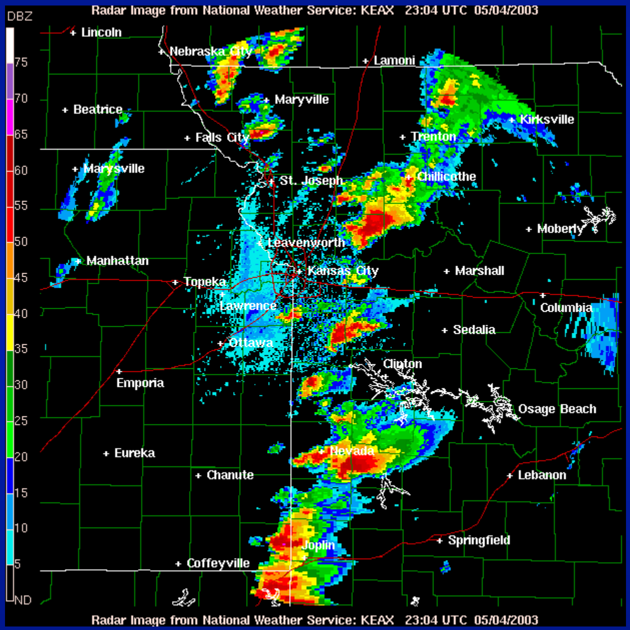
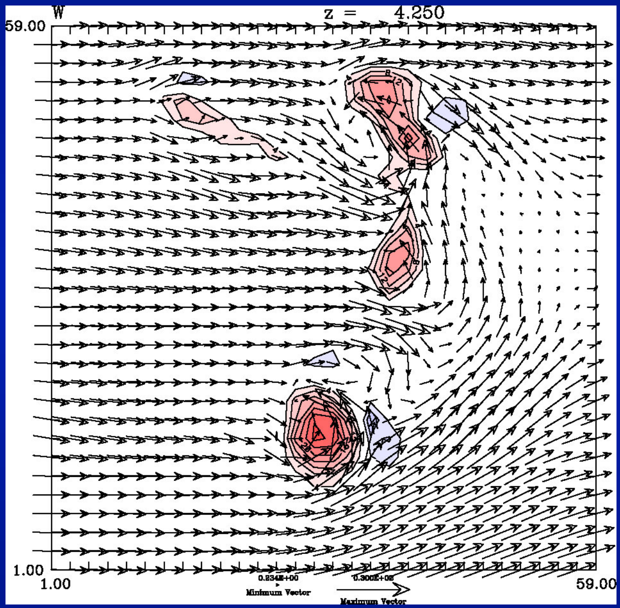
A Brief Tutorial in Convective-Scale Data Assimilation using the Ensemble Kalman Filter



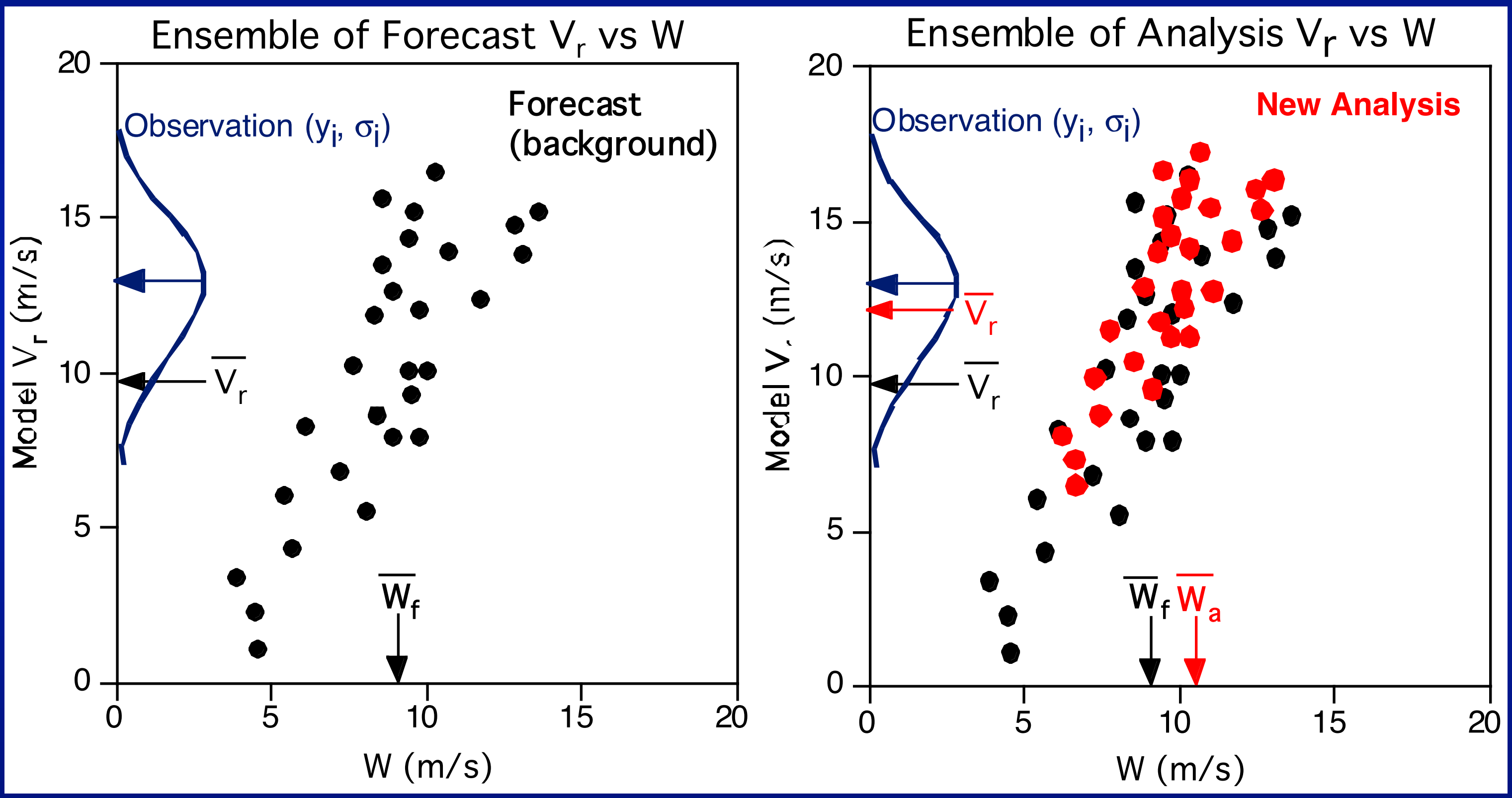


Louis Wicker

Aug 2008

How the EnKF Method "Works"

Interpolated forecast quantity



state variable

From Snyder and Zhang 2003

This Dummies' Guide to Ensemble Kalman Filtering…

- Least squares
- Kalman Filter
- Extended Kalman Filter (not going to talk about this…)
- Ensemble Kalman Filter

Similar, if not identical to variational approaches

Refs: Talagrand (J. Met. Soc. Japan, 1997), Maybeck (1979), Hansen and Smith (Tellus, 2001)

Start with a simple example…

Consider measuring a scalar, x^t, such as temperature with two different instruments (e.g., a mercury thermometer and a thermograph). Two measurements are made, z_i, and associated with each measurement is some error e_i.

$$z_1 = x^t + \varepsilon_1$$
$$z_2 = x^t + \varepsilon_2$$

The errors properties e are unbiased and that their variance is known:

$$Mean\left[\varepsilon_i\right] = 0$$
$$Var\left[\varepsilon_i^2\right] = \sigma^2$$

and to keep things simple, the correlation between the errors is zero.

$$Cor\left[\varepsilon_1 \varepsilon_2\right] = 0$$

How do we get the best "analysis", x^a?

Lets assume that a linear combination of the two measurements can give the best answer, i.e.,

$$x^a = a_1 z_1 + a_2 z_2$$

We need two constraints…..

$$a_1 + a_2 = 1$$

Unbiased estimate

$$Var\left[\left(x^a - x^t\right)^2\right] = \sigma^2$$

Minimize the variance

Adapted from: Talarand, 1997, "Assimilation of Observations, an Introduction

Working through this a bit…

$$Var\left[\left(a_1 z_1 + a_2 z_2 - x^t\right)^2\right] = \sigma^2$$

$$Var\left[\left(a_1\left(x^t + \varepsilon_1\right) + a_2\left(x^t + \varepsilon_2\right) - x^t\right)^2\right] = \sigma^2$$

$$Var\left[\left(\varepsilon_1 + a_2\left(\varepsilon_1 - \varepsilon_2\right)\right)^2\right] = \sigma^2$$

$$Var\left[\left(\varepsilon_1^2 - 2a_2\varepsilon_1^2 + 2a_2\varepsilon_1\varepsilon_2 + a_2^2\left(\varepsilon_1^2 - 2\varepsilon_1\varepsilon_2 + \varepsilon_2^2\right)\right)\right] = \sigma^2$$

$$\sigma_1^2\left(1 - 2a_2\right) + a_2^2\left(\sigma_1^2 + \sigma_2^2\right) = \sigma^2$$

$$\frac{\partial\sigma^2}{\partial a_2} = -2\sigma_1^2 + 2a_2\left(\sigma_1^2 + \sigma_2^2\right) = 0$$

$$a_2 = \frac{\sigma_1^2}{\left(\sigma_1^2 + \sigma_2^2\right)}$$

With some further algebra…

$$a_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$a_2 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

The physical interpretation is that the less accurate one observation is relative to another, more weight is given to the other observation.

The same problem can be written as a variational problem…

$$J(x) = \frac{(x - z_1)^2}{\sigma_1^2} + \frac{(x - z_2)^2}{\sigma_2^2}$$

By determining the minimum of the cost function J(x), we find where the analysis value $x=x^a$ satisfies the two previously defined constraints.

Interestingly we can convert either of these approaches, for the given assumptions, into a sequential estimator.

This sequential algorithm is the Kalman filter.

The Kalman Filter….

Since the observational errors are uncorrelated, one can process the observations sequentially.

For our problem, the first estimate of the analysis temperature and its variance are:

$$x_1^a = z_1$$
$$\langle \sigma_1^2 \rangle = \sigma_1^2$$

Our **first estimate** is simply the measurement from our first instrument.

Note the subscript now indicates the observation number, $z_1, z_2, \ldots, z_n$

The **second estimate** of the temperature and its variance is:

$$x_2^a = x_1^a + K_2 \left[z_2 - x_1^a \right]$$
$$\langle \sigma_2^2 \rangle = \langle \sigma_1^2 \rangle \left[1 - K_2 \right]$$

What is the “K_n”?

