Answers to Homework Assignment #1

1.1 $r_a = cT_s / 2$ and $v_a = \lambda / 4T_s$

where c is the velocity of the microwaves, $\lambda$ is its wavelength $\lambda = c / f$, $f$ is the wave's frequency, and $T_s$ is the pulse repetition time (PRT; i.e., the interval between echo samples)

1.2 The observed Doppler velocity $v_o$ equals the true Doppler velocity $v_t$ if $v_t$ lies in the unambiguous interval $\pm v_a$. If $v_t$ exceeds $+v_a$, the observed velocity $v_o$ aliases to negative velocities. Thus as $v_t$ increases beyond $+v_a$, the negative values of $v_o$ decrease as $v_t$ -2$v_a$.

1.3 First trip ground clutter from scatterers on the ground at ranges, for example, between zero and 20 km is overlaid on 2nd trip weather echoes if weather scatterers are at a range $r_a$ plus 0 to 20 km.

1.4 (a) Let the range error be denoted as $\Delta r$. Then

$$\Delta r = ct - vt = ct - \frac{c}{n}t = ct\left(1 - \frac{1}{n}\right)$$

where $t$ is the time after the microwave pulse is transmitted. But for a scatter at $r$ = 300 km, $t = r/v = rn/c$. Therefore $\Delta r = r(n-1)$. Given $n$ = 1.000300, and $r$ = 300 km,

$$\Delta r = 3 \times (3 \times 10^{-4}) = 90\,\text{m}$$

(b) The echo phase difference $\Delta\psi_e$ is

$$\Delta\psi_e = -2\pi f\left(\frac{2r}{c} - \frac{2r}{v}\right) = -4\pi f\frac{r}{c}(1-n) = 4\pi\frac{r}{\lambda}(n-1)$$

$$= \frac{4\pi}{0.1} \times 3 \times 10^4 (3 \times 10^{-4}) = 360\pi\,(\text{radians})$$

1.5 (a) $P_w$ is the water vapor pressure:

$$P_w = 6.11(10)^{7.5\left(\frac{T}{T+237.5}\right)}$$

N is the exponential reference profile of the refractivity:

$$N = 313\, e^{-0.1439h}$$

$N_d$ is the refractivity profile computed from the sounding neglecting the water vapor effect:

$$N_d = \frac{77.6P}{T + 273.15}$$

$N_w$ is the refractivity which has taken into account the water vapor effect:

