis constant, which is 7.29 x $10^{\wedge-5}$. The Coriolis force acts perpendicular to the wind at all times, it pulls the wind to the right in the Northern

Hemisphere and to the left in the Southern Hemisphere. The CF in the Northern Hemisphere is written as,

$$CF = fV$$

where V is the wind speed. The CF will only act on the wind, it cannot create the wind.

If the flow is curved, as in troughs and ridges, another force must be considered, the centrifugal force. This force is also known as the centripetal acceleration. The centrifugal force acts to push outward from the center of the turn. For example, if you are in a car and take a corner holding constant speed, you will feel like you are being pushed away from the turn. The centrifugal force can be written as,

$$CENTF = \frac{V^2}{R}$$

where R is the radius of curvature. The radius of curvature, R, is positive for cyclones and negative for anticyclones in the Northern Hemisphere. Therefore in low pressure the centrifugal force acts opposite direction to the PGF, but in high pressure it acts in the same direction.

Close to the surface there are many things blocking the flow of air from high to low pressure, such as trees, buildings, mountains, etc. We term this force, friction, and it always acts to slow the wind. Friction can be written as,

$$FF = -kV$$

where k represents the roughness of Earth's surface and will vary depending on what the surface looks like.

Finally, the last force to consider is the gravitation force. Gravity always acts downward and keeps things from accelerating in the vertical, and is assumed to equal 9.81 m s^{-2}.

All of these forces can now be combined into a number of different force balances, which can explain a large number of different flows in the atmosphere. We will next consider these force balances and the assumptions that form these atmospheric balances.

Force Balances

There are five main force balances that we will consider plus a balance of balances. The force balances that are presented are: hydrostatic, geostrophic, gradient, cyclostrophic, Guldberg-Mohn, and the thermal wind balance. Between these balances we can describe most of the flows that occur in the atmosphere.

Hydrostatic Balance

We begin with the hydrostatic balance, which is a vertical force balance between the vertical pressure gradient force and gravity. There is always a vertical pressure gradient as on average pressure is highest at the surface and decreases exponentially as you move vertically. This balance can be written as,

$$-\frac{1}{\rho}\frac{\Delta p}{\Delta z} = g\,.$$

On average, the atmosphere can be considered to be in hydrostatic balance (i.e., there is little vertical motion) as average vertical motions are on the order of centimeters per second. One atmospheric phenomenon that is not in hydrostatic balance is thunderstorms where vertical motions often exceed 1 m s^{-1}. By definition forces that are in balance will sum to zero.

Each force balance can also be visualized with a force balance diagram that depicts the different forces and how they are acting on an air parcels. The following is a an example of a force diagram for the hydrostatic balance.

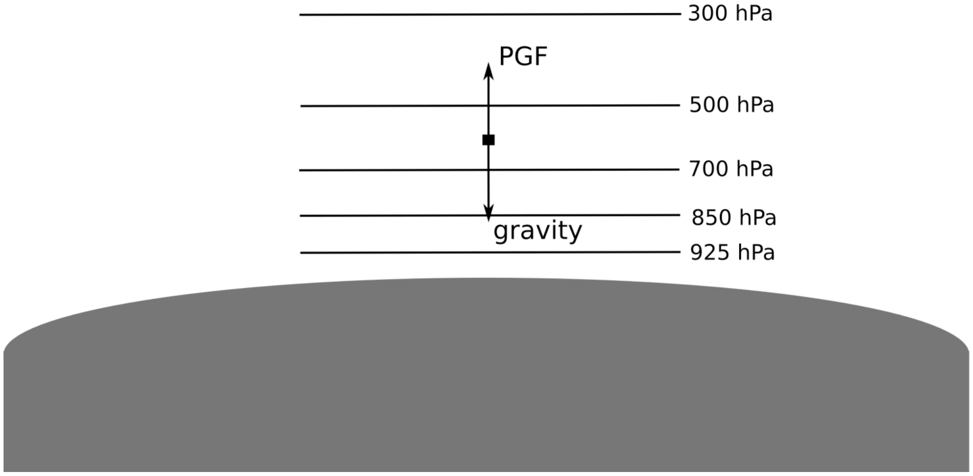


Figure 1. Force balance between the vertical pressure gradient force and gravity, the result is the hydrostatic balance.