It’s called the Kalman Gain

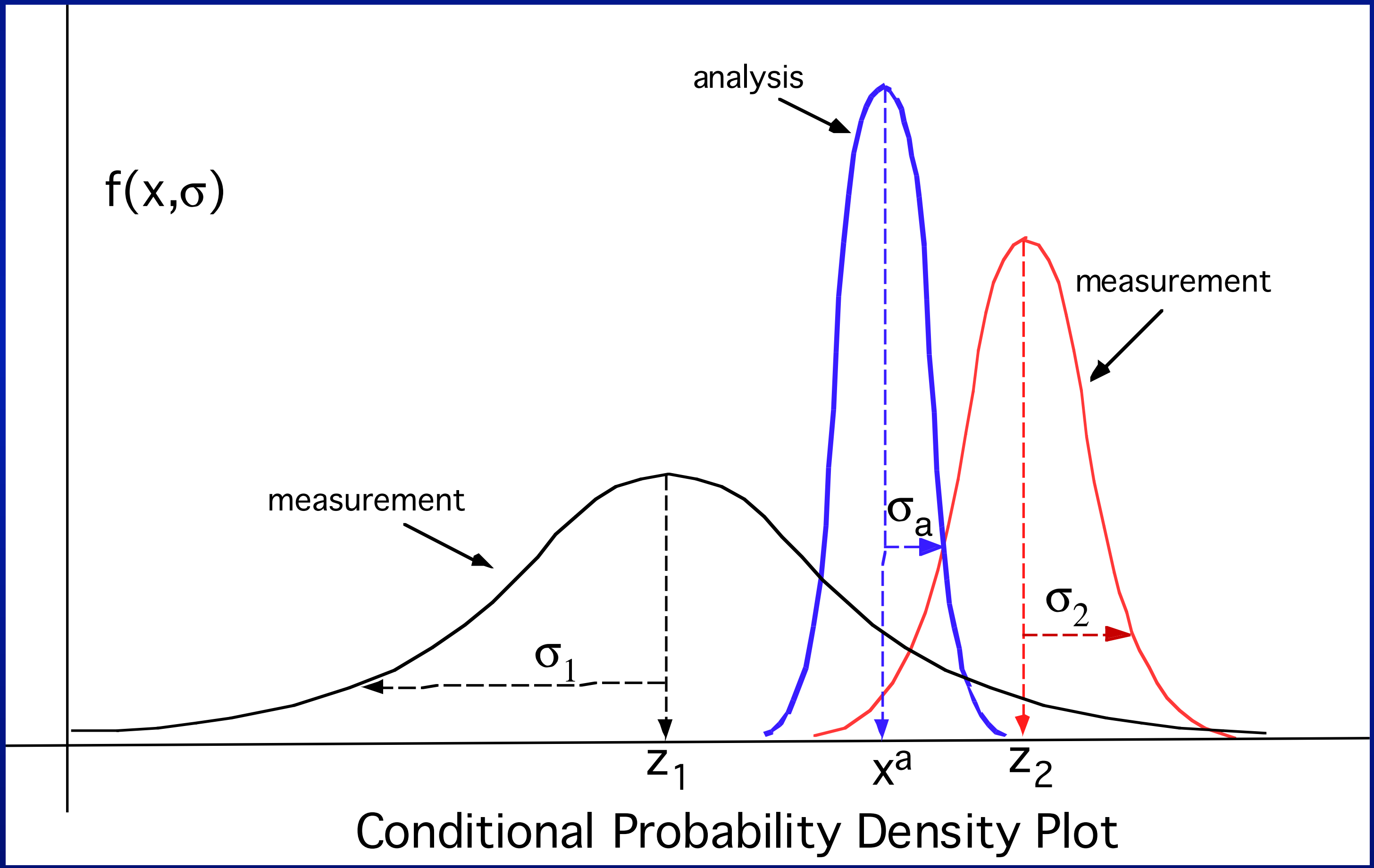
$$K_2 = \frac{\langle \sigma_1^2 \rangle}{\langle \sigma_1^2 \rangle + \sigma_2^2}$$

One can show that the Kalman filter solution to this problem is identical to our original approach.

Note that the analysis variance must decrease with each observation:

$$\langle \sigma_n^2 \rangle = \langle \sigma_{n-1}^2 \rangle \left[1 - K_n \right]$$

Graphical Interpretation



Third observation……

$$x_3^a = x_2^a + K_3\left[z_3 - H\left[x_2^a\right]\right]$$

$$H\left[x\right] = \mathit{forward}\ \text{model}$$

$$K_3 = \frac{\left\langle \sigma_2^2 \right\rangle}{\left\langle \sigma_2^2 \right\rangle + \sigma_3^2}$$

$$\left\langle \sigma_3^2 \right\rangle = \left\langle \sigma_2^2 \right\rangle \left[1 - K_3\right]$$

Analysis
Increment

We add one wrinkle, the *H(x)* forward operator: it transforms the model solution to the simulated observation at the observation point

How to generalize this?

$$Define : P_t^f = E\left[\left(x-\overline{x}_t^f\right)\left(x-\overline{x}_t^f\right)^T\right]$$

$$Define\ \ a\ \ Model$$

$$\overline{x}_{t+\Delta t}^f = A\overline{x}_t^a + Bu_{k-1}$$

$$P_{t+\Delta t}^f = AP_t^a A^T + Q_{noise}$$

$$if\ :\ x = \begin{bmatrix} u \\ \theta \end{bmatrix};\ \ P_t^f = \begin{bmatrix} \langle u'u' \rangle & \langle u'\theta' \rangle \\ \langle u'\theta' \rangle & \langle \theta'\theta' \rangle \end{bmatrix} \approx \begin{bmatrix} \partial u\,\partial u & \partial u\,\partial\theta \\ \partial u\,\partial\theta & \partial\theta\,\partial\theta \end{bmatrix}$$

$$Analysis$$

$$K = \frac{P_t^f H^T}{HP_t^f H^T + R}$$

$$\overline{x}^a = \overline{x}^f + K\left(z_o - H\left[\overline{x}^f\right]\right)$$

$$P^a = \left(I - KH\right)P^f$$

What the heck?

Now starting with Snyder & Zhang 2003…pg 1666

$$P_t^f H^T = Cov\left[\left(x - \overline{x}_t^f\right)\left(x - \overline{x}_t^f\right)^T\right] H^T = Cov\left[\left(x - \overline{x}_t^f\right) H\left(x - \overline{x}_t^f\right)^T\right]$$

$\mathbf{P}^f\ \mathbf{H}^T$ is the forecasted covariance of the model state variables
with
the observed variables.

There is MAGIC here - this means if you have an radar observation of radial velocity,
$\mathbf{P}^f\ \mathbf{H}^T$ tells you how *to increment the temperature field at that location!*

Continuing….with Snyder & Zhang 2003…pg 1666

$$P_t^f H^T = P_{xy}^f$$

$$H = \begin{bmatrix} N_{\text{var}} & . & . \\ & & \\ . & . & N_{obs} \end{bmatrix}$$

P^f H^T is the forecasted covariance of the model state variables with the observed variables.

$$if \;\; no.\, of\, obs = 1$$

$$HP_t^f = \begin{bmatrix} z_1^f \\ z_2^f \\ \cdot \\ z_n^f \end{bmatrix} z_o - H[\overline{x}^f] = c_i$$

$$HP_t^f H^T + R = d$$

$$\overline{x}_i^f = \overline{x}_i^f + \frac{c_i}{d}\left[z_o - H\left[\overline{x}_i^f\right]\right]$$

$$P^a = P^f - \frac{cc^T}{d}$$

This result corresponds heuristically to the first figure (setting x_i to be the vertical velocity and z_o to be the radial velocity). The updated w differs from the prior w by an amount proportional to the increment * Cov(x, H[x]). For example, if the observed radial velocity is greater than its forecast radial velocity and if radial velocity in the model is positively correlated with **w**, then the analyzed w should be greater than the forecast w.

Quick Review

- Kalman filter uses three pieces of information, the observation, its error variance, and the error variance of previous state estimate ("the background state") to create the analysis.

- A dynamical model can be thought of not only propagating the observations, but the error variance as well.

- Using the Kalman filter with a dynamical model requires that we need to know:

    - observational error variance
    - model forecast error variance

Kalman filter data assimilation within an NWP model

- Multi-dimensional and multi-variable!
- P^f is an ~ N^2 matrix ($N \sim 10^8$) where N is the number of degrees of freedom (Nx*Ny*Nz*Nvar)
- Direct evolution of $P^f(t)$ is computational prohibitive for NWP models!
- Approximate P^f using an ensemble (Evensen 1994)

Ensemble Kalman Filter

P^f is estimated from an ensemble of model forecasts where ensemble mean is used as estimate of the true state.

