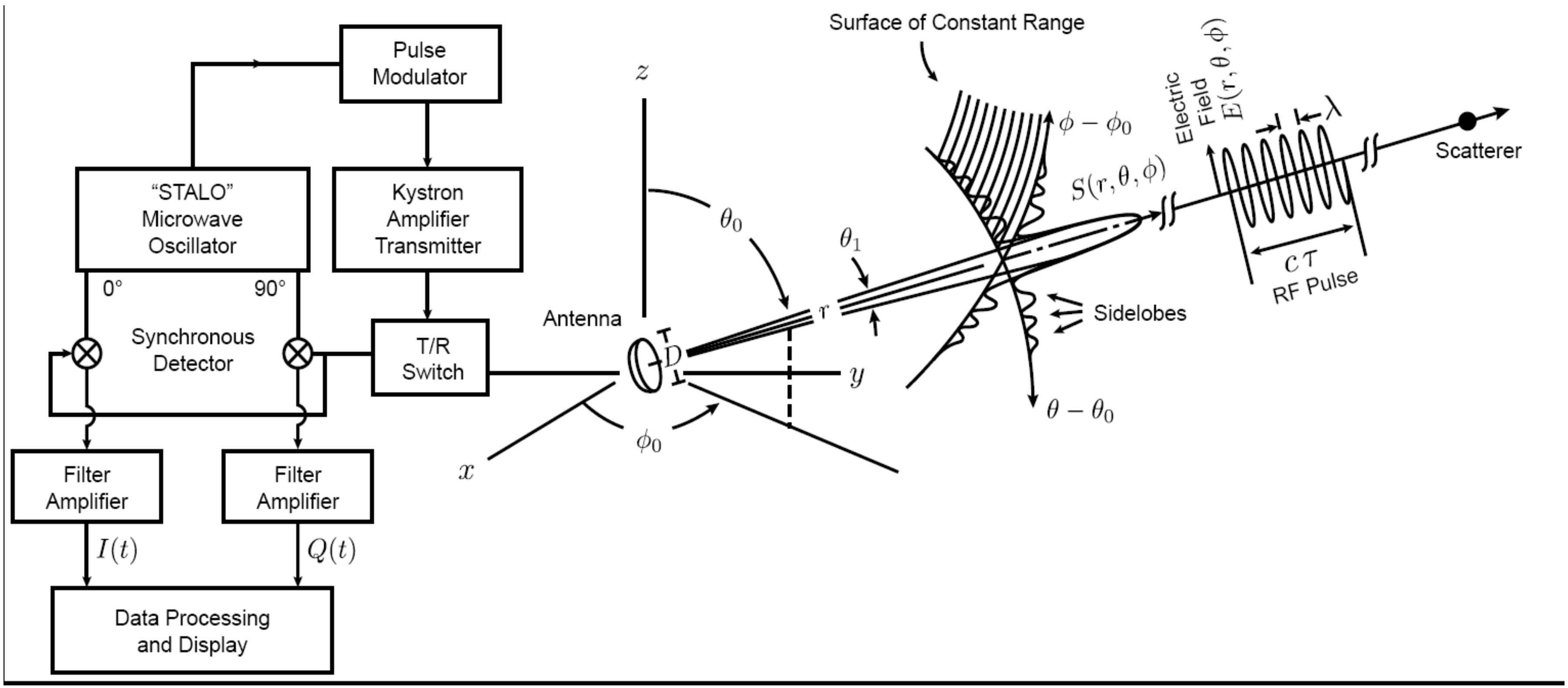
Review

Doppler Radar (Fig. 3.1)

A simplified block diagram



$$E_i = \frac{A_i(\theta,\varphi)}{r}\exp\left[j2\pi f\left(t-\frac{r}{c}\right)+j\psi_t\right]$$

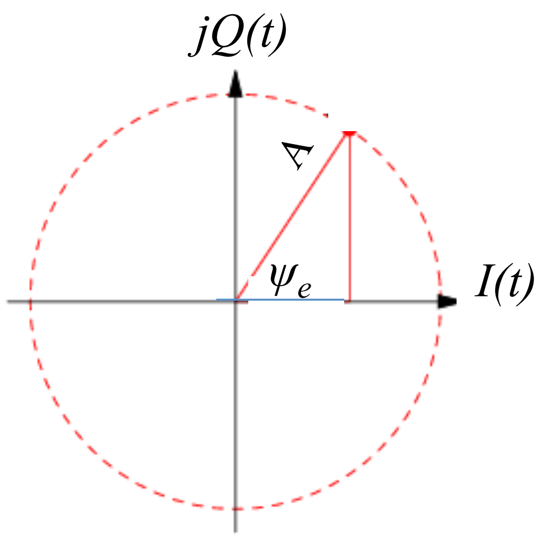
Electric field incident on scatterer

$$\boldsymbol{E}_r = \frac{\boldsymbol{A}_r(\theta,\varphi)}{r^2}\exp\left[j2\pi f\left(t-\frac{2r}{c}\right)+j\psi_t\right]$$

Reflected electric field incident on antenna

$$V_i = A_i \exp\left[j2\pi ft - j\frac{4\pi r}{\lambda}+j\psi_t\right]$$

Voltage input to the synchronous detectors;
This pair of detectors shifts the frequency *f* to 0