Geostrophic Balance

One very common horizontal force balance is the geostrophic balance. Often flows in the atmosphere are over large enough temporal and spatial scales that bring the Coriolis force into play. In the upper-levels of the atmosphere (generally above 700 hPa) the atmosphere can be approximated to be in geostrophic balance. The balance is between the pressure gradient (height gradient) force and the Coriolis force and can be written as,

$$\begin{aligned} 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta x} + fV \\ 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta y} - fU \end{aligned},$$

where the subscript *g* signifies the geostrophic wind. At upper-levels we can re-write the right hand side of the force balance for geopotential height as,

$$\begin{aligned} 0 &= -\frac{\Delta \Phi}{\Delta x} + fV_g \\ 0 &= -\frac{\Delta \Phi}{\Delta y} - fU_g \end{aligned},$$

where ΔΦ = gΔz and is known as geopotential height. Solving for V_g yields an equation for the geostrophic wind,

$$\begin{aligned} V_g &= \frac{1}{f}\frac{\Delta\Phi}{\Delta x} \\ U_g &= -\frac{1}{f}\frac{\Delta\Phi}{\Delta y} \end{aligned}.$$

Gradient Balance

In curved flow, geostrophic balance does not apply because of the addition of the centrifugal force. Therefore, the gradient wind balance is three-way balance between the pressure gradient force, Coriolis force, and the centrifugal force. This balance applies in upper-level troughs and ridges and can be written as,

$$\begin{aligned} 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta x} + fV + \frac{V^2}{R} \\ 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta y} - fU - \frac{U^2}{R} \end{aligned}.$$

This equation can be solved using the quadratic formula. If the pressure gradient were equal around both a high and a low, gradient wind balance would dictate that winds around a high would be supergeostrophic (more than the geostrophic balance would predict) and subgeostrophic (less than the geostrophic balance would predict).

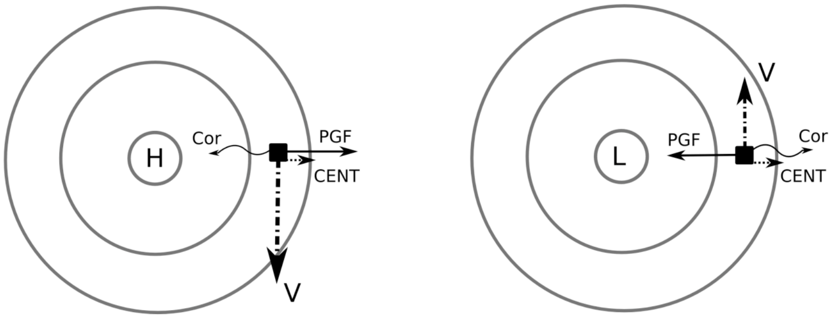


Figure 2. Gradient wind balance force diagram.

Cyclostrophic Balance

When there is curved flow, but not on large temporal or spatial scales, the Coriolis force is no longer a large factor and the wind balance is between the pressure gradient force and the centrifugal force. The meteorological phenomenon that exhibits this balance is the tornado. Often lasting for short amounts of time and on spatial scales much less than 1000 km, the balance can be written as,

$$\begin{aligned} 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta x} + \frac{V^2}{R} \\ 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta y} - \frac{U^2}{R} \end{aligned}.$$

Guldberg-Mohn Balance

At and near the surface, friction begins to affect atmospheric flows by slowing the wind. As a result of the slowing of the wind, the Coriolis force acting on the slower wind does not turn the wind as much and subsequently you have flow crossing the isobars. The Guldberg-Mohn force balance can be written as,

$$\begin{aligned} 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta x} + fV - kU \\ 0 &= -\frac{1}{\rho}\frac{\Delta p}{\Delta y} - fU - kV \end{aligned}.$$

The greater the roughness of the surface, the more the flow will cross the isobars.

Thermal Wind Balance

The thermal wind is a combination of the hydrostatic balance and geostrophic balance and explains why winds become more westerly with increasing height in the mid-latitude regions. This balance brings together the relationship between temperature and winds. Cooler surface temperatures means a shorter vertical distance to a given pressure level, whereas higher surface temperatures would mean a longer vertical distance to a given pressure level. This results in strong pressure gradients aloft, which is in association with the strong jet stream winds observed in the upper-troposphere.

Vertical Motions

We know that the pressure gradient force is always pointed from high pressure to low pressure, but what happens to the air once it reaches the low pressure? Air coming into an area of low pressure at the surface can't go into the ground, so it rises. Additionally, if we heat the air at the surface the warmer air has a tendency to rise and lower surface pressure. The opposite is true of colder air and high pressure. Similarly at upper-levels, when air converges it cannot go up into the stratosphere, so it must sink.