$$x_i^f(t) = \text{forecast model output}, \quad i = 1, n$$

$$\bar{x}^f(t) = \frac{1}{n}\sum_i x_i^f(t)$$

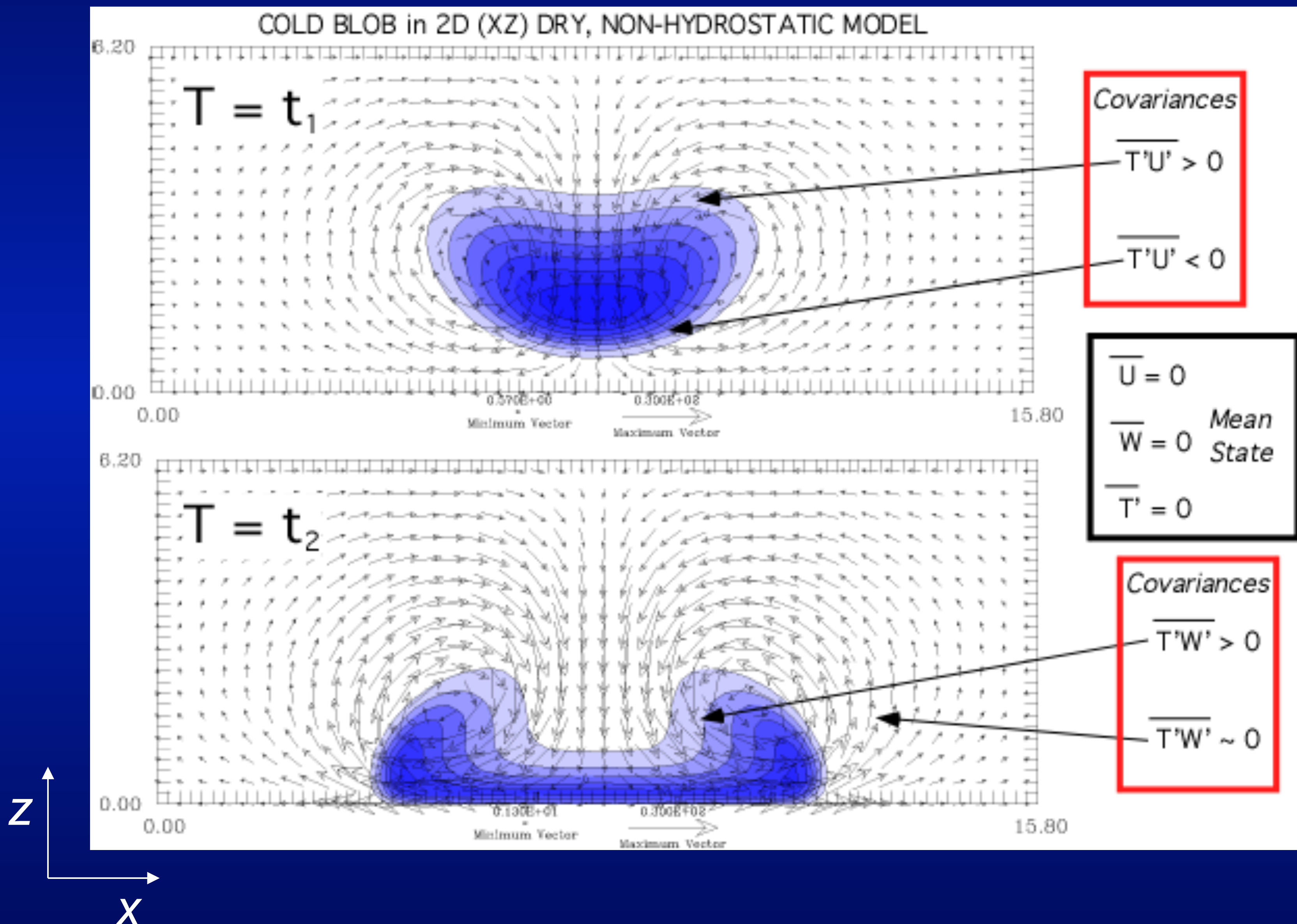
$$K = \frac{\frac{1}{n-1}\sum_i \left(\left[x_i^f(t) - \bar{x}(t)\right]\left[H(x_i^f) - \bar{H}(x_i^f)\right]^T\right)}{\sigma_{obs}^2 + \frac{1}{n-1}\sum_i \left(\left[H(x_i^f) - \bar{H}(x_i^f)\right]^T\right)}$$

$$\bar{x}^a(t) = \bar{x}^f(t) + K\left[z_o(t) - H\left[\bar{x}^f(t)\right]\right]$$

$$x_i^a(t) = \bar{x}^a(t) + \left(x_i^f - \bar{x}^f\right)K\beta\left[\bar{H}\left(x^f\right) - H\left(x_i^f\right)\right]$$

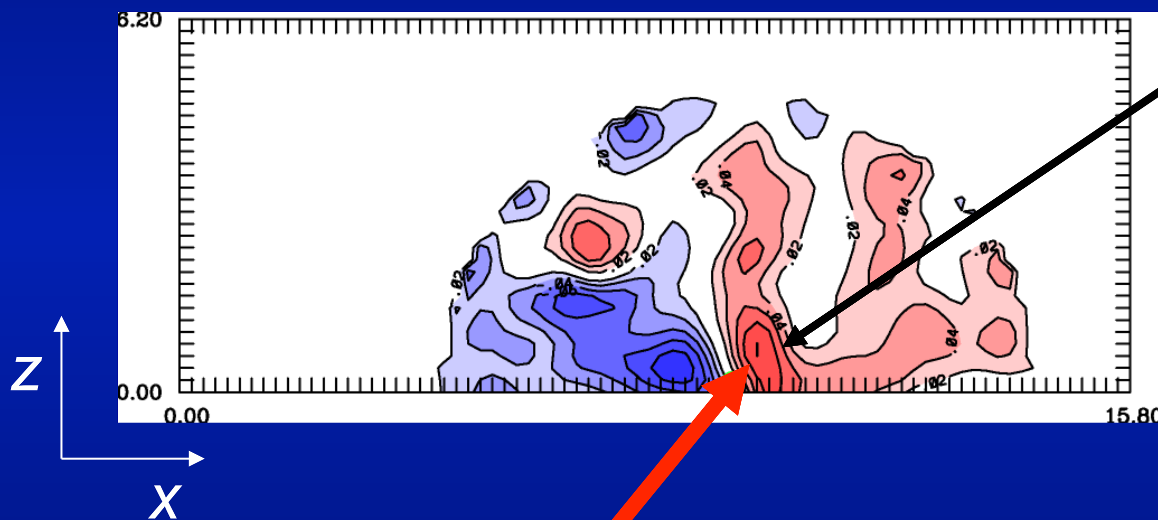
$$\beta = \left(1 + \left(\frac{\sigma_{obs}^2}{\sigma_{obs}^2 + \frac{1}{n-1}\sum_i \left(\left[H(x_i^f) - \bar{H}(x_i^f)\right]^T\right)}\right)^{1/2}\right)^{-1}$$

How the EnKF Method “Works”



How EnKF "Works"

Normalized covariance of **MODEL** temperature perturbations and **MODEL** radial velocity (v_r) at the point shown at $T=t_n$



OBSERVATION

$\overline{T' V_r'} > 0$ then

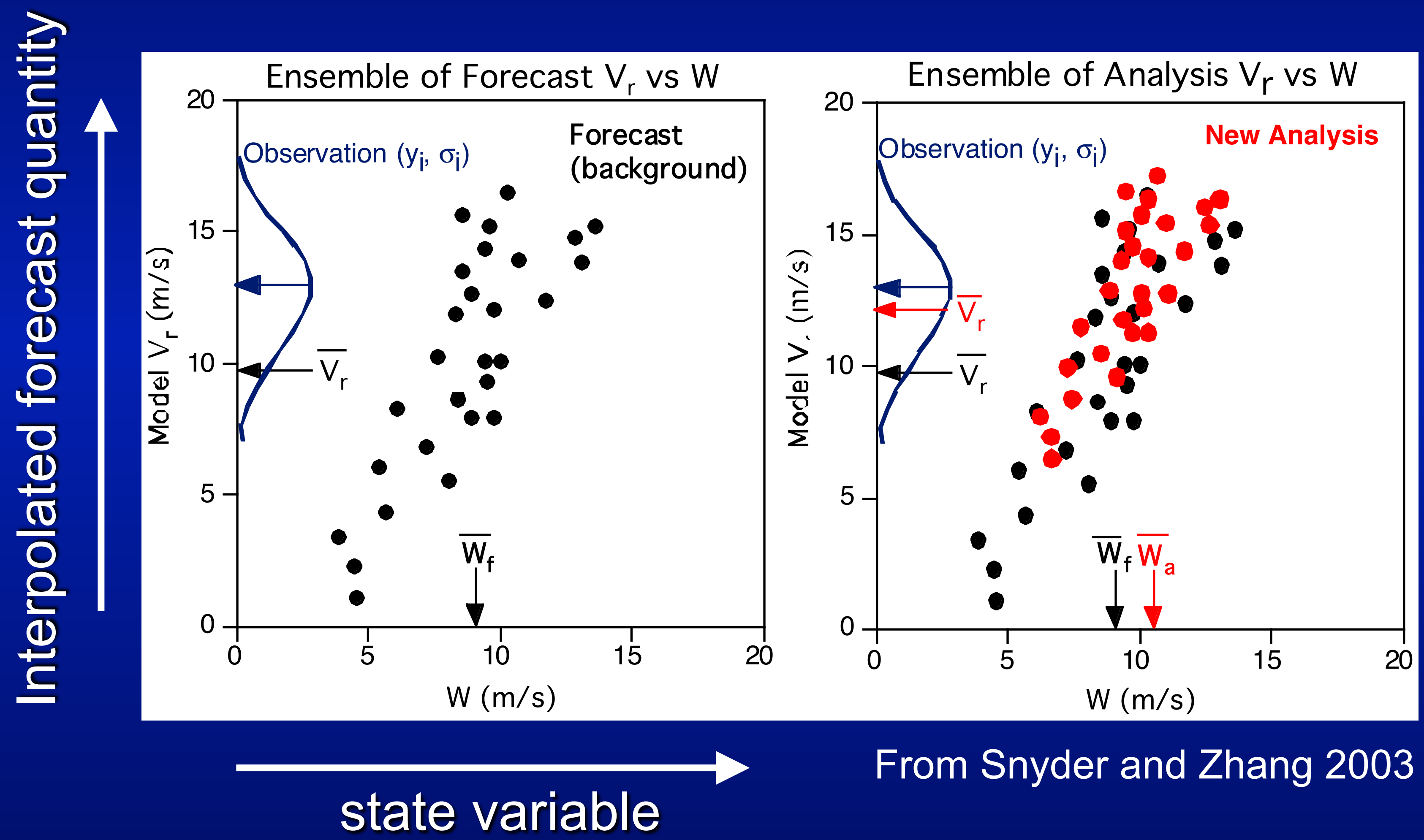
IF V_r (radar) > 0, adjust $T' > 0$

IF V_r (radar) < 0, adjust $T' < 0$

MODEL $P^f(t)$

$$x_i^a(t) = x_i^f(t) + K(t)\left[y_k(t) - H\left[x_i^f(t)\right]\right]$$

How the EnKF Method "Works"



From Snyder and Zhang 2003

Ensemble Kalman Filter (EnKF) Data-Assimilation Method

- Implementation is straightforward.
  - The data-assimilation code is typically easier to develop than the forecast model.
  - Serial (one-at-a-time) observation processing is possible if observation errors are uncorrelated.
- First tests were for large-scale flows; tests for deep, moist convection are potentially more challenging.
  - Lack of geostrophic balance
  - Importance of precipitation microphysical processes
  - Non-Gaussian forecast errors

Storm-Scale EnKF

- Snyder and Zhang 2003, *Monthly Weather Review*
- Focus on radar observations (only volumetric observations on ~1 km scale)
  - Doppler velocity
  - Reflectivity (*)
- Emphasis on retrieval of unobserved fields
  - Vertical velocity, temperature, cloud water, etc.

