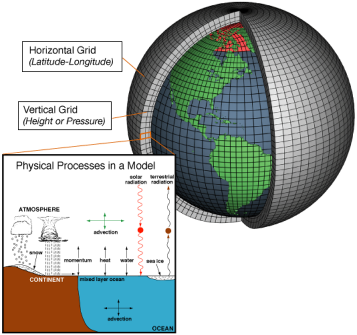
Numerical Weather Prediction



Steven Cavallo
12 November 2012

History of numerical weather prediction (NWP)

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History of numerical weather prediction (NWP)

- **Numerical weather prediction**: The use of mathematical models of the atmopshere and oceans to predict the weather
- 1904: Vilhelm Bjerknes first recognized that the governing equations could be solved to obtain future states of the atmosphere.
  - ➔ Electronic computers did not exist yet.
  - ➔ He envisioned splitting up the calculations into tasks, with different groups of people focusing on different parts of it.

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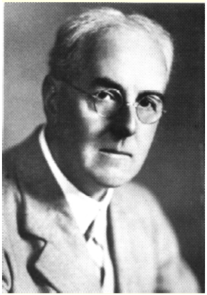
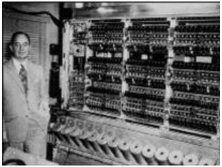
  - ➔ Calculation was done *by hand* and took over 6 weeks!
  - ➔ Forecasts were not so great; the 6-hour pressure error was over 100 hPa.
  - ➔ Initial data was too ‘noisy’ and needed to be filtered before integrating.
  - ➔ He estimated that he would need about 64,000 “human” computers to keep pace with the weather.



L.F. Richardson

History of numerical weather prediction (NWP)

- Little progress was made in NWP until WWII, when electronic computers were developed.
  - ➔ Electronic computers could be used to simulate the behavior of a system of processes.
  - ➔ Computer simulations were first used during the Manhattan Project to simulate the process of nuclear detonation.
- The first electronic computer was called the ENIAC, developed in 1946-1947 by John von Neumann. It was funded during WWII by the U.S. Army.



John Von Neumann in front of the ENIAC.

History of numerical weather prediction (NWP)

- John von Neumann believed the problem of weather forecasting was good use for his computing machinery.
  - ➔ He assembled a group of theoretical meteorologists together to develop a simplified, filtered system of equations for weather forecasting.
  - ➔ The group included recognizable names such as Jule Charney and Carl-Gustaf Rossby.
  - ➔ The group constructed the first successful weather model in 1950.

Charney

Rossby

History of numerical weather prediction (NWP)

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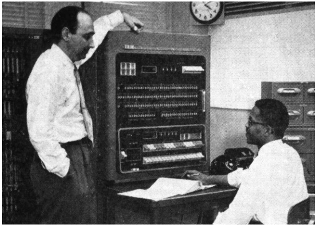
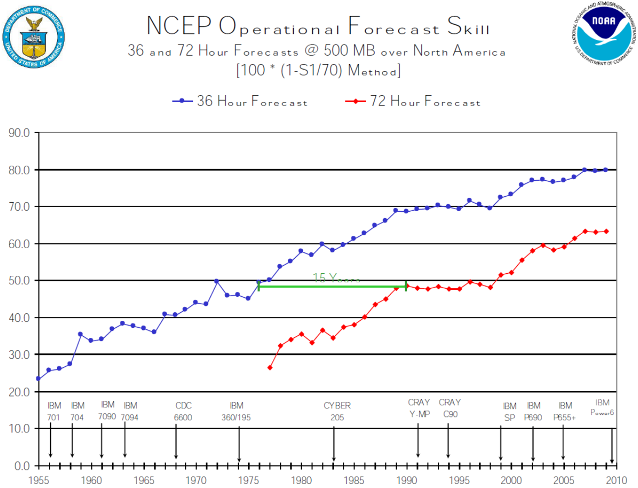
- By 1958, the forecasts being produced began to show steadily increasing and useful skill.



IBM 701

History of numerical weather prediction (NWP)

The use of numerical weather prediction models became routine in the 1950's, and we have since continued to improve them.