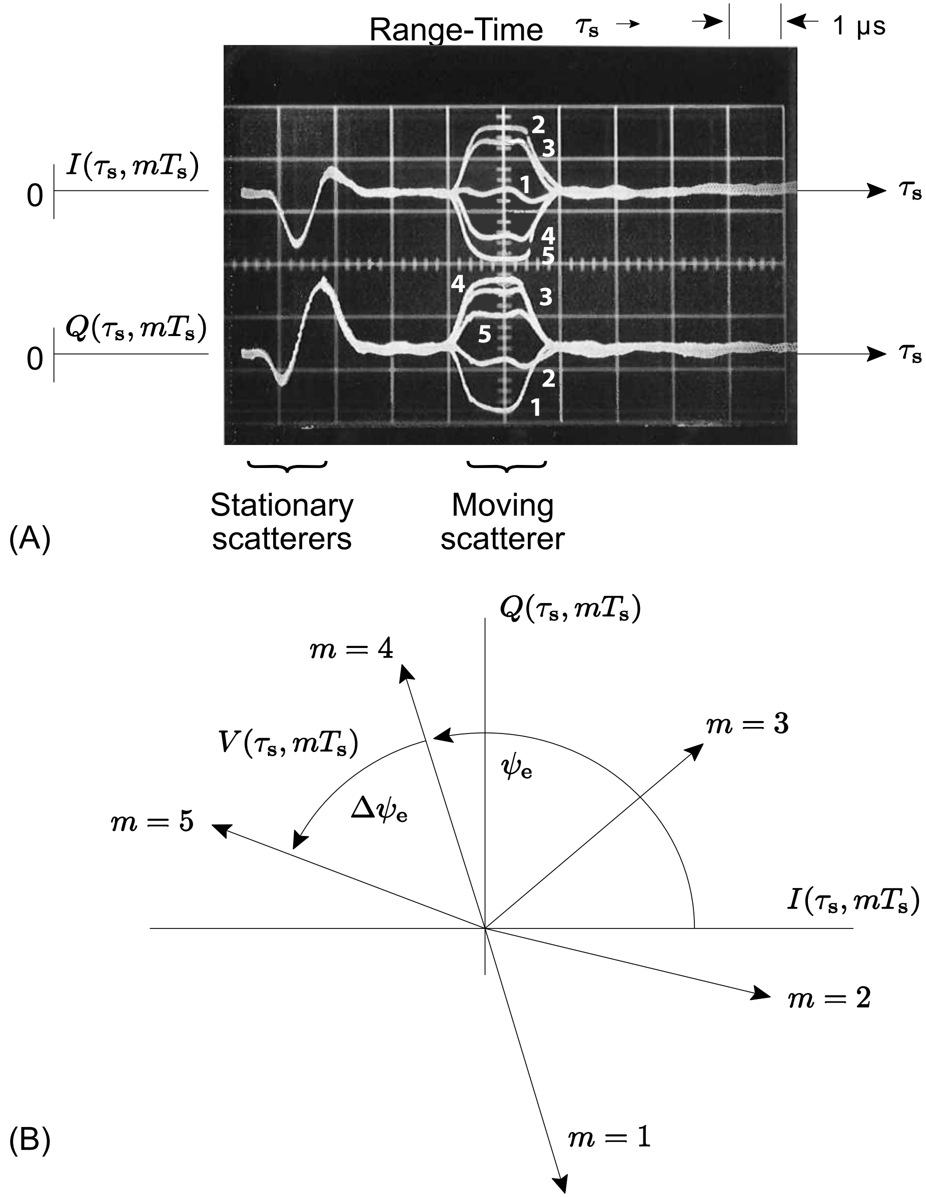
Complex plane
(Phasor diagram)

$$V_o = I + jQ \propto A_o \exp\left[-j4\pi r/\lambda + j\psi_t\right]$$

Echo phase at the output of the detectors and filters

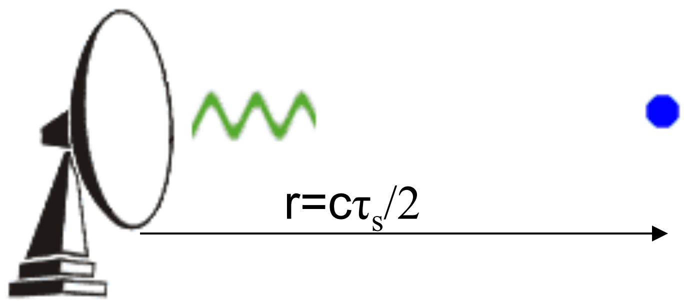
$$\psi_e = (-4\pi r/\lambda) + \psi_t$$

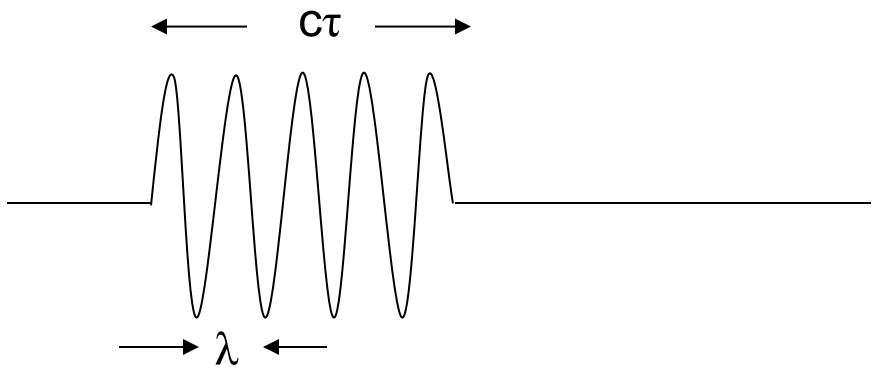
If the range *r* of the scatterer is fixed, phasor A is fixed.
But if scatterer has a radial velocity, Phasor A rotates about the origin at the Doppler frequency f_d.



Range-Time
τ_s
1 µs
$I(\tau_s, mT_s)$
0
τ_s
$Q(\tau_s, mT_s)$
0
τ_s
Stationary scatterers
Moving scatterer
(A)
$Q(\tau_s, mT_s)$
$m = 4$
$m = 3$
$V(\tau_s, mT_s)$
ψ_e
$m = 5$
$\Delta\psi_e$
$I(\tau_s, mT_s)$
$m = 2$
$m = 1$
(B)