Snyder and Zhang's Synthetic-Data Experiments

- Produce a reference simulation ("truth").
  - Splitting supercell in Sun and Crook (1997) model
  - Grid spacing: 2 km horizontal, 0.5 km vertical

- Extract observations (velocity component with respect to "radar") from the reference simulation.
  - Add random errors with 1 $m\ s^{-1}$ standard deviation.
  - Extract observations only where there is precipitation.

- Initialize and advance an ensemble, assimilating observations when available.
  - Ensemble Square-Root Filter (EnSRF) data-assimilation scheme (Whitaker and Hamill 2002)

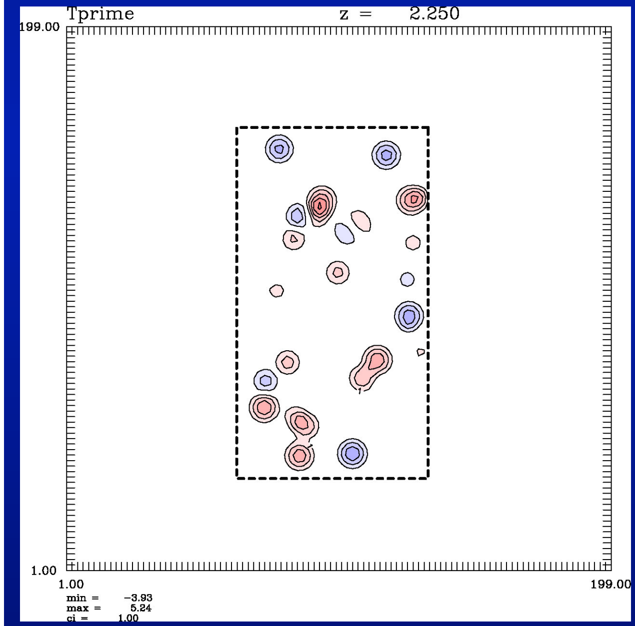
Snyder and Zhang's Synthetic-Data Experiments

- 50 ensemble members
  - Base state plus random noise
  - 20 min integration before first observation is assimilated
- Assimilation of synthetic radar data every 5 min
- Localization of filter update
  - Sphere of radius 4 km around each observation
- "Perfect model" experiments
  - Exact environmental state estimate
  - Exact model resolution and physics

Ensemble Initialization

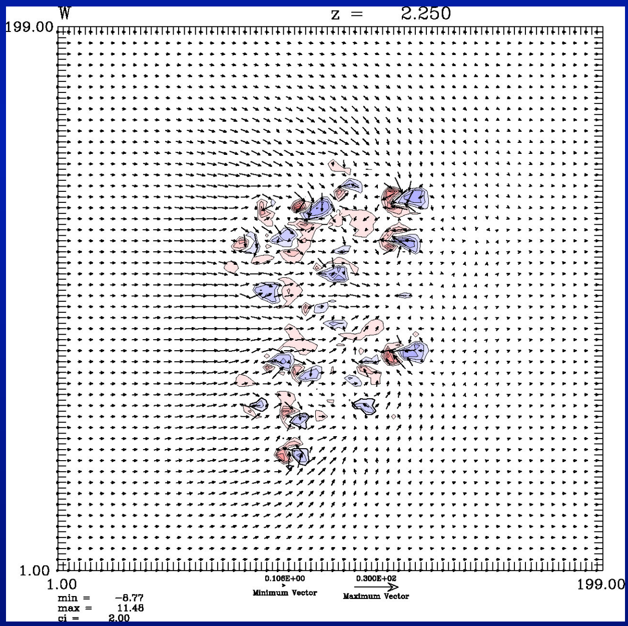
- Each ensemble member is initialized with low-level temperature perturbations in random locations, then integrated 30 min before the first data assimilation.

Ensemble Member



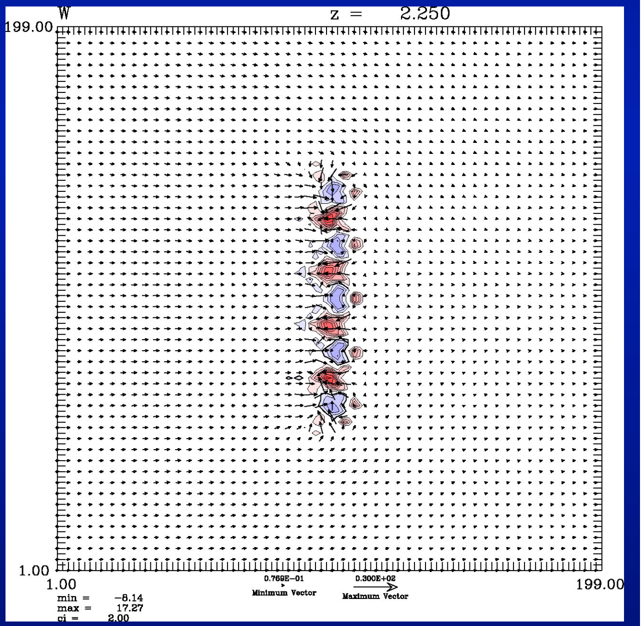
Perturbation temperature
2.25 km AGL
at initialization time

Ensemble Member



Velocity
2.25 km AGL
at first ob. time

Truth

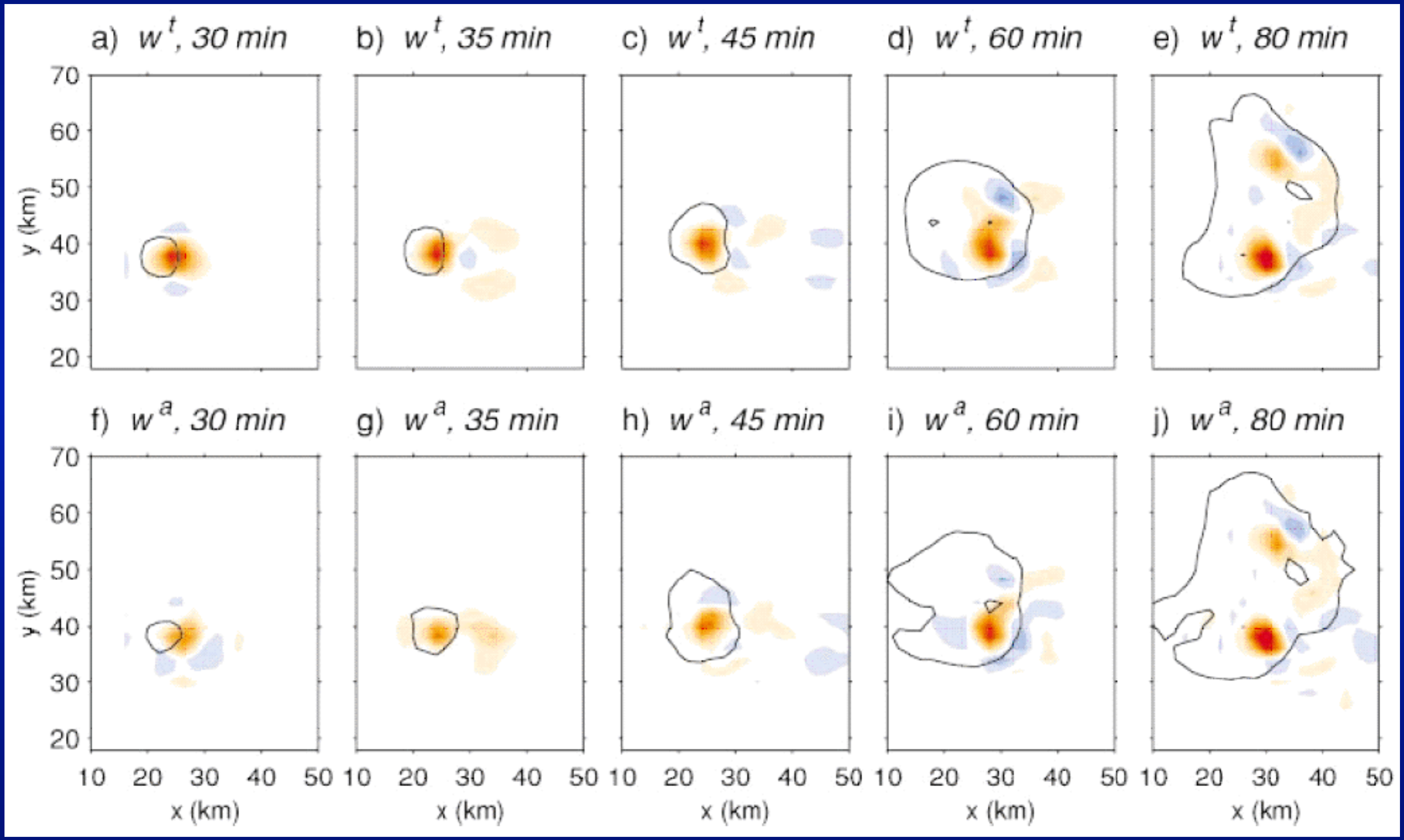


Velocity
2.25 km AGL
at first ob. time

Vertical Velocity (6 km AGL) in Snyder and Zhang's Experiments

truth

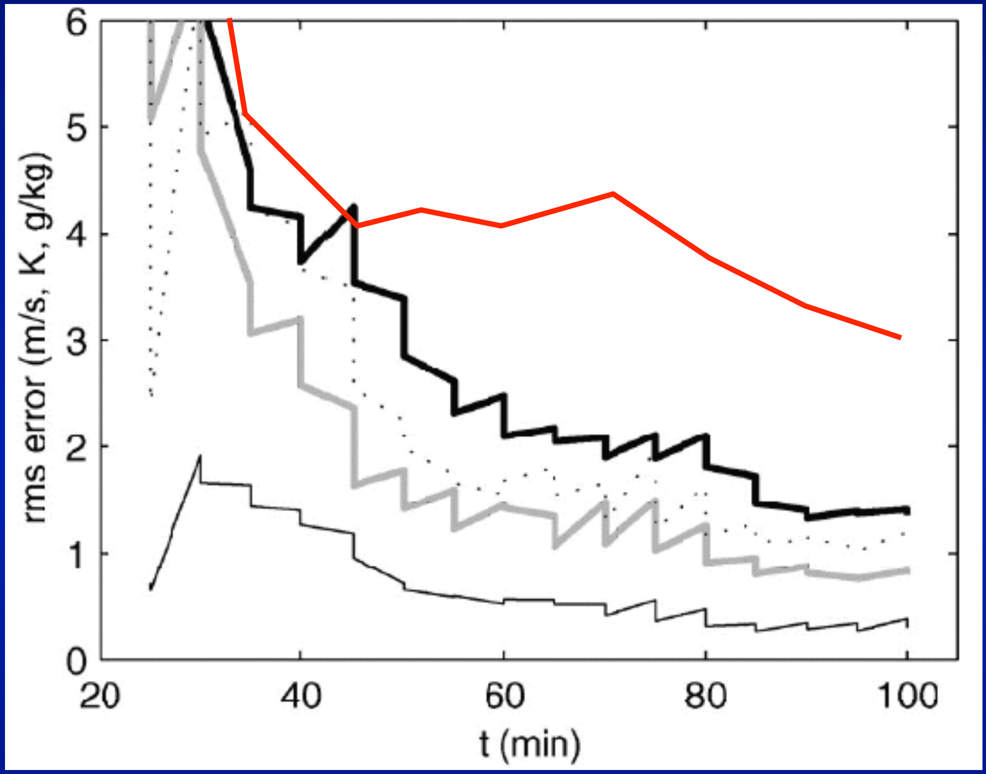
EnKF analysis



3 volumes assimilated

13 volumes assimilated

RMS Errors of Ensemble Mean in Snyder and Zhang's Experiments



- Gray, dotted, thick black, and thin black lines indicate errors in w, q_l, v_h, and q_r respectively.
- The red line indicates errors in *w* for an assimilation experiment in which the filter was used to update *only velocity* in the model.

