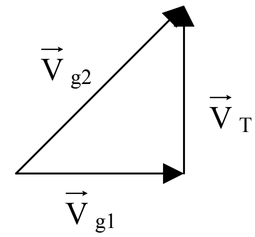
Thermal Wind and Thickness Field

Definition: The *thermal wind* is a vector difference between the geostrophic wind at two levels.



$$\vec{V}_T = \vec{V}_{g2} - \vec{V}_{g1} \qquad (1)$$

$$\therefore \quad \vec{V}_{g2} = \vec{V}_{g1} + \vec{V}_T :$$

The thermal wind can be deduced directly from the thickness field. To prove this, write Equation (1) in component form:

$$u_T = u_{g2} - u_{g1} \qquad V_T = V_{g2} - V_{g1}$$

and apply the geostrophic wind relations:

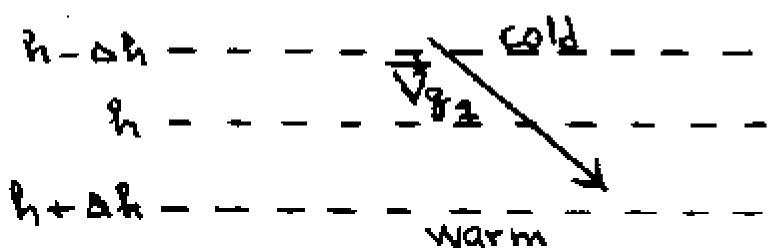
$$u_T = -\frac{g\partial}{f\,\partial y}(z_2 - z_1) \qquad V_T = \frac{g\,\partial}{f\,\partial x}(z_2 - z_1)$$

or in vector form:

$$\vec{V}_T = \frac{g}{f}\hat{k} \times \nabla_p h \qquad (2)$$

Therefore if you know the thickness field, the thermal wind "blows" parallel to the thickness contours with cold thickness to the left in the Northern Hemisphere.

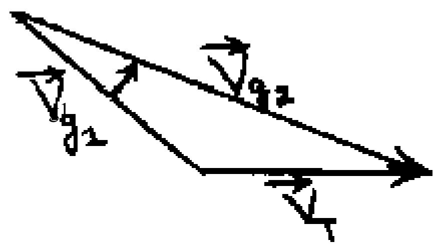
Equations (1) and (2) can be used together to deduce the relationship between temperature advection and change of wind direction with height. Suppose we have *cold thickness advection* near the surface. Therefore, we are *given*:



From Equation (2), we know $\vec{V}_T$ is parallel to the thickness lines directed from 270°:

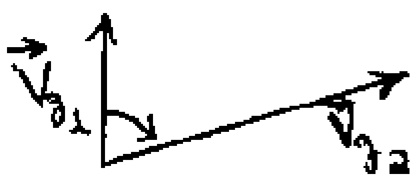
$\vec{V}_T$

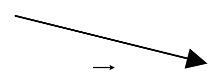
From Equation (1), we *deduce* $\vec{V}_{g2}$ from:



Since the wind direction of $\vec{V}_{g2}$ is counterclockwise from $\vec{V}_{g1}$, the wind has backed with height. Therefore *cold advection* is associated with *backing* of the winds with height. Conversely, the student should deduce that *warm advection* is associated with *veering* of the winds with height.

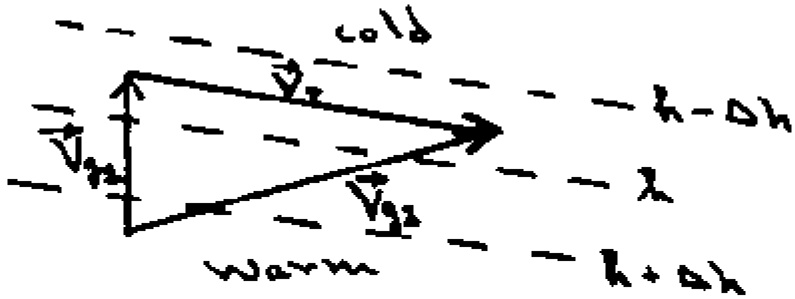
You should also be able to construct the "proof" in the other direction; e.g., *given that the winds veer with height*, deduce the sign of thickness advection. Here we *start* with





and we deduce from Equation (1) that the thermal wind is $\vec{V}_T$

Now Equation (2) tells us that thickness lines are parallel to $\vec{V}_T$, so we have:



Since $\vec{V}_{g1}$ blows from warm to cold thickness, we have deduced that *veering* of the winds is associated with *warm advection*.

Recall that the *thermal wind equation*

$$\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{p}\,\hat{k} \times \nabla_p T$$

is an expression for the *vertical shear* of the *geostrophic wind*, which says that the *rate of change* of geostrophic winds with height (the shear) is directly proportional to the horizontal temperature gradient. Note that in finite difference form:

$$\frac{\partial \vec{V}_g}{\partial p} \sim \frac{\Delta \vec{V}_g}{\Delta p} = \frac{\vec{V}_T}{\Delta p} \qquad (3)$$

Thus Equation (3) can be integrated with respect to pressure to obtain a formula for $\vec{V}_T$. If one uses the hypsometric equation:

$$h = \frac{R\overline{T}_v}{g} \ln\left(p_1/p_2\right) \qquad (4)$$

then you can also derive equation (2) in this manner (student exercise).

Recall also that for a 1000 – 500 mb thickness calculation, Equation (4) is written as:

$$h = K\overline{T}_v \qquad \text{where } K \approx 20.3 \text{ mK}^{-1}$$

Thus all thickness contours can be re-labeled as mean virtual temperatures of the 1000 – 500 mb layer. For example, the 540 dam thickness contour corresponds to the 266 K mean isotherm.