Pulsed Radar Principle



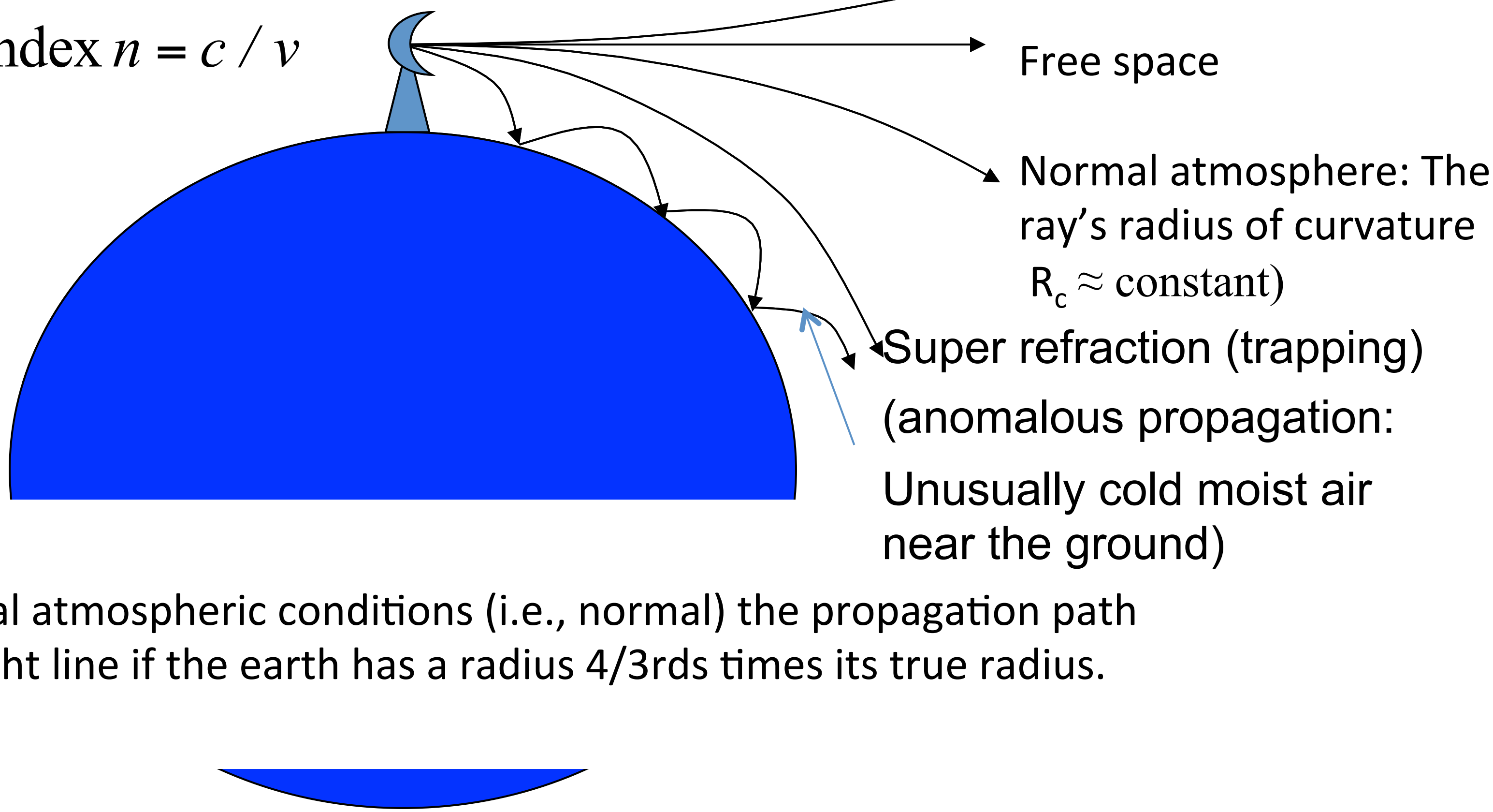


c = speed of microwaves
= c_h for H and = c_v for V waves
τ = pulse length
λ = wavelength
= λ_h for H and λ_v for V waves
τ_s = time delay between transmission of a pulse and reception of an echo.

Normal and Anomalous Propagation

$$n^2 = 1 + C_d P_d / T + C_{w1} P_w / T + C_{w2} P_w / T^2$$

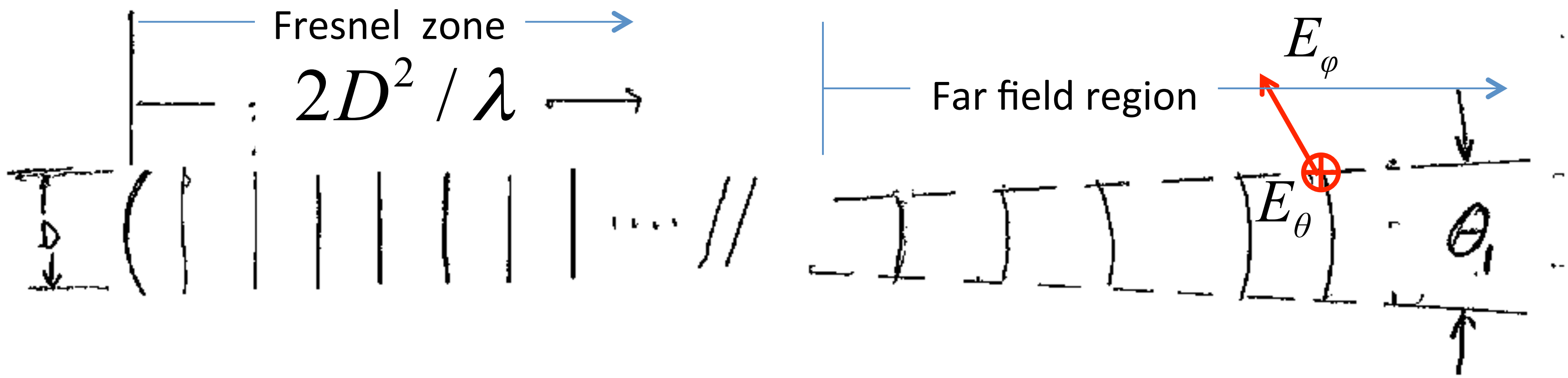
refractive index $n = c / v$



For typical atmospheric conditions (i.e., normal) the propagation path is a straight line if the earth has a radius 4/3rds times its true radius.