Conclusions: Snyder and Zhang 2003

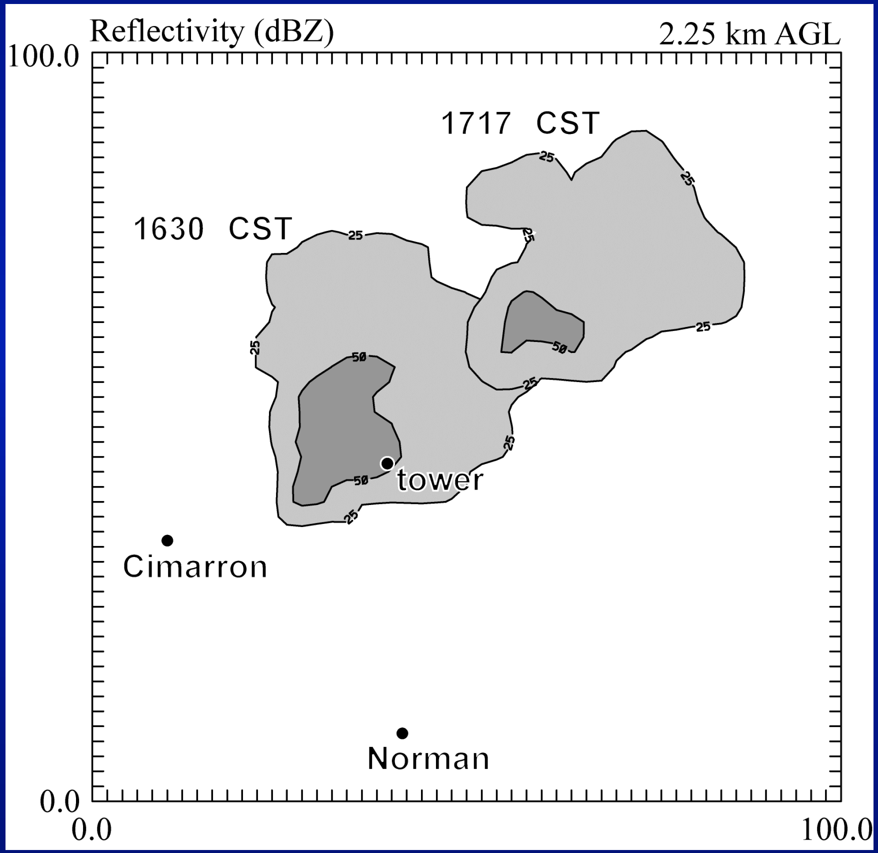
- The model state in the reference simulation was reproduced well after several Doppler volumes were assimilated (over approximately 30 min).
- Ensemble covariances between observed quantities and unobserved fields provided useful information.
- Results were sensitive to initialization.

- Suggested topics for further research:
  - Performance relative to other retrieval methods
  - Other observation types (e.g., reflectivity)
  - Uncertainty in environmental state
  - Errors in parameterizations of moist processes

Real-Data Experiments

- Dowell et al., *Monthly Weather Review*, summer 2004

17 May 1981 Arcadia, Oklahoma supercell



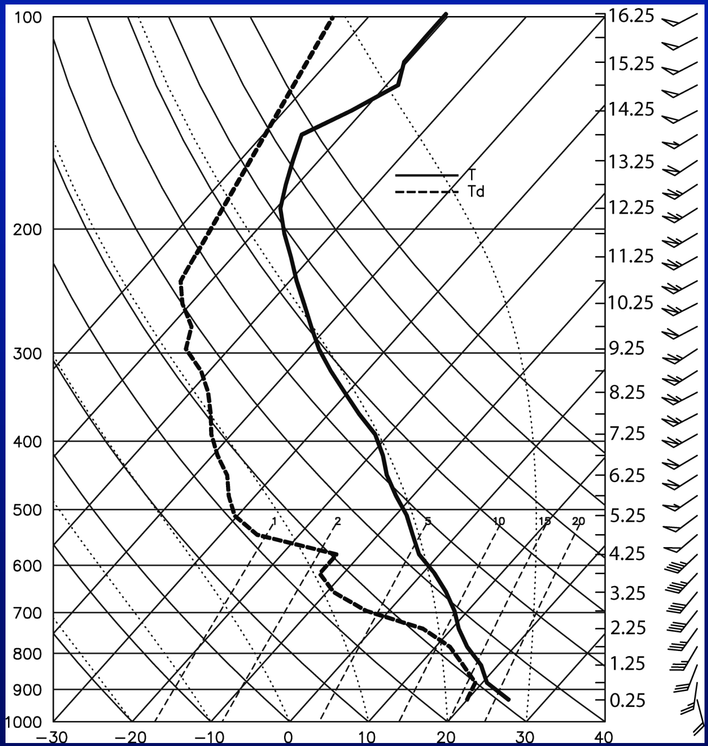
Cimarron animation (lowest tilt)

Radar-Data Characteristics

- One of the few good dual-doppler radar data sets of a tornadic supercell available.
- Observations from both radars for 1 hour during mature storm phase.
- Volumes typically every 4-5 min.
- 12-15 sweeps per volume.
- Removal of noisy data, unfolding of aliased velocities, and objective analysis before assimilation.

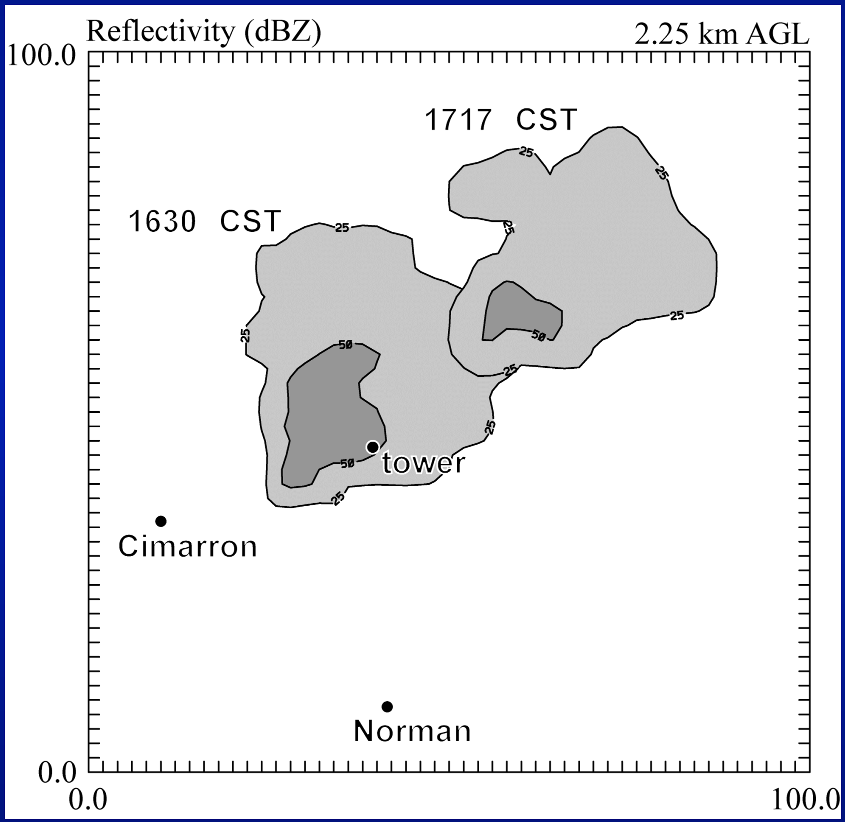
Forecast-Model Characteristics

- Anelastic model (Sun and Crook 1997, 1998)
- Warm-rain microphysical scheme
- Domain: 100 km x 100 km x 17 km
- Grid spacing: 2 km horizontal, 500 m vertical
- Base state: 3400 J kg^{-1} CAPE, 23 m s^{-1} change in wind from 250 to 5250 m AGL



EnKF Assimilation Experiments: Real Data

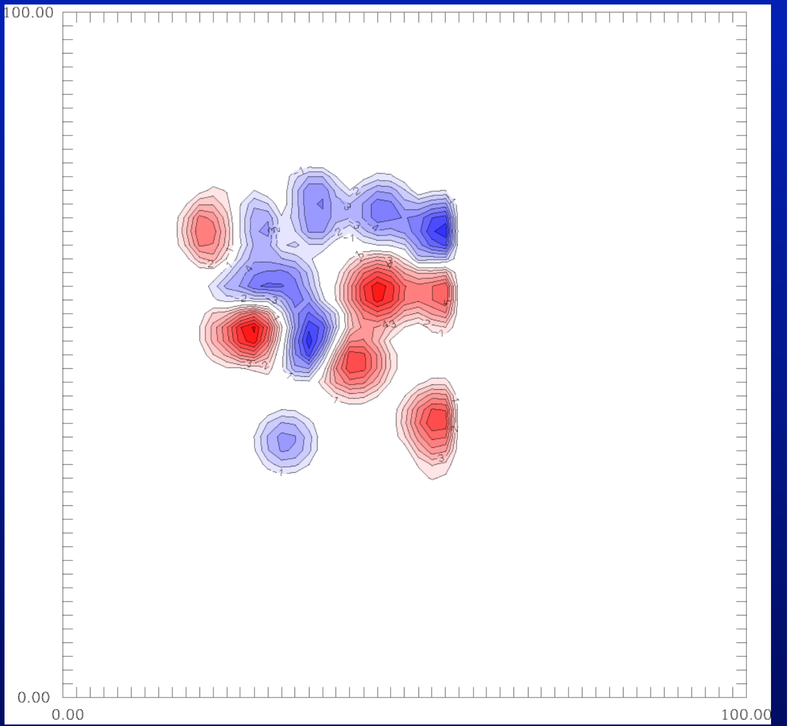
- Assimilated data
  - Cimarron radar observations
- ***Data not assimilated:***
  - Norman radar observations
  - WKY 440 m tall instrumented tower data
  - *Used for independent validation*