NWP models

- Major operational weather models today:
  - ➔ GFS: The U.S. NCEP Global Forecast System
  - ➔ FNO: The U.S. Navy Fleet Numerical Meteorology and Oceanography Center
  - ➔ ECM: European Center for Medium-Range Weather Forecasts
  - ➔ UKM: The United Kingdom Met Office
  - ➔ CMC: The Canadian Meteorological Center
  - ➔ CDAS: The frozen version of GFS used for the NCEP/NCAR Reanalysis Project

NWP model assessment

Models are assessed using ***skill scores***: a scaled representation of forecast error that relates the forecast accuracy of a particular forecast model to some reference model.
One skill score is called the *anomaly correlation*

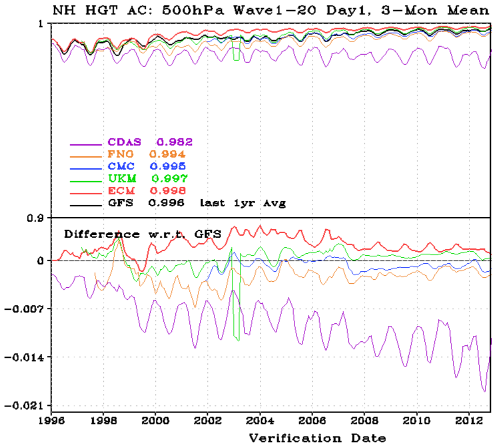
$$AC = \frac{\overline{f'o'}}{\sigma_f \sigma_o}$$

where f' is the deviation of the forecast from the long-term climatology, o' is the deviation of the observation from the long-term climatology, σ means standard deviation with respect to the long-term climatology, and the overbar is the mean over all model gridpoints.

AC is a correlation coefficient between the forecasts and observations. A perfect correlation is 1, implying perfect forecasts. Random forecasts will result in $AC \simeq 0.5$, and forecasts are subjectively considered by forecasters to have skill when $AC \geq 0.6$.

NWP model assessment

Anomaly correlation skill score comparisons of the 5 major operational models:



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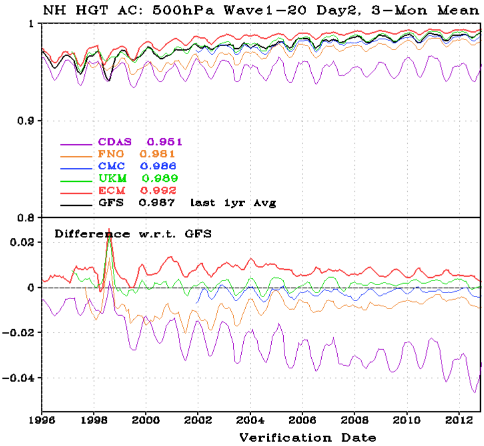

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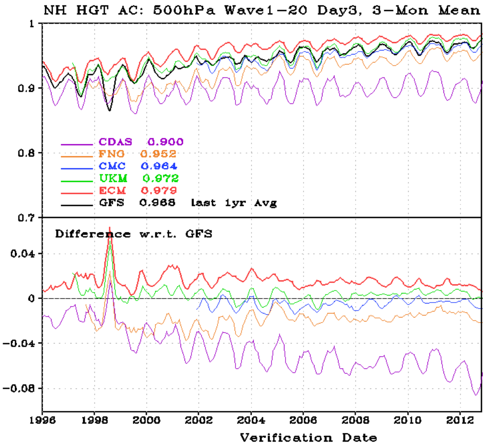

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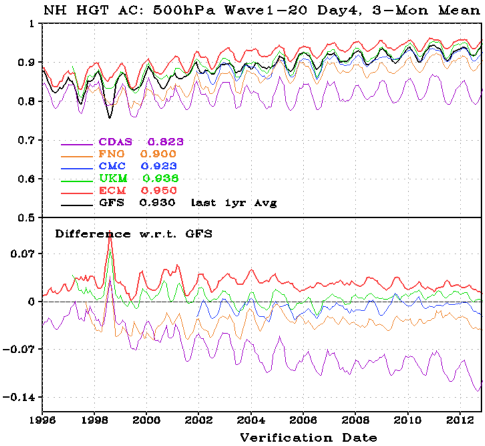