Angular Beam Formation

(the transition from a circular beam of constant diameter to an angular beam of constant angular width)



$$2D^2/\lambda\,;\ 1.5\ \text{km};$$

$$\text{WSR-88D: } D\,;\ 8.53\ \text{m}; \lambda=10\ \text{cm}$$

$$\theta_1 = 1.27\ \lambda/\text{D (radians)}$$

Antenna (directive) Gain g_t

The defining equation:

$$S_i = \frac{P_t}{4\pi r^2} g_t f^2(\theta,\phi) \qquad \text{Eq. (3.4)}$$

S_i(W m^{-2}) = power density incident on scatterer

r = range to measurement

$f^2(\theta,\phi)$= radiation pattern (= 1 on beam axis)

P_t = transmitted power (W)

Wavenumbers for H, V Waves

Horizontal polarization: $k_{\mathrm{h}} = (k + k'_{\mathrm{h}}) = 2\pi/\lambda_{\mathrm{h}}$

Vertical polarization: $k_{\mathrm{v}} = (k + k'_{\mathrm{v}}) = 2\pi/\lambda_{\mathrm{v}}$

where k = free space wavenumber = 3.6×10^6 (deg./km)

for $\lambda = 10$ cm

(e.g., for R= 100 mm h^{-1}, k'_{h} = 24.4°km^{-1}, k'_{v} = 20.7° km^{-1})

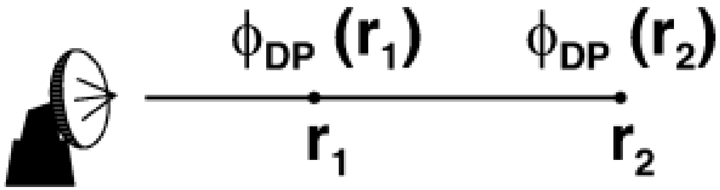
Therefore: $c_h < c_v$; $\lambda_h < \lambda_v$; $k_h > k_v$

Specific differential phase:

$K_{\mathrm{DP}} = k'_{\mathrm{h}} - k'_{\mathrm{v}} = 3.7(\mathrm{deg.\ km}^{-1})$ (for R = 100 mm h^{-1})

(K_{DP}: an important polarimetric variable related to rainrate)

Specific Differential Phase



(Fig.6.17)

DIFFERENTIAL PHASE SHIFT ϕ_{DP} (two-way) = Phase Lag of H – Phase Lag of V

Echo phase of H = $\phi_h = 2k_h r$

SPECIFIC DIFFERENTIAL PHASE K_{DP} (one-way!)

$$K_{DP} = \frac{d\phi_{DP}}{2\,dr} \approx \frac{\phi_{DP}(r_2) - \phi_{DP}(r_1)}{2\,(r_2 - r_1)} \qquad \text{Eq. (6.60)}$$

Backscattering Cross Section, σ_b for a Spherical Particle

Rayleigh condition on a spherical particle of diameter D:

$$D < \lambda / 16; \qquad \lambda \equiv \text{ wavelength}$$

$$\boldsymbol{\sigma_b = \frac{\pi^5}{\lambda^4} | K_m |^2 D^6};$$

$K_m \equiv \dfrac{m^2 - 1}{m^2 + 2}$; Dielectric Factor of the medium filling the sphere; Eq.3.6

$m = n - jn\kappa$ = the complex index of refraction

$|K_m|^2 \rightarrow |K_w|^2 = 0.90 \Rightarrow 0.93$ for water, and

$|K_m|^2 \rightarrow |K_i|^2 = 0.18$ for ice (density = 0.917 g m^{-3})

Backscattered Power Density Incident on Receiving Antenna

$$S_r(r,\theta,\phi) = \frac{\overbrace{P_t g_t f^2(\theta,\phi)}^{S_i}}{4\pi r^2 l} \bullet \sigma_b \bullet \frac{1}{4\pi r^2 l} \qquad (3.13a)$$

where l is the loss factor (due to attenuation)

$$l = \exp\left(\int_0^r (k_g + k)\,dr\right) \qquad (3.13b)$$

Echo Power P_r Received

$$P_r = S_r(r,\theta,\phi)A_e(\theta,\phi) \quad (3.20)$$

A_e is the effective area of the receiving antenna for radiation from the θ,φ direction. It is shown that:

$$A_e = g_r f_r^2(\theta,\phi)\lambda^2 / 4\pi \qquad (3.21)$$

If the transmitting antenna is the same as the receiving antenna then:

$$g_r f_r^2(\theta,\phi) = g_t f_t^2(\theta,\phi) \equiv g f^2(\theta,\phi)$$

The Radar Equation
(point scatterer/discrete object)

$$P_r = \frac{P_t g f^2(\theta,\varphi)}{4\pi r^2 l} \bullet \frac{\sigma_b}{4\pi r^2 l} \bullet \frac{g\lambda^2 f^2(\theta,\varphi)}{4\pi} \quad (3.24)$$

Example:

$\lambda = 0.1\,\mathrm{m}; \quad r = 20\,\mathrm{km}\,(2\mathrm{x}10^4\mathrm{m}); \; P_r(\mathrm{min}) = 10^{-14}(\mathrm{W});$

$P_t = 10^6(\mathrm{W;\ peak}); \quad g = 3x10^4; \; l = 1(\text{no path loss})$

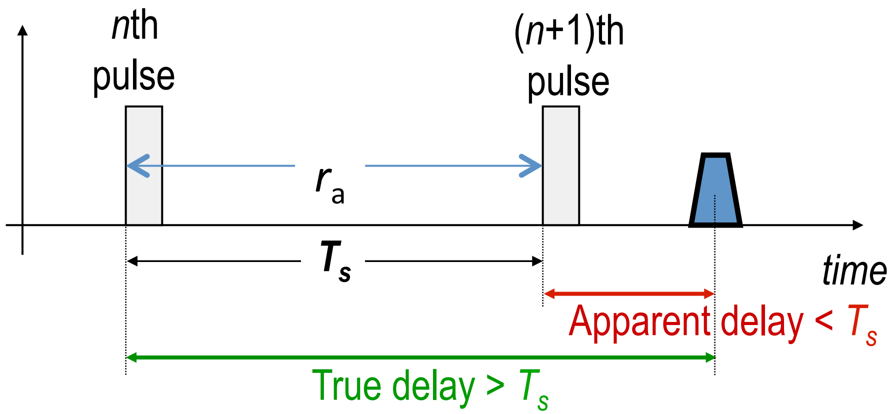
Calculating the minimum detectable backscattering σ_b :

$\sigma_b(\mathrm{min}) = 2x10^{-7}\ \mathrm{m}^2 = \sigma_b$ for a 6.3 mm drop!