EnKF Assimilation Method: Real Data

The assimilation method is similar to that employed by Snyder and Zhang for their synthetic-data experiments, except:

- Ensemble initialized with “blob” perturbations
- Reflectivity data in precipitation core used to update model q_r
  - Truncation at 55 dBZ

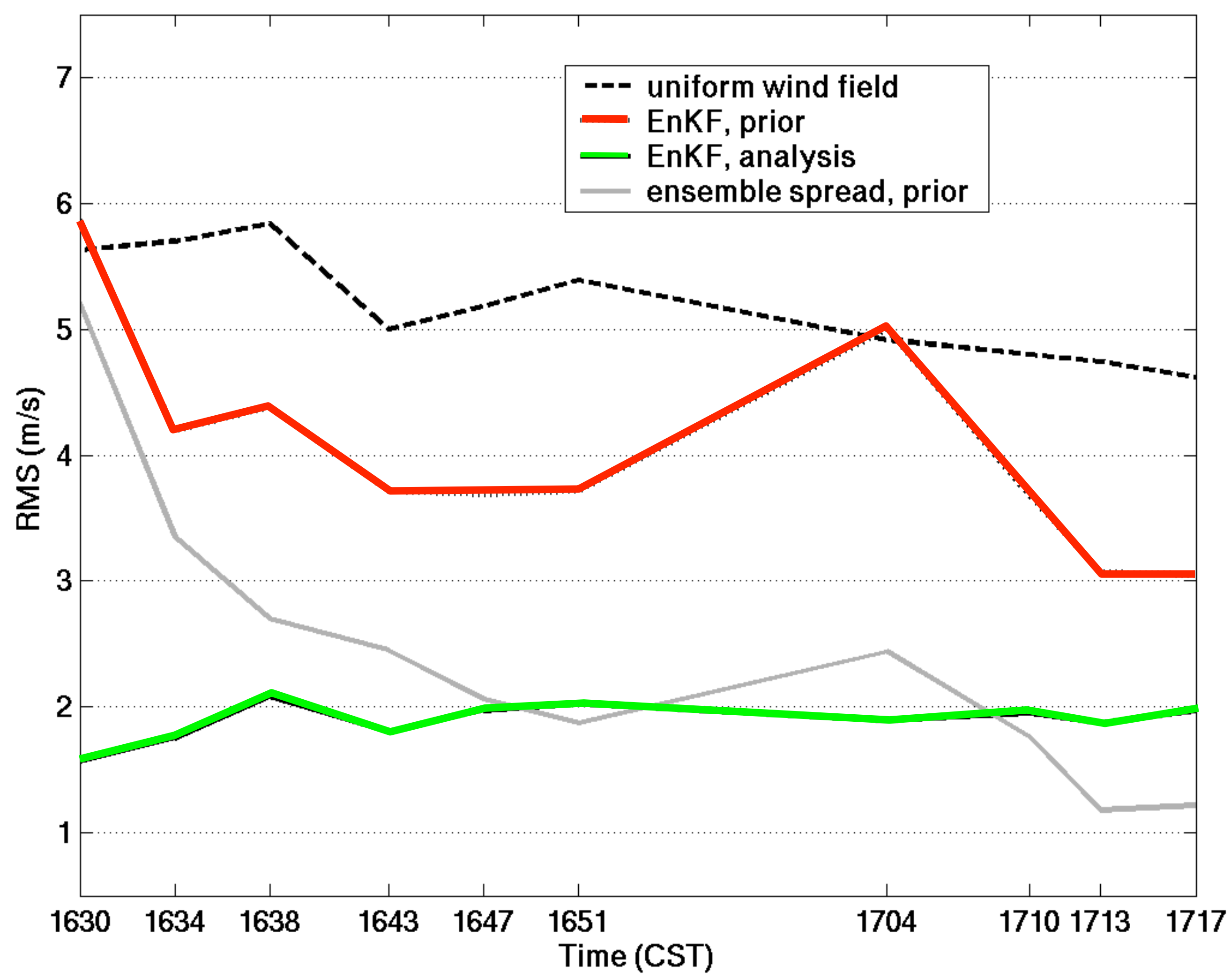


- Horizontally homogeneous environment
- No systematic attempt to correct model biases

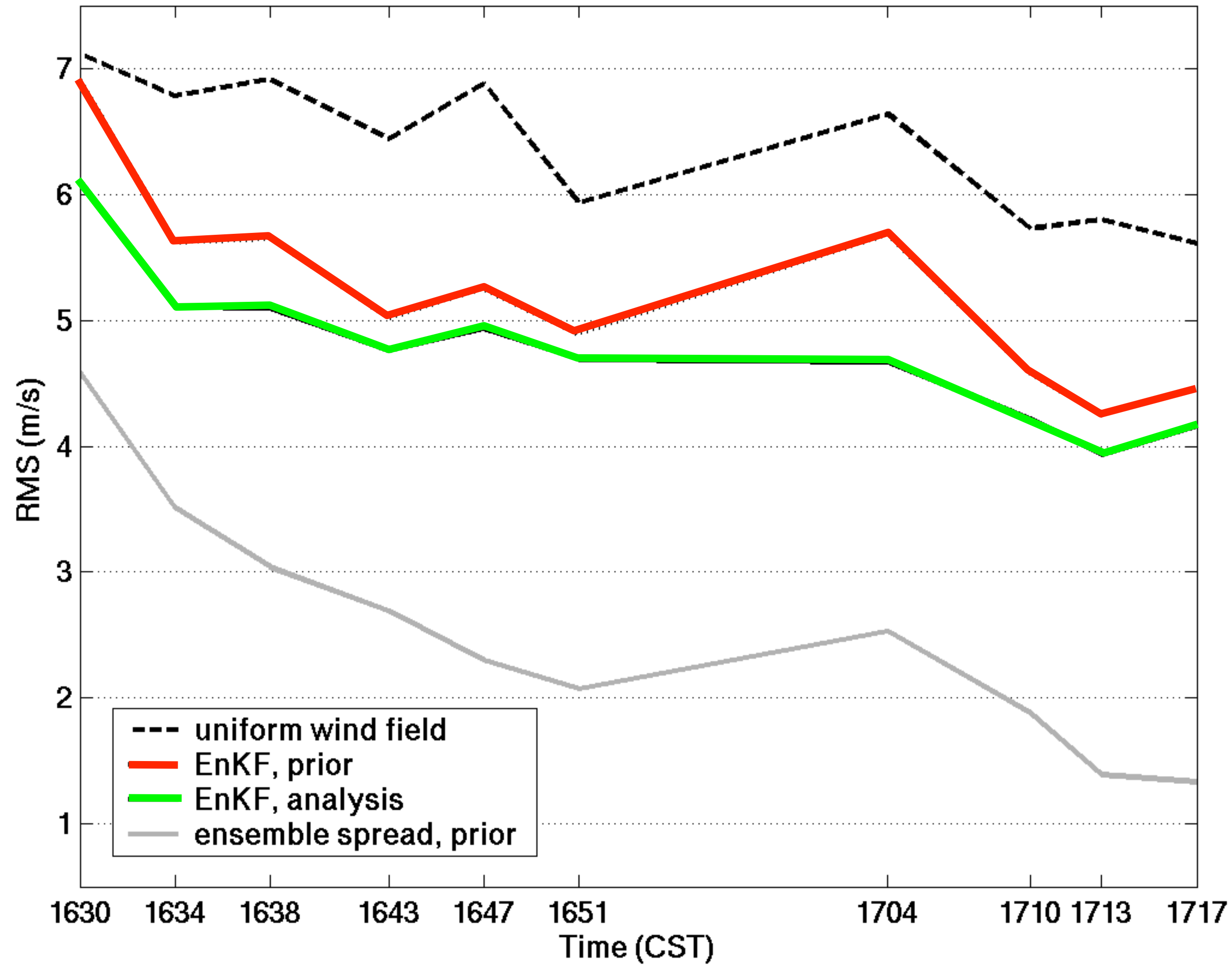
Verification of Real-Data Experiments

- Large errors in data used for "verification"
  - Cimarron and Norman Doppler observations: typical errors 2 to 4 m s^{-1} (?)
  - Crossbeam horizontal wind component in dual-Doppler analysis: 50 to 100% errors
  - Vertical velocity in dual-Doppler analysis: even larger errors
- Interest in isolated phenomena
  - RMS differences between EnKF and dual-Doppler analyses in updraft region (not downstream anvil)
  - Subjective analyses of updraft, downdraft, and mesocyclone characteristics

RMS Differences between Cimarron Doppler Observations (which were assimilated) and EnKF Radial Winds

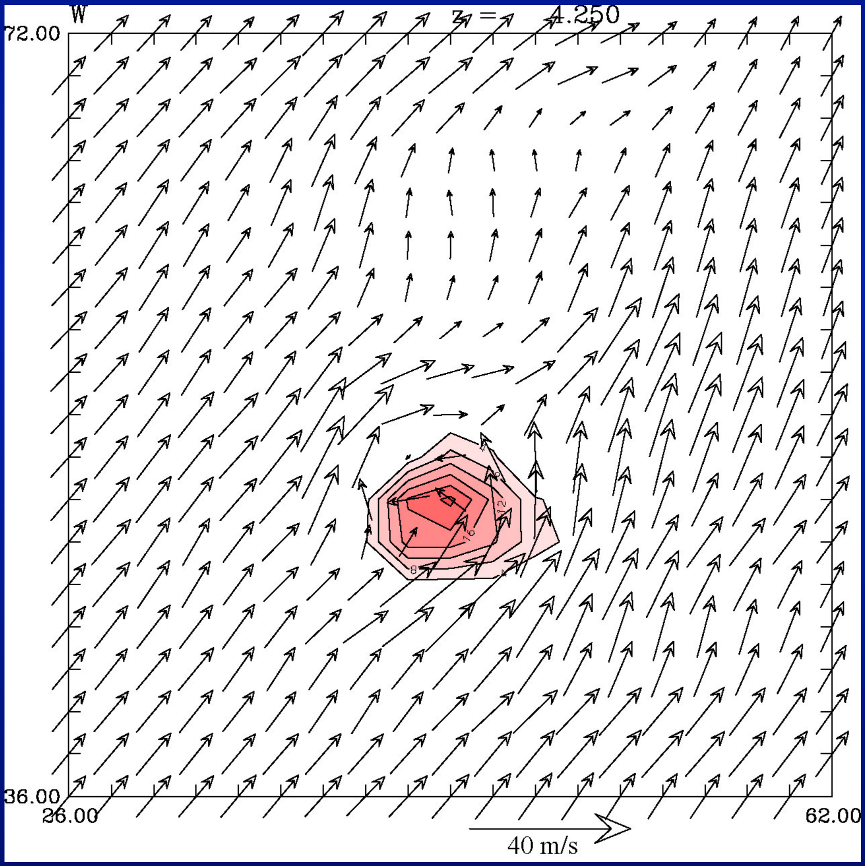


RMS Differences between Norman Doppler Observations (which were not assimilated) and EnKF Radial Winds