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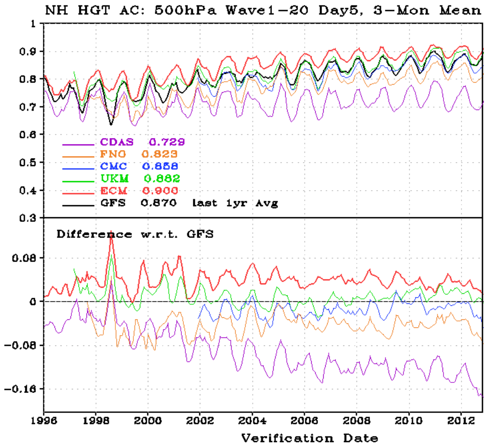

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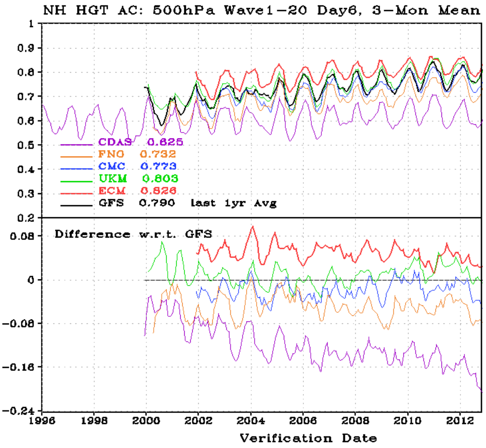

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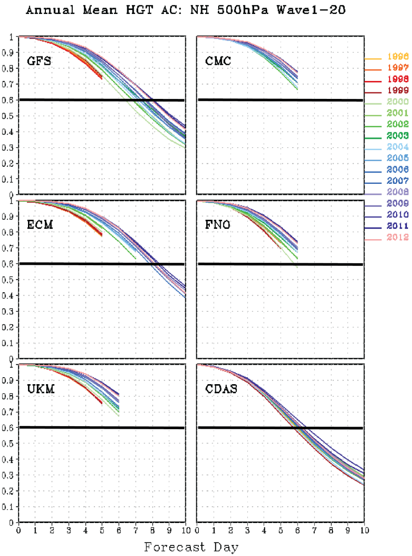
Anomaly correlation skill score comparisons of the 5 major operational models as a function of forecast lead time:



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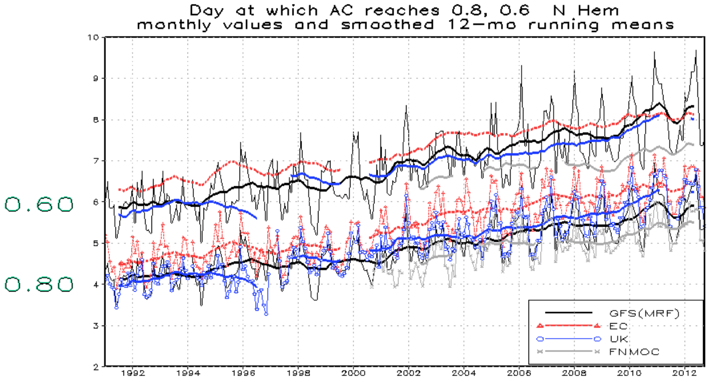


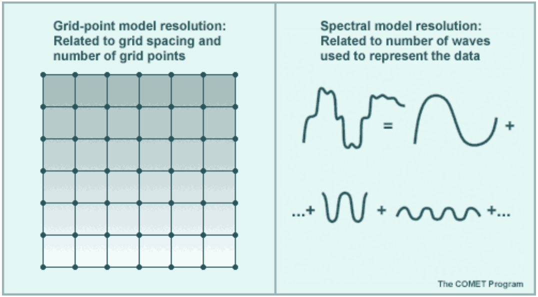
Different types of numerical prediction models

NWP vs. climate models

NWP	Climate
Global or regional domain	Global domain
Higher resolution	Lower resolution
Short-term forecasts (0-10 days)	Long-term statistics (decades, centuries)
Extremely sensitive to initial conditions	Initial conditions not as relevant
Couplings with ocean not as important	Extremely important to couple with all Earth System components