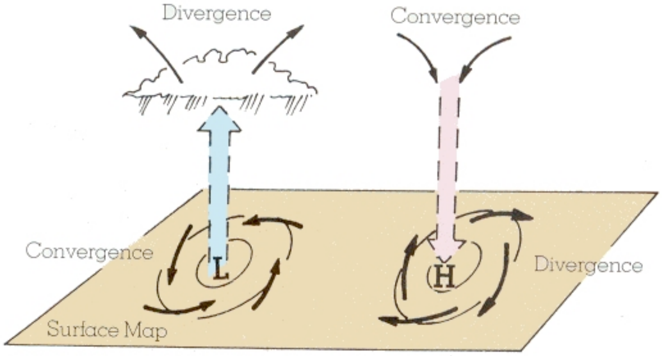
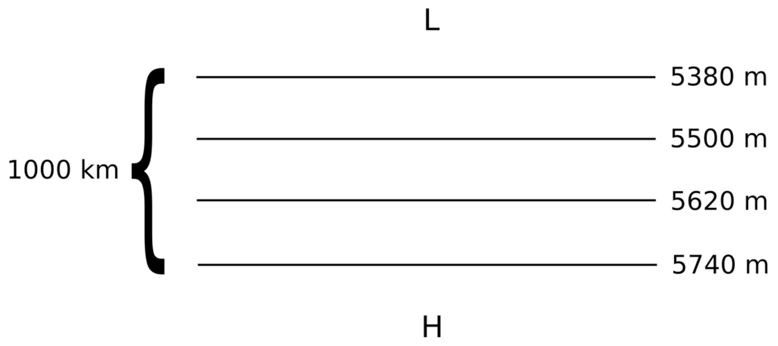


Figure 3. Convergence and divergence schematic in an idealized atmosphere. Graphic courtesy of Wunderground. (http://bimedia.ftp.clickability.com/wshmwebftp/WebStuff/Convergence_Divergence.jpg)

ASSIGNMENT

1. (a) Draw the force balance diagram for the geostrophic wind for the following pressure gradient.



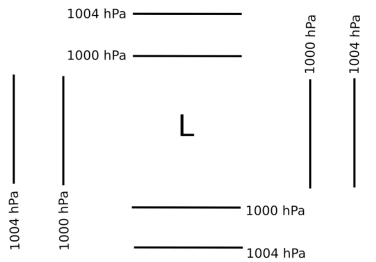
(b) Calculate the geostrophic wind in question 1a. Show all your work and don't forget UNITS!

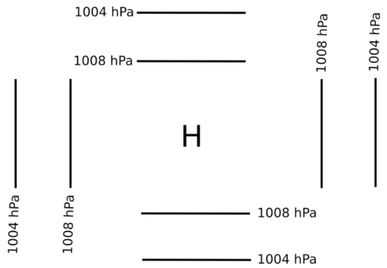
(c) Where in the atmosphere is the geostrophic wind balance valid? Find an example on an actual weather map (either created by yourself or found on the web) and post the map with a description of geostrophic wind balance below the image on your webpage.

2. Solve the gradient wind balance equation for V. Show all of your work.

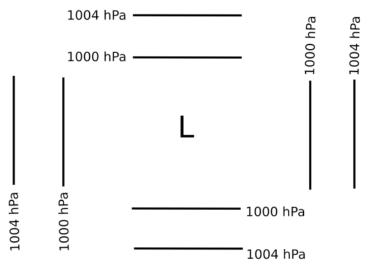
3. How can you determine where a low-pressure system (in the Northern Hemisphere) is at the surface if you are outside and have no access to the internet to get a weather map? How would your answer change if you were an exchange student taking meteorology classes in Australia?

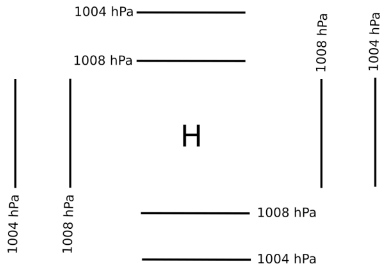
4. Draw force balances around the following surface low- and high-pressure systems,
(a) which are situated in the Northern Hemisphere.





(b) which are situated in the Southern Hemisphere





5. What force is common to all force balances? Why is that the case?

6. Estimate the strength of a tornadoes wind speed if there is a 100-hPa pressure difference from outside of the tornado to its center and the diameter of the tornado is 500 m. Show all your work.

7. Find an example of wind that exhibits the Guldberg-Mohn force balance by either making a map of your own or finding one on the internet. Post the image to your website and write a description of surface wind balance below the image.

8. If the surface wind blows toward the north, in what direction does the frictional force act on it? If the wind reverses direction and blows toward the south, in what direction does the frictional force act now?

9. Using your knowledge of forces, force balances, and vertical motions related to high and low pressure, describe how a sea breeze develops during the daytime over the Florida peninsula. Use figures and words to describe the formation of the sea breeze.

10. On the map handed out in class, circle the strongest areas of height gradient and explain the relationship between the gradient and wind speed.