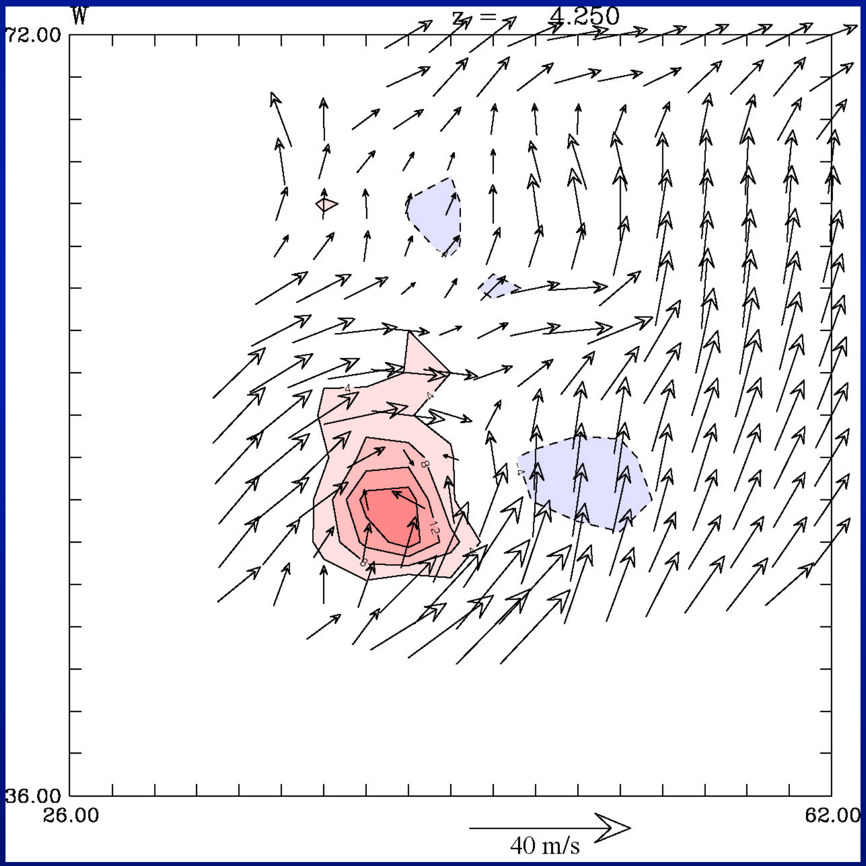


Vertical Velocity and Horiz. Wind at 4.25 km AGL

EnKF analysis
5 Cimarron volumes assimilated

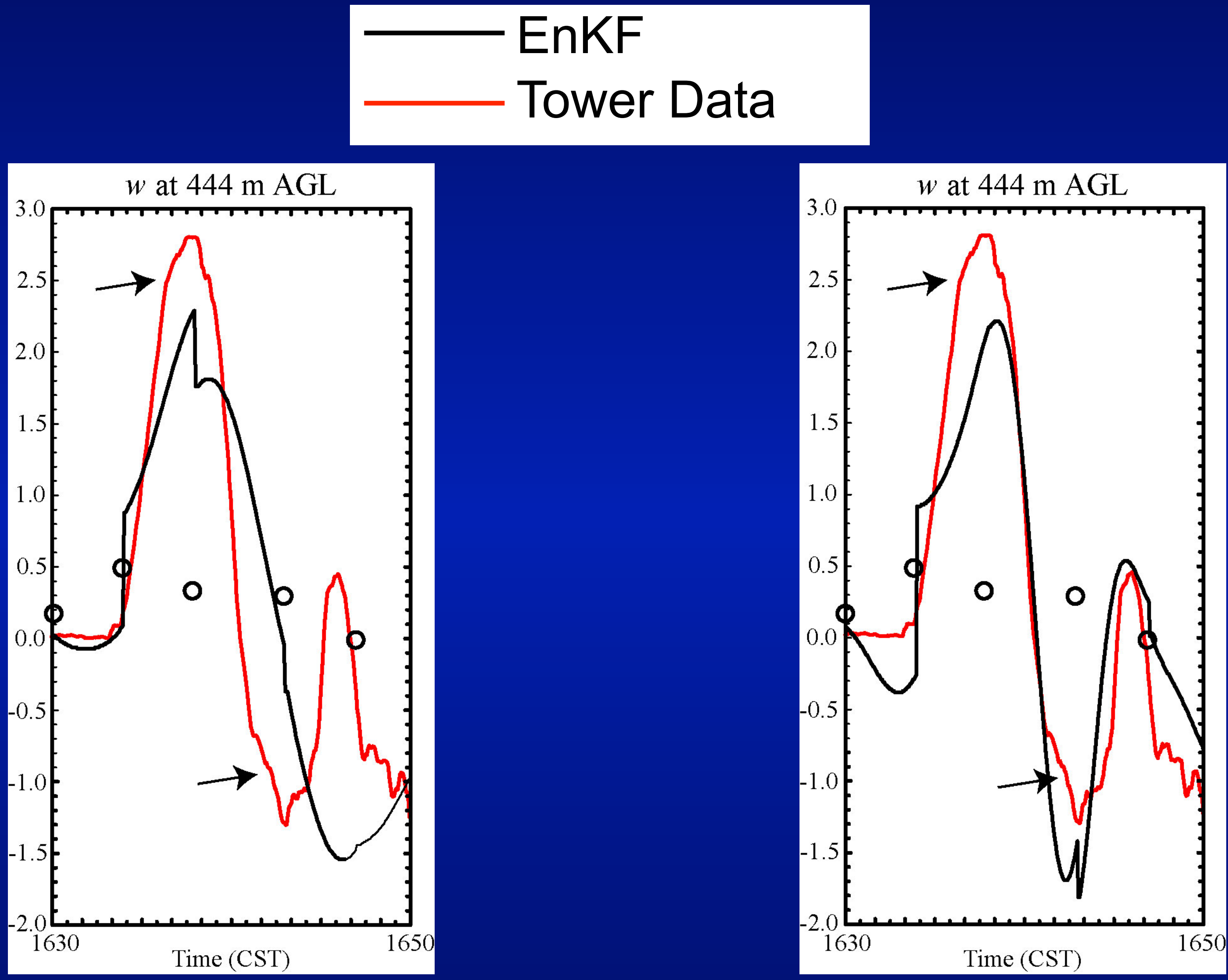


Dual-Doppler analysis



Contour interval: 4.0 m s^{-1}

Vertical Velocity (m s^{-1}) at Tower Location (444 m AGL)

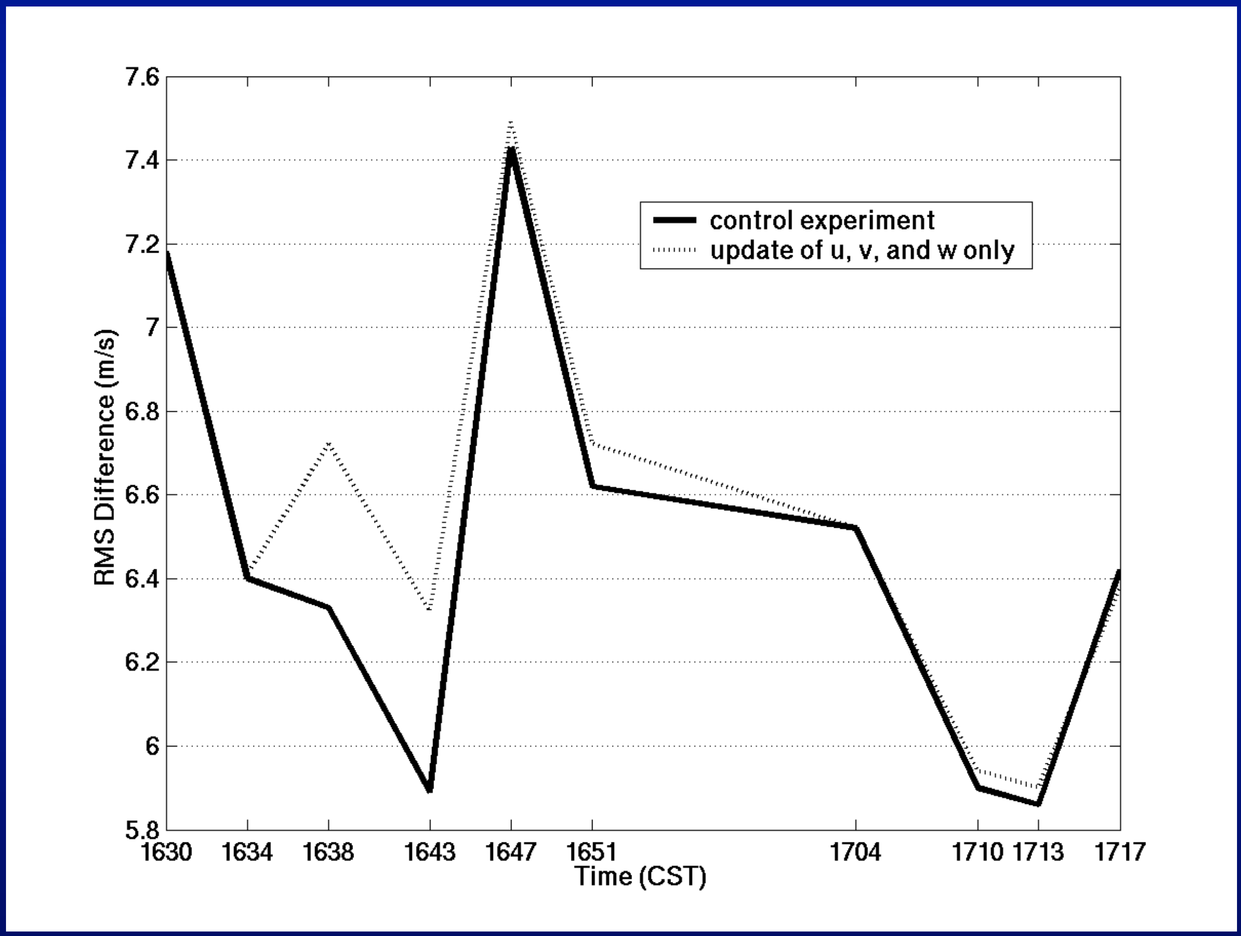


Control Assim. Experiment

Higher-Resolution Assim. Experiment (D*x* = D*y* = 1 km)