Types of NWP models

Grid point vs. spectral model



Types of NWP models

Grid point vs. spectral models

1. Spectral
   - ➔ Data represented by wave functions, resolution is a function of the number of waves resolved.
   - ➔ Equations of motion can be integrated with little error.
   - ➔ *Periodic* boundary conditions (BCs) —must be global.
   - ➔ Computationally expensive to resolve more waves.
2. Grid point
   - ➔ Data represented by fixed set of grid points—regularly spaced grid.
   - ➔ Good for high resolution, regional forecasts.
   - ➔ BCs of regional model must be from another model, thus it is highly sensitive to the model chosen for BCs.
   - ➔ Finite difference approximations used to compute model derivatives; this introduces significant model truncation error.
3. Next generation

NCEP regional models

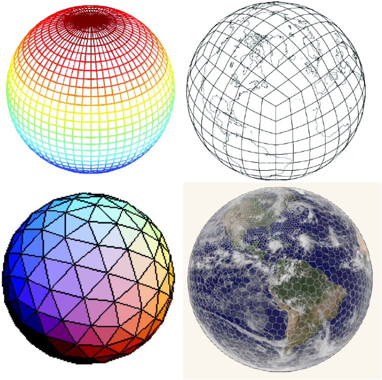
- Routine model runs by the National Centers for Environmental Prediction (NCEP):
  - ➔ NAM: North American Mesoscale (12,4 km)
  - ➔ SREF: Short range ensemble forecast
  - ➔ RAP: Rapid Refresh (recently replaced RUC) (13 km)
  - ➔ HRRR: High-resolution Rapid Refresh (3 km)
  - ➔ GEFS: Global ensemble forecast system
  - ➔ NAEFS: North American ensemble forecast system
  - ➔ WW3: Wave Watch III
  - ➔ POLAR: Polar ice drift
  - ➔ HWRF: Hurricane Weather Research and Forecasting (seasonal)
  - ➔ GHM: GFDL hurricane model

Model comparision matrix:

`http://www.meted.ucar.edu/nwp/pcu2/index.htm`

Next generation: Bridging climate with NWP

- Latitude–longitude grids: Grid converges at poles, so fields must be filtered at high latitudes to maintain model stability.
- Cubed-sphere grids (include finite volume, spectral elements)
- Icosahedral grids
- Cubed-sphere and icosahedral grids are comparable, and the winner is TBD.
- Some properties:
  - ➔ Global energy conservation and a reasonable kinetic energy spectrum are required.
  - ➔ Desirable for atmospheric mass and transport operators (tracers) to be conserved.



Numerical methods for meteorology

The governing equations (a.k.a primitive equations, Navier-Stokes equations) form a closed set of differential equations. We can solve them to estimate the future of state of the atmosphere. The equations are of the form:

$$\underbrace{\frac{d\phi}{dt}}_{\text{"Dynamics"}} = \underbrace{F_\phi}_{\text{"Physics"}} \tag{1}$$

where ϕ is any scalar. In meteorology, ϕ can be u, v, w, θ, and mass.

Methods used to solve the left-hand side of (1) are usually considered the model "dynamics" while solving the right-hand side of (1) is considered the model "physics."

Numerical methods for meteorology

To advance ϕ in time, (1) is rearranged to solve for the time tendency:

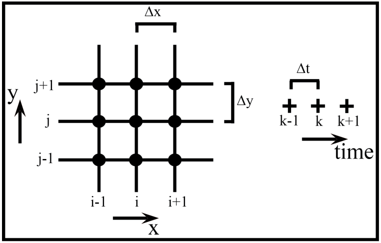
$$\underbrace{\frac{\partial \phi}{\partial t}}_{\text{Time tendency}} = \underbrace{-\vec{U} \cdot \nabla \phi}_{\text{Advection}} + \underbrace{F_\phi}_{\text{Physics}} \tag{2}$$

where $\vec{U}$ is the 3-D wind vector.