Unambiguous Range r_a

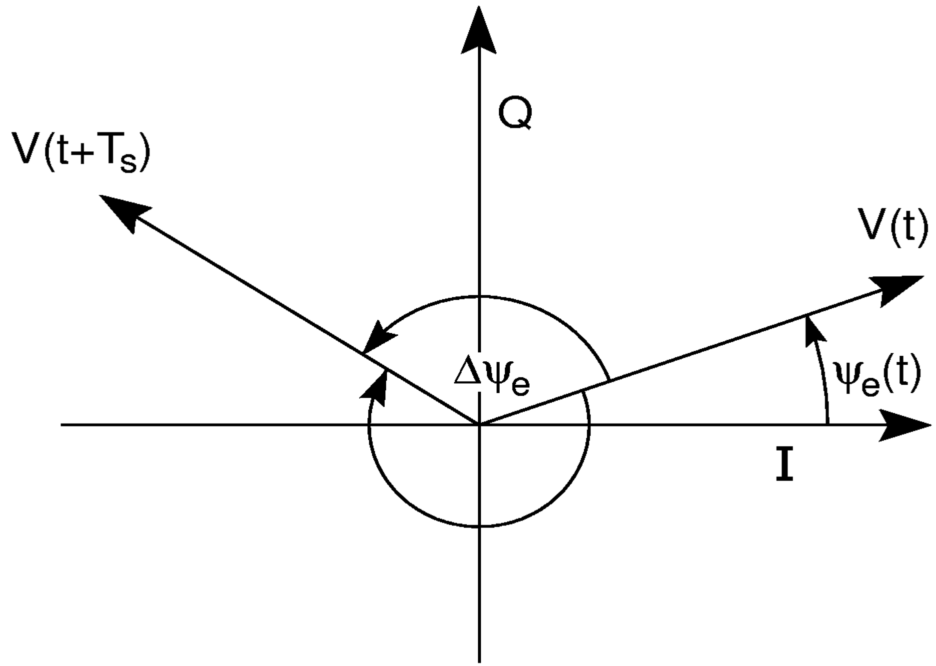
- If targets are located beyond $r_a = cT_s/2$, their echoes from the n^{th} transmitted pulse are received after the $(n+1)^{th}$ pulse is transmitted. Thus, they appear to be closer to the radar than they really are!
  - This is known as **range folding**

- T_s = PRT



- **Unambiguous range**: $r_a = cT_s/2$
  - Echoes from scatterers between 0 and r_a are called **1st** trip echoes,
  - Echoes from scatterers between r_a and $2r_a$ are called **2nd trip** echoes, Echoes from scatterers between $2r_a$ and $3r_a$ are called **3rd trip** echoes, etc

Ambiguous Doppler Shifted Echoes (Fig. 3.14)



frequency aliases are:

$$\frac{\Delta\psi_e}{2\pi T_s} + \frac{n}{T_s} \quad \frac{\text{cycles}}{\text{sec}}$$

$n = 0, \pm 1, \pm 2, \ldots..$

Unambiguous Velocity

- A pulsed Doppler radar measures radial Doppler velocity by <u>keeping track</u> of phase changes between samples that are T_s (pulse repetition time) apart
- Recall that echo phase shift is $\psi_e = -4\pi r/\lambda$. Then, the phase change from pulse to pulse is $\Delta\psi_e = -4\pi\Delta r/\lambda = -4\pi v_r T_s/\lambda$
  - Note that only phase changes between $-\pi$ and π can be unambiguously resolved
- Therefore, the unambiguous velocity is:
  - $4\pi v_a T_s/\lambda = \pi \Rightarrow v_a = \lambda/4T_s$
  - This is related to the **Nyquist** sampling theorem:
    Doppler velocities outside the $\pm v_a$ interval will be aliased!

$\Delta r = v_r T_s$ is the change in range of the scatterer between successive transmitted pulses

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Another PRT Trade-Off

- Correlation of pairs: $\rho(T_s) = \exp\left[-8\left(\pi\sigma_v T_s / \lambda\right)^2\right]$
  - This is a measure of signal coherency
- Accurate measurement of power requires long PRTs
  - $\lim_{T_s \to \infty} \rho(T_s) = 0$
  - More **independent samples** (low coherency)
- But accurate measurement of velocity requires short PRTs
  - $\lim_{T_s \to 0} \rho(T_s) = 1$
  - High correlation between sample pairs (high coherency)
  - Yet a large number of **independent sample pairs** are required

Signal Coherency

- How large a T_s can we pick?
  - Correlation between *m = 1* pairs of echo samples is:

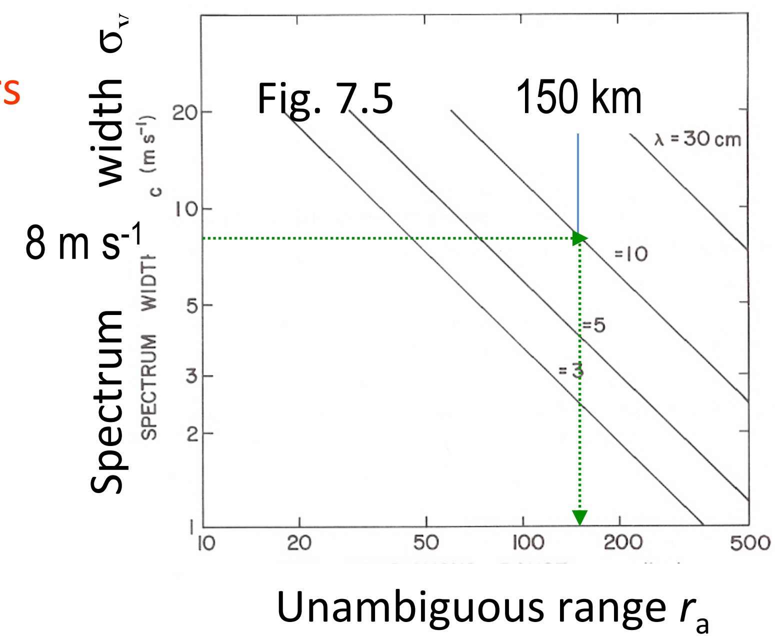
$$\rho(T_s) = \exp\left[-8(\pi\,\sigma_v T_s/\lambda)^2\right]$$

