Advection

Recall the difference between the Eulerian and Lagrangian view points in measuring the change of some field variable Q:

Lagrangian: following the motion: $\frac{dQ}{dt} = \text{forcing}$ (1)

Eulerian: at a fixed point: $\frac{\partial Q}{dt} = \text{advection and forcing}$ (2)

Relation between the two:

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \vec{U} \cdot \nabla_3 Q$$

$\vec{U}$ is 3-D velocity.

or

$$\frac{dQ}{dt} = \underbrace{\frac{\partial Q}{\partial t}}_{\substack{\text{Local} \\ \text{change}}} + \underbrace{\vec{V_h} \cdot \nabla_p Q}_{\substack{\text{Horizontal} \\ \text{advection}}} + \underbrace{\omega \frac{\partial Q}{\partial p}}_{\substack{\text{Vertical} \\ \text{advection}}} \quad \text{(in pressure coordinates)} \tag{3}$$

Recall also that $\vec{V_h} \cdot \nabla_p Q = u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y}$

In meteorology, it is easier to determine an instantaneous "snapshot" or distribution of the field variables (via rawinsondes, satellites, profiler network) than to follow many individual parcels for a long time. Therefore we usually chose the Eulerian framework; - i.e., - use (3) in (1) to get

$$\frac{\partial Q}{\partial t} = -\vec{V_h} \cdot \nabla_h Q - \omega \frac{\partial Q}{\partial p} + \text{forcing} \tag{3.5}$$

Note: <u>advection</u> is <u>always</u> defined with a – sign so that <u>positive advection</u> ($-\vec{V} \cdot \nabla Q > 0$) will increase the value of Q $\left(\frac{\partial Q}{\partial t} > 0 \right)$

Qualitative Evaluation of Advection on Weather Maps

Quantitative evaluation of advection is routinely performed by computers (via, e.g., finite difference expressions) but it would also be useful to have a qualitative method to estimate the sign and magnitude of temperature and vorticity advections. We will now derive an expression for advection that can be estimated visually from standard weather charts.

Let A_Q represent horizontal advection in natural coordinates, i.e.,

$$A_Q = -\vec{V} \cdot \nabla Q = -V_s \frac{\partial Q}{\partial s} \tag{4}$$

where V_s is the wind speed in the s direction. Now assume that the geostrophic wind Vg is a good approximation for V_s so that

$$V_s \approx V_g = -\frac{g}{f}\frac{\partial z}{\partial n} \qquad \text{and}$$

$$A_Q = +\frac{g}{f}\frac{\partial z}{\partial n}\frac{\partial Q}{\partial s} \approx \frac{g}{f}\frac{\Delta z}{\Delta n}\frac{\Delta Q}{\Delta s} \tag{5}$$

Since all weather maps use fixed contour intervals, Δz, ΔQ, are f at any point are constant and we can write (5) as

$$A_Q \approx \frac{K}{\Delta n \Delta s} \text{ or } A_Q \alpha \frac{1}{\Delta n \Delta s} \tag{6}$$

Thus the magnitude of advection is inversely proportional to the area of the quadrilateral enclosed by intersecting pairs of z and Q lines. We shall call this quadrilateral an "advection box".

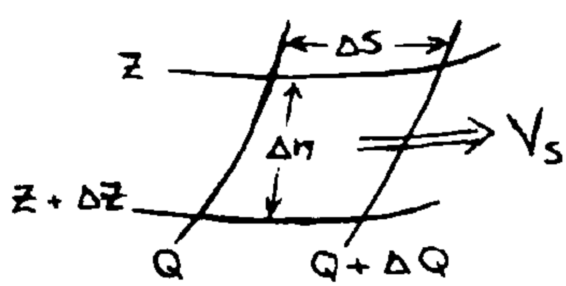


Fig. 1

The smaller Δn and Δs are, the smaller the advection box, the larger the (geostrophic) advection is. In the diagram above, the advection is negative since smaller values of Q are being advected downstream $\left(-Vs\frac{\partial Q}{\partial s} < 0\right)$.

From (4) or (5) we can state that:

(a) $A_Q = 0$ when:

(i) $V_s = 0$

(ii) $\vec{V} \cdot \nabla Q = 0$ or streamlines and contours of Q are parallel

(iii) $\frac{\partial Q}{\partial s} > 0$, $V_s \neq 0$ (Q is a maximum or minimum)

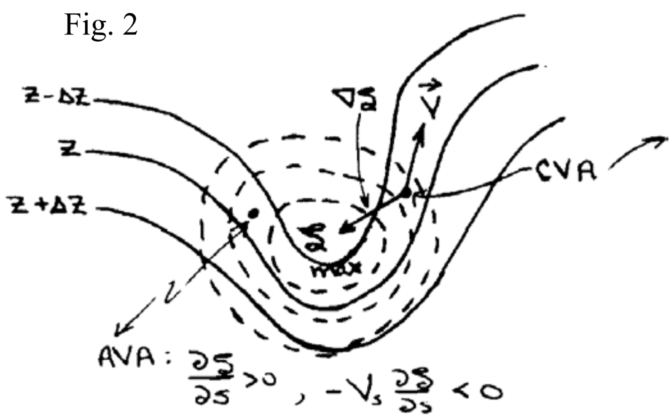
(iv) $\frac{\partial z}{\partial n} = 0$ (z is a maximum or minimum)

(b) $A_Q > 0$ when $\frac{\partial Q}{\partial s} < 0$ i.e., Q increases upstream

(c) $A_Q < 0$ when $\frac{\partial Q}{\partial s} > 0$ i.e., Q increases downstream as in Fig. 1.

The deduction of positive and negative vorticity advection is very important in weather forecasting. In Fig. 2 below, solid lines are contours and dashed lines are isopleths of absolute vorticity $(\varsigma+f) = \varsigma_a$.

Fig. 2



Now $-\vec{V} \cdot \nabla\zeta \equiv -V_s \frac{\partial \zeta}{\partial s} > 0$

Since $\frac{\partial \zeta}{\partial s} < 0$ or

$\vec{V} \cdot \nabla\zeta < 0$ $(\cos\theta < 0)$

Smallest boxes – greatest advection – usually when angle of intersection ~ 90°

Finite Difference Calculations on Weather Maps

Once scalar analysis of a field variable has been performed, one can use finite difference approximations directly on the analysis to calculate various quantities:

Temperature advection - $\vec{V} \cdot \nabla T$

If we don't have an isotach analysis for $|\vec{V}|$, we can assume $\vec{V}_g \approx \vec{V}$ and use the z field.

$$T_A = -\overrightarrow{V_g} \cdot \nabla T = -\left(u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} \right)$$

where

$$u_g = -\frac{g}{f_o}\frac{\partial z}{\partial y} \qquad vg = \frac{g}{f_o}\frac{\partial z}{\partial x}$$

<u>Vorticity</u>

Again assume geostrophic flow:

$$\boldsymbol{\zeta}_{\mathbf{g}} = \hat{k} \cdot \nabla x \overrightarrow{V_g} = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \qquad \frac{g}{f_o}\left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right]$$

Note that if $\Delta x = \Delta y$, $\quad \zeta_g = \frac{g}{f_o} \nabla^2 z$

Think about how you might calculate vorticity advection.