The "dynamics" now consists of two terms: The (1) time tendency and (2) the advection term. Both can be solved using Finite differencing.

Grid point models

In a grid point model, the equation is *discretized*, or broken down so that it can be represented by a finite number of grid points.



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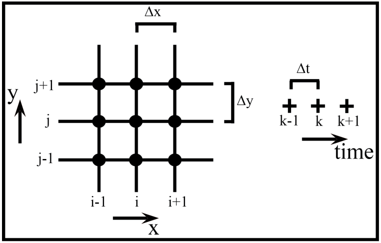


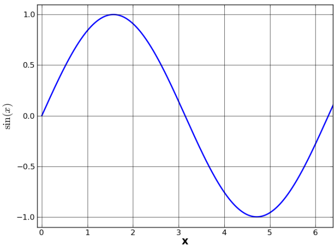
If we consider a 1-D grid advection problem with no outside forcings, then we can discretize (2) like:

$$\frac{\phi^{k+1} - \phi^{k-1}}{2\Delta t} = -u\frac{\phi^k_{i+1} - \phi^k_{i-1}}{2\Delta x}.$$

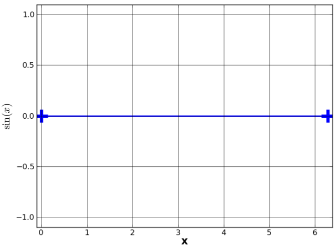
Grid point models

How many grid points does it take to resolve a wave?

True solution



2 Grid points?



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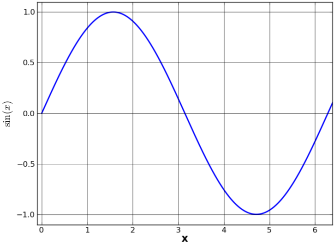
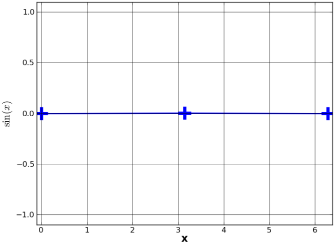


3 Grid points?



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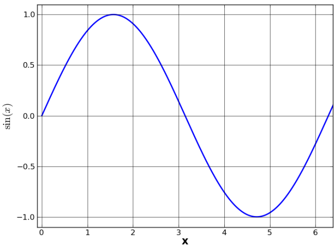
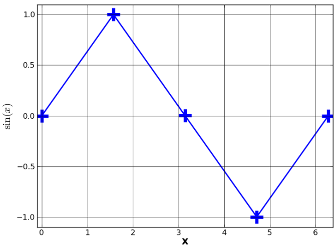


5 Grid points?



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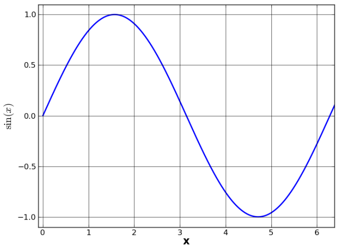
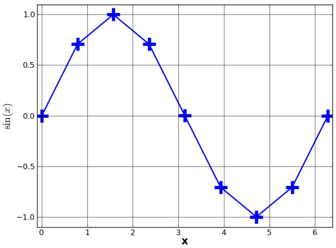


9 Grid points?



Grid point models

The distance between grid points is the length scale of a wave, divided by the number of grid points:

$$\Delta x = \frac{L}{n_{gpts}}$$

where Δx is called the *grid spacing* or *resolution*, L is the length of the entire wave or feature, and n_{gpts} is the number of grid points.

We just saw that we can roughly resolve a wave using 5 grid points.

Therefore, to resolve something of length L, we must have a MINIMUM of 5 grid points.