$$Nw = \frac{77.6}{T + 273.15}\left[P + \frac{4810P_w}{T + 273.15}\right]$$

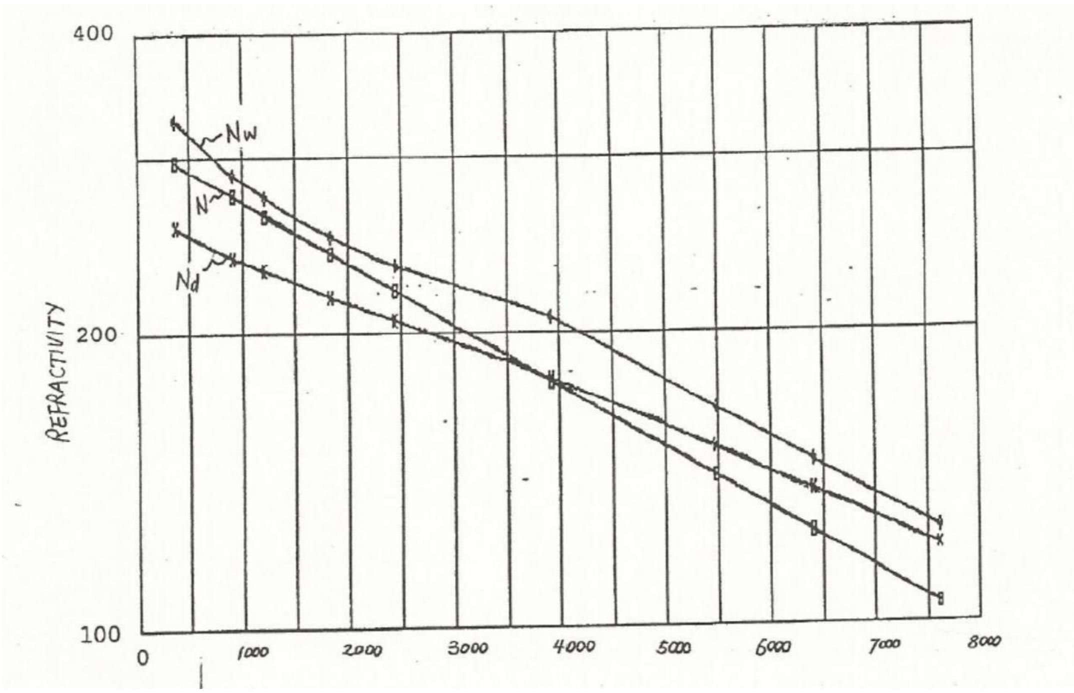



Fig.1. Profiles of N, $N_d$ and $N_w$ are plotted vs height

(b) The departure of $N_d$ from the reference profile (N) may be attributed to the temperature and water vapor variations in the real atmosphere.

The difference between $N_d$ and $N_w$ is caused by the water vapor effect. In the lower part of the atmosphere where moisture is abundant, the difference between $N_d$ and $N_w$ is the largest.

(c) Use Fig. 1 to calculate $\frac{dN}{dh}$ near the surface.

For the moist atmosphere:

$$\frac{dN}{dh} \approx -7.64(10)^{-2}(m^{-1})$$

$$\frac{dn}{dh} \approx -7.64(10)^{-5}(km^{-1})$$

The corresponding effective earth's radius is:

$$a_e = \frac{a}{1 + a\left(\frac{dn}{dh}\right)} = \frac{6370}{1 + 6370[-7.64(10^{-5})]}$$

$$a_e = 12409\ km$$

(d) Again, use Fig. 1 to calculate $\frac{dN}{dh}$ near the surface.

For exponential atmosphere:

$$\frac{dN}{dh} \approx -4.11(10)^{-2}(m^{-1})$$

$$\frac{dn}{dh} \approx -4.11(10)^{-5}(km^{-1})$$

The corresponding effective earth's radius is:

$$a_e = \frac{6370}{1 + 6370[-4.11(10^{-5})]}$$

$$a_e = 8629\ km$$

For the dry atmosphere:

$$\frac{dN}{dh} \approx -3.30(10)^{-2}(m^{-1})$$

$$\frac{dn}{dh} \approx -3.30(10)^{-5}(km^{-1})$$

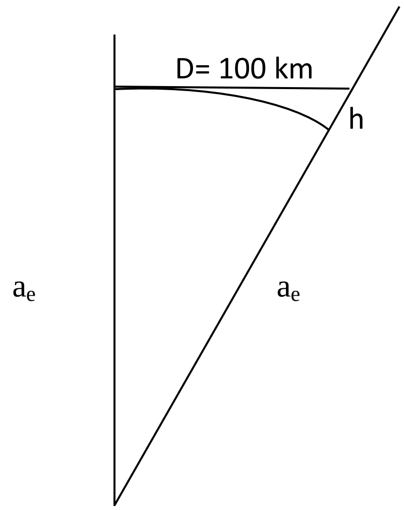
The corresponding effective earth's radius is:

$$a_e = \frac{6370}{1 + 6370[-3.30(10^{-5})]}$$

$$a_e = 8065\ km$$

(e) The $a_e$ for the exponential reference atmosphere agrees well with $\frac{4}{3}a$. But the $a_e$ obtained from the actual sounding is much larger than the $a_e$ for the $\frac{4}{3}$ earth radius model.

(f)




$$(a_e + h)^2 = a_e{}^2 + D^2$$

$$a_e{}^2 + 2a_e h + h^2 = a_e{}^2 + D^2$$

but $h^2 \ll 2a_e h$, therefore

$$h \cong \frac{D^2}{2a_e}$$

For the observed atmosphere:

$$h = \frac{10^4}{2(12409)} km = 403\ m$$

For the $\frac{4}{3}$ earth radius model:

$$h = \frac{10^4}{2\left(\frac{4}{3}\right)6370} km = 589\ m$$

Therefore,

$$\Delta h \cong 200\ m$$