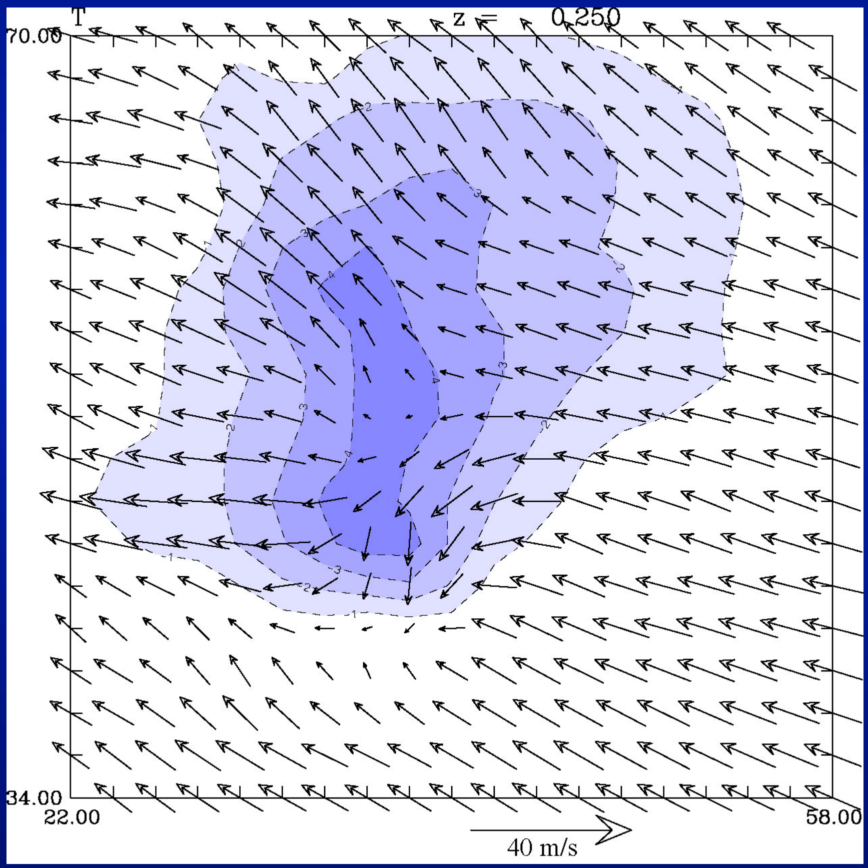
RMS Differences between Crossbeam Horiz. Wind Components in EnKF and Dual-Doppler Analyses

- How valuable are the covariances between radial velocity and unobserved fields (q_l, q_r, q_t)?

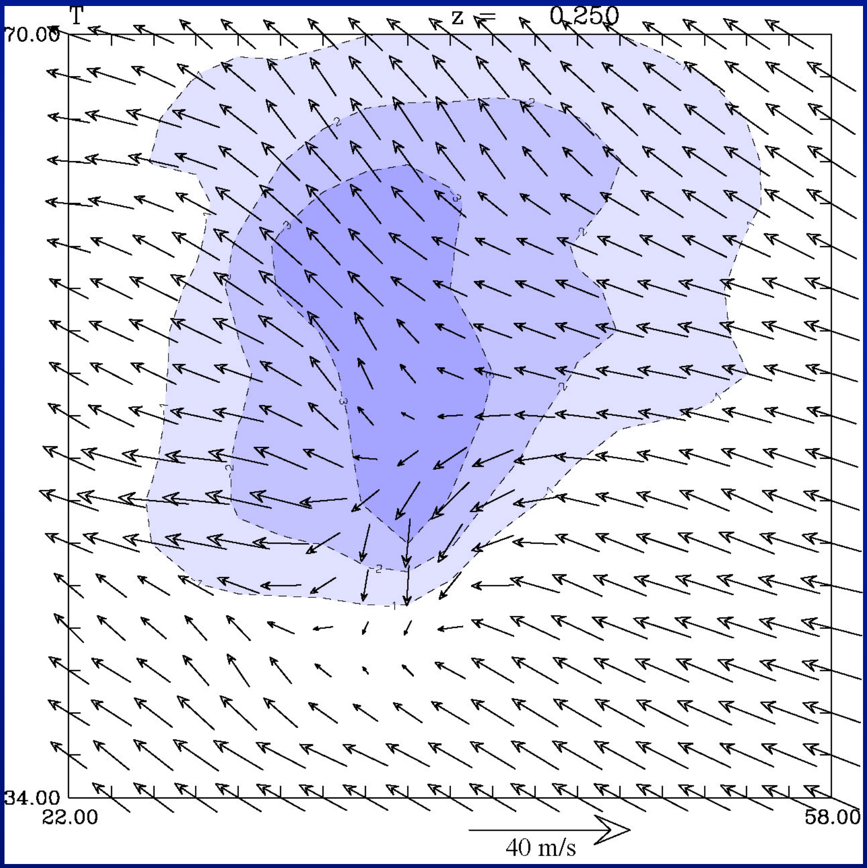


Perturbation Temp. (K) and Horiz. Wind at 0.25 km AGL (3 Cimarron volumes assimilated)

Control assimilation experiment (all model fields updated)

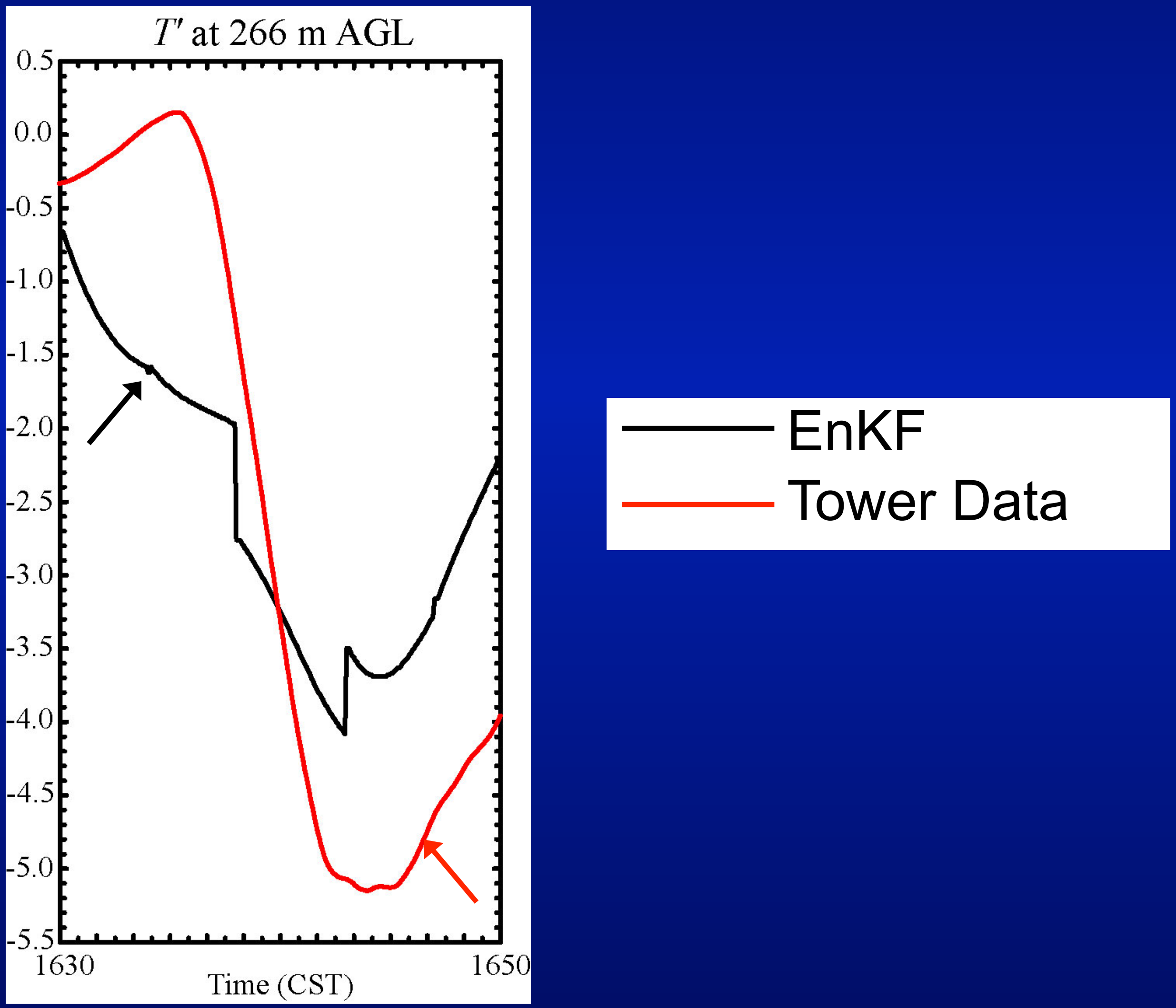


Only *u*, *v*, and *w* updated with filter



Contour interval: 1.0 K

Perturbation Temperature (K) at 266 m AGL: Control Assimilation Experiment vs. Tower Data



Conclusions: Real-Data Experiments

- Verification of assimilation results is difficult; nevertheless, the retrieved velocity fields are somewhat encouraging.
  - Agreement between EnKF and dual-Doppler analyses about locations and strengths of main updraft and mesocyclone
  - Similar low-level vertical-velocity traces in the assimilation results and tower data

- Analyses of the cold pool highlight challenges in retrieving temperature at low levels.
  - Observational limitations: lowest scans at approximately 400 m AGL for a storm that is 30 km from the radar
  - Model uncertainty: precipitation microphysical parameterizations