Grid point models

In general, in a model with a horizontal grid spacing of Δx, the smallest resolvable feature is

$$L_{smallest} = 5\Delta x.$$

Horizontal resolution in some well-known weather models today:

- GFS (23 km): $\rightarrow$ $L_{smallest}$ = 115 km
  - ➔ After 2013, GFS (12 km): $\rightarrow$ $L_{smallest}$ = 60 km
- ECMWF (10 km): $\rightarrow$ $L_{smallest}$ = 50 km
- NAM and NSSL WRF (4 km): $\rightarrow$ $L_{smallest}$ = 20 km
- NCEP/NCAR Reanalysis (215 km): $\rightarrow$ $L_{smallest}$ = 1075 km

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Scales of Motion

Scale	Time	Distance	Example
Planetary	Weeks, or longer	1000 to 40,000 km	Westerlies, trade winds
Synoptic	Days to weeks	100 to 5000 km	Cyclones
Mesoscale	Minutes to hours	1 to 100 km	Tornado, T-storm
Microscale	Seconds to minutes	$<$ 1 km	Turbulence, wind gusts

Numerical differentiation

Spatial numerical differentiation-Some common derivative methods:

$$\textit{Centered} \quad \frac{\partial \phi}{\partial x} = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

$$\textit{Forward} \quad \frac{\partial \phi}{\partial x} = \frac{\phi_{i+1} - \phi_{i}}{\Delta x}$$

$$\textit{Backward} \quad \frac{\partial \phi}{\partial x} = \frac{\phi_{i} - \phi_{i-1}}{\Delta x}$$

Numerical integration

Back to the 1-D grid advection equation:

$$\frac{\phi^{k+1} - \phi^{k-1}}{2\Delta t} = -u\frac{\phi^k_{i+1} - \phi^k_{i-1}}{2\Delta x}.$$

Solving for the next time step $K + 1$ gives:

$$\begin{aligned}
\phi^{k+1} &= \phi^{k-1} - \frac{u\Delta t}{\Delta x}\left(\phi^k_{i+1} - \phi^k_{i-1}\right) \\
\phi^{k+1} &= \phi^{k-1} - \mu\left(\phi^k_{i+1} - \phi^k_{i-1}\right)
\end{aligned}$$

where μ can be used to determine model stability.

The Courant-Fredrich-Lewy (CFL) condition (1928) requires that in order for a model to remain numerically stable, the following must be true:

$$\mu = \frac{u\Delta t}{\Delta x} \leq 1. \tag{3}$$

Numerical integration

In most numerical models, the dynamics and physics tendencies are computed seperately, and added together into one “tendency” term:

$$\begin{aligned}
\frac{\phi^{k+1} - \phi^{k-1}}{2\Delta t} &= -u\left(\frac{\phi_{i+1}^{k} - \phi_{i-1}^{k}}{2\Delta x}\right) + F_{\phi}^{k} \\
\frac{\phi^{k+1} - \phi^{k-1}}{2\Delta t} &= f(\phi^{k}) \qquad (4)
\end{aligned}$$

where $f(\phi^k)$ are all model tendencies at time k.

Physics tendencies (F_{ϕ}^{k}) are usually parameterized (which we will cover later) because these processes generally occur at smaller scales than the model grid spacing. Numerical methods primarily focus on the numerical calculations of the dynamics terms.

There are many different methods to integrate ϕ. The one we have set up above is the *Leapfrog* method.

Numerical integration

Let's compare some different time integration schemes. Using centered differencing in time is called the **Leapfrog** method:

$$\begin{aligned}\frac{\phi^{k+1}-\phi^{k-1}}{2\Delta t} &= f(\phi^k)\\ \phi^{k+1} &= \phi^{k-1}+2\Delta t\ f(\phi^k)\end{aligned}$$

Similarly, the **forward** method uses forward differencing:

$$\begin{aligned}\frac{\phi^{k+1}-\phi^{k}}{\Delta t} &= f(\phi^k)\\ \phi^{k+1} &= \phi^{k}+\Delta t\ f(\phi^k)\end{aligned}$$

and the **backward** method uses tendencies at the future time step (this is called an implicit scheme):

$$\begin{aligned}\frac{\phi^{k+1}-\phi^{k}}{\Delta t} &= f(\phi^{k+1})\\ \phi^{k+1} &= \phi^{k}+\Delta t\ f(\phi^{k+1})\end{aligned}$$

Numerical integration

Of course, there are a lot of other methods (Durran 1999):

