REVIEW OF HYDROSTATIC CONCEPTS

Recall the hydrostatic equation, gas law, and definition of virtual temperature:

$$(1) \qquad dp/dz = -\rho g \quad , \qquad \rho = p/RT_v \quad , \qquad T_v \approx (1 + .61w)$$

To derive an expression for obtaining heights from temperature (and moisture) data, we first separate variables:

$$dz = -\frac{RT_v}{g}\frac{dp}{p}$$

and integrate over an atmospheric layer:

z_2 ——————— p_2

z_1 ——————— p_1

$$\int_{z_1}^{z_2} dz = -\frac{R}{g}\int_{p_1}^{p_2} T_v d(\ln p)$$

Let T_v be the mean virtual temperature between p_1 and p_2.

Then $$z_2 - z_1 = -\frac{R\overline{T}_v}{g}\int_{p_1}^{p_2} d(\ln p) = -\frac{R\overline{T}_v}{g}\ln p_2/p_1$$

$$\text{or} \quad z_2 = z_1 + \frac{R\overline{T}v}{g}\ln p_1/p_2 \qquad (2)$$

which is the hypsometric equation. If we define $z_2 - z_1$ as the thickness h, then the "thickness equation" is

$$h = \frac{R\overline{T}v}{g}\ln p_1/p_2$$

For a fixed lower pressure (p_1) and upper pressure (p_2) – e.g., for the 1000-500 mb layer, $R/g \ln p_1/p_2$ is a constant K. Thus

$$h = K\overline{T}_v \qquad (3)$$

which emphasizes the point that the thickness of a layer is solely due to its mean virtual temperature.

We can use equation (3) to estimate expected thickness errors due to radiosonde temperature errors. First take differentials:

$$\delta h = K\delta\overline{T}_v$$

If we assume a 1 °C systematic error in the radiosonde thermistor, we can construct the following table for the corresponding thickness errors:

Layer	δh (m)
1000 – 850 mb	5
1000 – 700 mb	11
1000 – 500 mb	20
1000 – 300 mb	35
1000 – 200 mb	47
1000 – 100 mb	68

You can allow for about 50% of these values as the margin of error in the reported height values when adjusting your contours for smoothness and obeying geostrophic wind spacing.

You should be able to prove to yourself the following applications of hydrostatic concepts:

(i) troughs (lows) tilt toward cold air

(ii) ridges (highs) tilt toward warm air

(iii) cold lows intensify with height

(iv) warm lows weaken with height

(v) warm highs intensify with height

(vi) cold highs weaken with height

You should use the above rules to develop vertical consistency between contour analysis at different levels.