  - Correlated pairs: $\rho(T_s) \approx 1 \Rightarrow \frac{\pi\sigma_v T_s}{\lambda} << 1 \Rightarrow \frac{\lambda}{\pi T_s} >> \sigma_v$

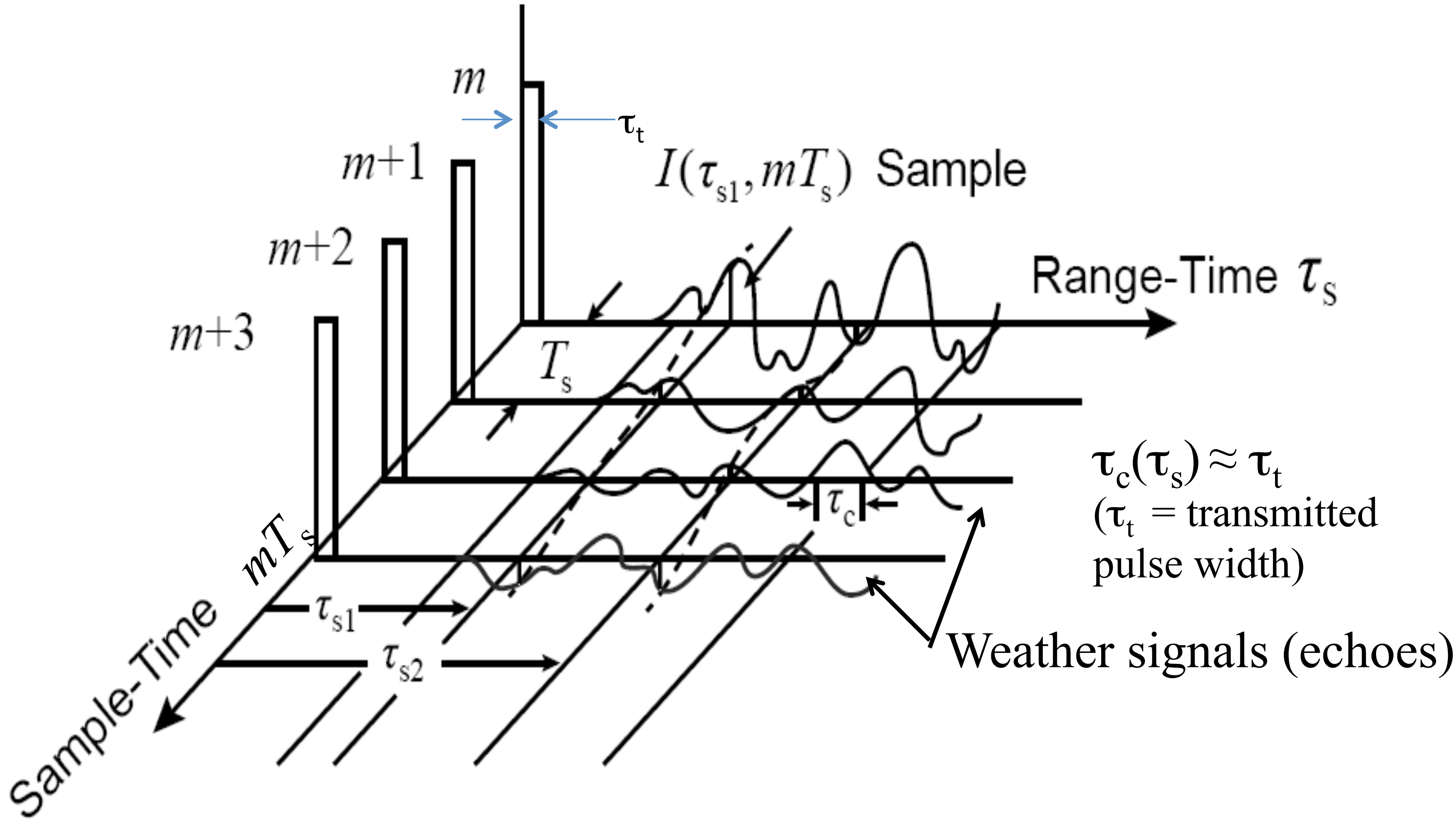
    (i.e., Spectrum width σ_v must be much smaller than unambiguous velocity $v_a = \lambda/4T_s$)
- Increasing T_s decreases correlation exponentially
  - $\text{Var}[\hat{v}]$ and $\text{Var}[\hat{\sigma}_v]$ also increases exponentially!
- Pick a threshold:
  - $\rho(T_s) \geq e^{-0.5} \Rightarrow -8\left(\pi\sigma_v T_s/\lambda\right)^2 = 0.5 \Rightarrow \sigma_v \leq v_a/\pi$
  - Violation of this condition results in very large errors of estimates!

Signal Coherency and Ambiguities

- Range and velocity dilemma: $r_a v_a = c\lambda/8$
- Signal coherency: $\sigma_v \leq v_a/\pi$
- r_a constraint: $$r_a \leq \frac{c\lambda}{8\pi\sigma_v}$$ Eq. (7.2c)
  - This is a more basic constraint on radar parameters than the first equation above
    - Then, σ_v and not v_a imposes a basic limitation on Doppler weather radars
  - Example: **Severe storms have a median σ_v ~ 4 m/s and 10% of the time σ_v > 8 m/s**. If we want accurate **Doppler** estimates 90% of the time with a 10-cm radar (λ = 10 cm); then, $r_a \leq$ 150 km. This will often result in range ambiguities