Method	*Order*	*Formula*
Forward	1	$\phi^{n+1} = \phi^n + hF(\phi^n)$
Backward	1	$\phi^{n+1} = \phi^n + hF(\phi^{n+1})$
Asselin Leapfrog	1	$\phi^{n+1} = \overline{\phi^{n-1}} + 2hF(\phi^n)$ $\overline{\phi^n} = \phi^n + \gamma(\overline{\phi^{n-1}} - 2\phi^n + \phi^{n+1})$
Leapfrog	2	$\phi^{n+1} = \phi^{n-1} + 2hF(\phi^n)$
Adams–Bashforth	2	$\phi^{n+1} = \phi^n + \frac{h}{2}\left[3F(\phi^n) - F(\phi^{n-1})\right]$
Trapezoidal	2	$\phi^{n+1} = \phi^n + \frac{h}{2}\left[F(\phi^{n+1}) + F(\phi^n)\right]$
Runge–Kutta	2	$q_1 = hF(\phi^n), \quad \phi_1 = \phi^n + q_1$ $q_2 = hF(\phi_1) - q_1, \quad \phi^{n+1} = \phi_1 + q_2/2$
Magazenkov	2	$\phi^n = \phi^{n-2} + 2hF(\phi^{n-1})$ $\phi^{n+1} = \phi^n + \frac{h}{2}\left[3F(\phi^n) - F(\phi^{n-1})\right]$
Leapfrog–Trapezoidal	2	$\phi_1 = \phi^{n-1} + 2hF(\phi^n)$ $\phi^{n+1} = \phi^n + \frac{h}{2}\left[F(\phi_1) + F(\phi^n)\right]$
Adams–Bashforth	3	$\phi^{n+1} = \phi^n + \frac{h}{12}\left[23F(\phi^n) - 16F(\phi^{n-1}) + 5F(\phi^{n-2})\right]$
Adams–Moulton	3	$\phi^{n+1} = \phi^n + \frac{h}{12}\left[5F(\phi^{n+1}) + 8F(\phi^n) - F(\phi^{n-1})\right]$
ABM Predictor–Corrector	3	$\phi_1 = \phi^n + \frac{h}{2}\left[3F(\phi^n) - F(\phi^{n-1})\right]$ $\phi^{n+1} = \phi^n + \frac{h}{12}\left[5F(\phi_1) + 8F(\phi^n) - F(\phi^{n-1})\right]$
Runge–Kutta	3	$q_1 = hF(\phi^n), \quad \phi_1 = \phi^n + q_1/3$ $q_2 = hF(\phi_1) - 5q_1/9, \quad \phi_2 = \phi_1 + 15q_2/16$ $q_3 = hF(\phi_2) - 153q_2/128, \quad \phi^{n+1} = \phi_2 + 8q_3/15$
Runge–Kutta	4	$q_1 = hF(\phi^n), \quad q_2 = hF(\phi^n + q_1/2)$ $q_3 = hF(\phi^n + q_2/2), \quad q_4 = hF(\phi^n + q_3)$ $\phi^{n+1} = \phi^n + (q_1 + 2q_2 + 2q_3 + q_4)/6$

Numerical integration

- As mentioned earlier, numerical approximations introduce model error via. These approximations are the model error that arises from the **dynamics**. Other model error can still arise from the **physics**.
- The table on the previous slide has a column that says “order,” which refers to the accuracy of the numerical scheme.
- The higher the order, the lower the error.

Numerical integration

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- The table on the previous slide has a column that says “order,” which refers to the accuracy of the numerical scheme.
- The higher the order, the lower the error.
- We looked at the forward, backward, and Leapfrog methods. Which method is most accurate?

Numerical integration

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Important points for review

- What are some differences between “NWP” and “Climate” models?
- What are the two basic types of numerical models?
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- If you setup a NWP experiment using a grid point model with a horizontal grid spacing of 30 km, what size is the smallest resolvable feature? What are some atmospheric phenomena that will not be resolved?
- Suppose that after reading this, you are inspired to write your own numerical model that uses the Leapfrog method. You run an experiment without physics and compare your model’s solutions to the Weather Research and Forecasting model, which uses a Runge-Kutta scheme that is 3rd order accurate in time. Which model would you expect better results from and why?