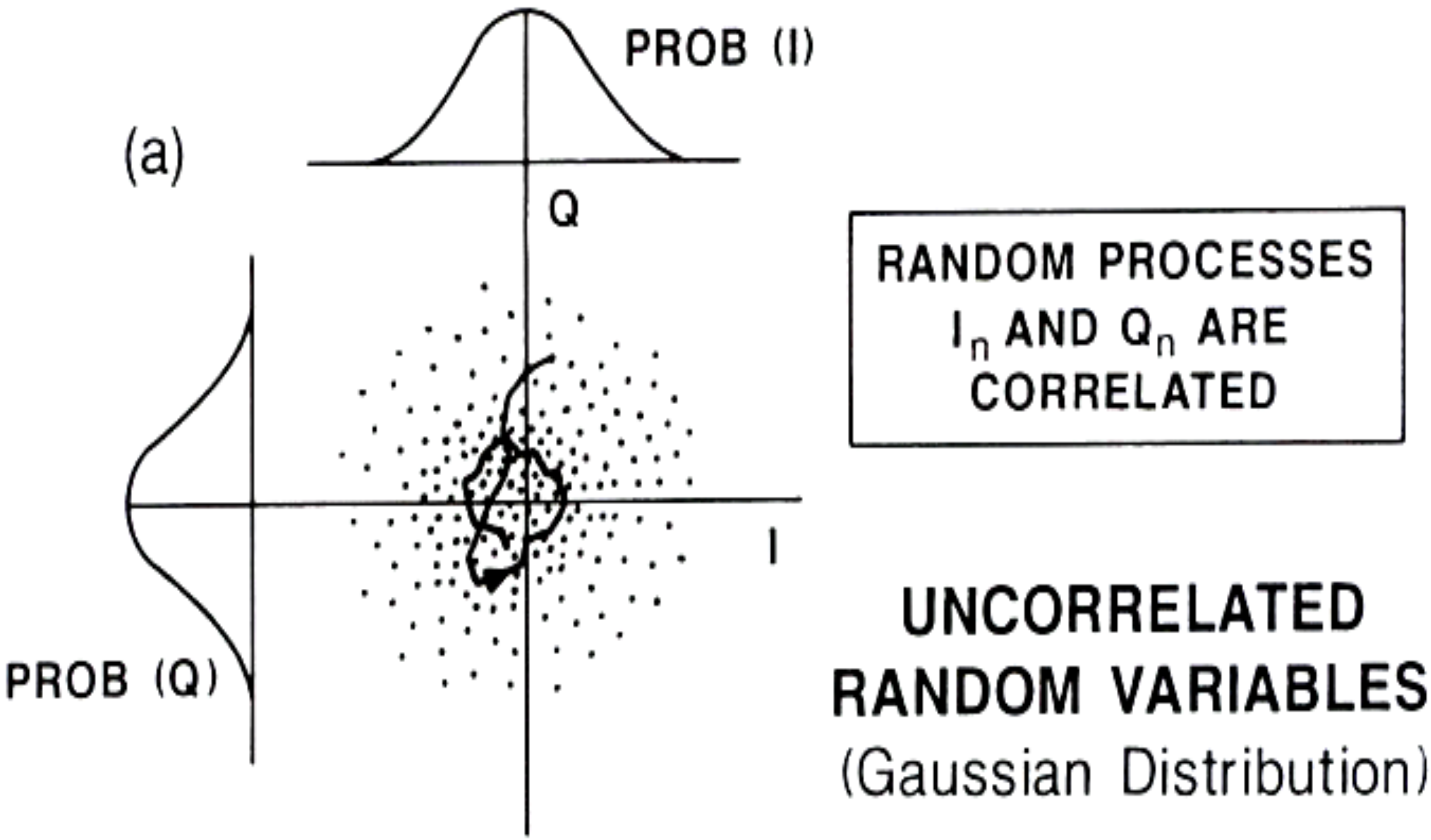


Echoes (I or Q) from Distributed Scatterers (Fig. 4.1)

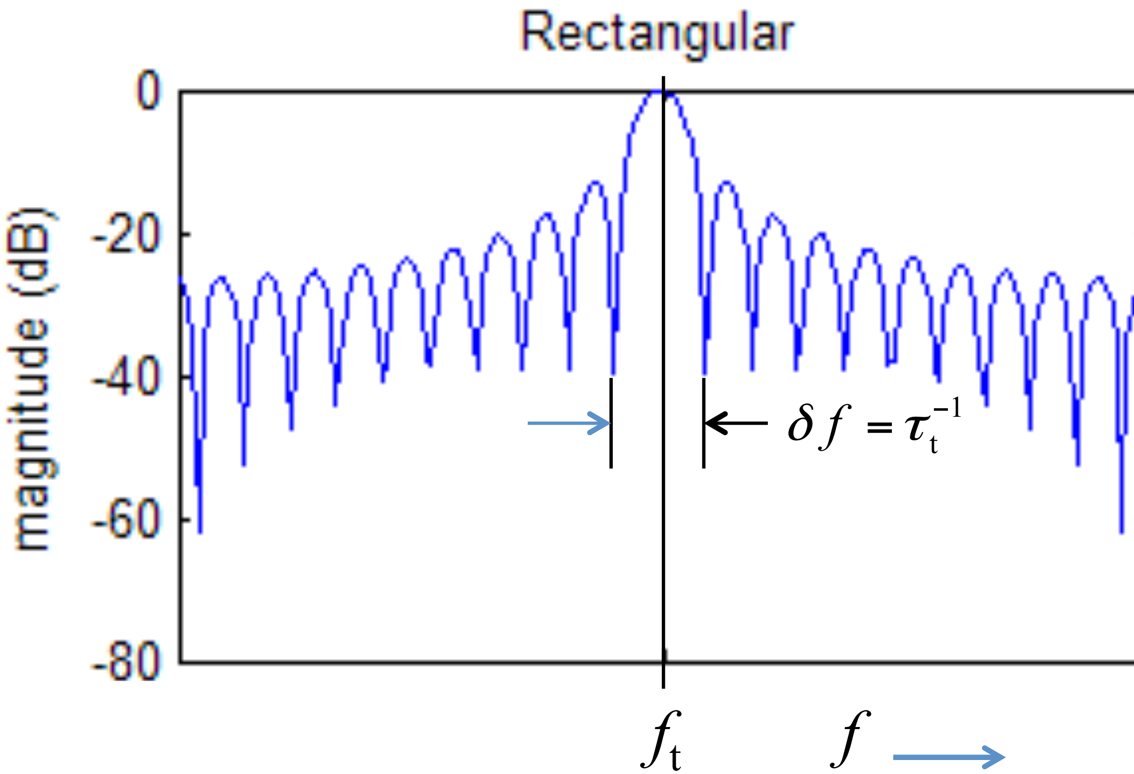


$\tau_c(\tau_s) \approx \tau_t$
(τ_t = transmitted pulse width)

Weather Echo Statistics (Fig. 4.4)



Spectrum of a transmitted rectangular pulse



If receiver frequency response is matched to the spectrum of the transmitted pulse (an ideal matched filter receiver), some echo power will be lost. This is called the finite bandwidth receiver loss L_r. For and ideal matched filter L_r = 1.8 dB (ℓ_r= 1.5).

Reflectivity Factor Z

(Spherical scatterers; Rayleigh condition: $D \le \lambda/16$)

$$\eta(\boldsymbol{r}) = \frac{\pi^5}{\lambda^4} |K_\mathrm{m}|^2 Z(\boldsymbol{r}) \qquad (4.31)$$

where

$$Z(\boldsymbol{r}) \equiv \frac{1}{\Delta V} \sum_i D_i^6 = \int_0^\infty N(D,\boldsymbol{r}) D^6 dD \qquad (4.32)$$

$$\eta(\boldsymbol{r}) = \frac{\pi^5}{\lambda^4} |K_\mathrm{w}|^2 Z_\mathrm{e}(\boldsymbol{r}) \qquad (4.33)$$

for water drops : $|K_\mathrm{w}|^2 \approx 0.93$ independent of T($^\mathrm{o}$C);

for ice particles : $|K_\mathrm{i}|^2 \approx 0.16$ dependent on T and ice density.

Differential Reflectivity

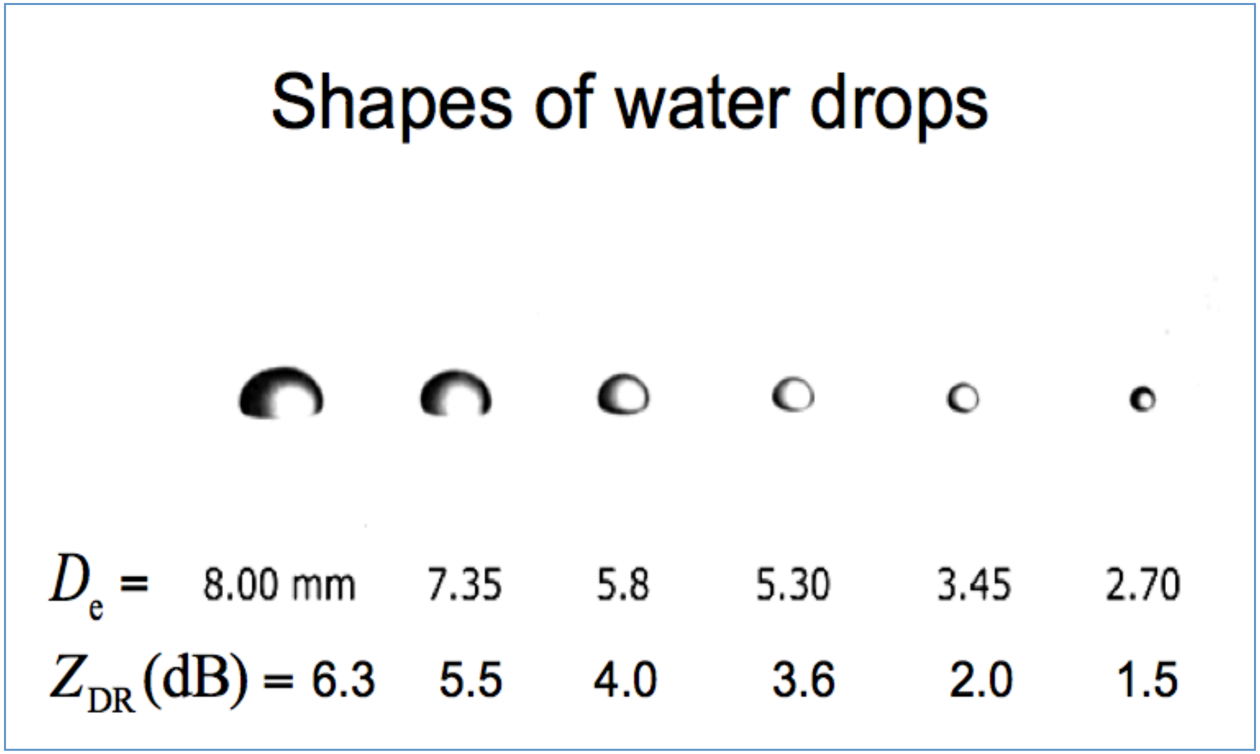
in dB units:

$Z_{DR}(\mathrm{dB}) = Z_h(\mathrm{dBZ}) - Z_v(\mathrm{dBZ})$

in linear units:

$Z_{dr} = Z_h(\mathrm{mm^6m^{-3}})/Z_v(\mathrm{mm^6m^{-3}})$

- is independent of drop concentration N_0
- depends on the shape of scatterers



Shapes of raindrops falling in still air and experiencing drag force deformation.

D_e is the equivalent diameter of a spherical drop. Z_{DR} (dB) is the differential reflectivity in decibels (Rayleigh condition is assumed). Adapted from Pruppacher and Beard (1970)

The Weather Radar Equation

A form of the weather radar equation for echo power from rain is:

$$E\left[P(\boldsymbol{r}_0)\right](\text{mW}) = \frac{\pi^5 10^{-17} P_t(\text{W}) g^2 g_s \tau(\mu\text{s}) \theta_1^2(\text{deg.}) \mid K_w \mid^2 Z_w(\text{mm}^6 \text{m}^{-3})}{6.75 \times 2^{14} (\ln 2) r_0^2(\text{km}) \lambda^2(\text{cm}) l^2 l_r} \qquad (4.35)$$

$E\left[P(\boldsymbol{r}_0)\right] \equiv$ Expected peak weather signal power in milliwatts;

$P_t \equiv$ Peak transmitted pulse power (typically 500 kW)

$g_s \equiv$ net power gain of the echo in going from the antenna to the radar output. τ = pulse width

$\theta_1 \equiv$ one-way half-power beamwidth; $\mid K_w \mid^2 \equiv$ dielectric factor of water

$Z_w \equiv$ reflectivity factor for water spheres; $r_0 \equiv$ range (in km) to the center of the resolution volume V_6

$l \equiv$ one-way loss factor (a number $\geq$ 1) incurred for propagation through a rain filled atmosphere.

$l_r \equiv$ loss factor due to the finite bandwidth of the receiver; $\lambda \equiv$ wavelength of the transmitted radiation