METR 5004
A SERIES OF SHORT COURSES ON THE FUNDAMENTALS OF ATMOSPHERIC SCIENCE

THIS SHORT COURSE IS ON:
BASICS OF POLARIMETRIC-DOPPLER RADAR
AND
WEATHER OBSERVATIONS

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For theory and more information on weather radar:

ACADEMIC PRESS, 2nd edition, 3rd & 4th Prts.
or
DOVER PUBLICATIONS INC
June 2006

(Book has been translated into Russian and Chinese)

Book errata and supplements at:
www.nssl.noaa.gov/papers/books.html

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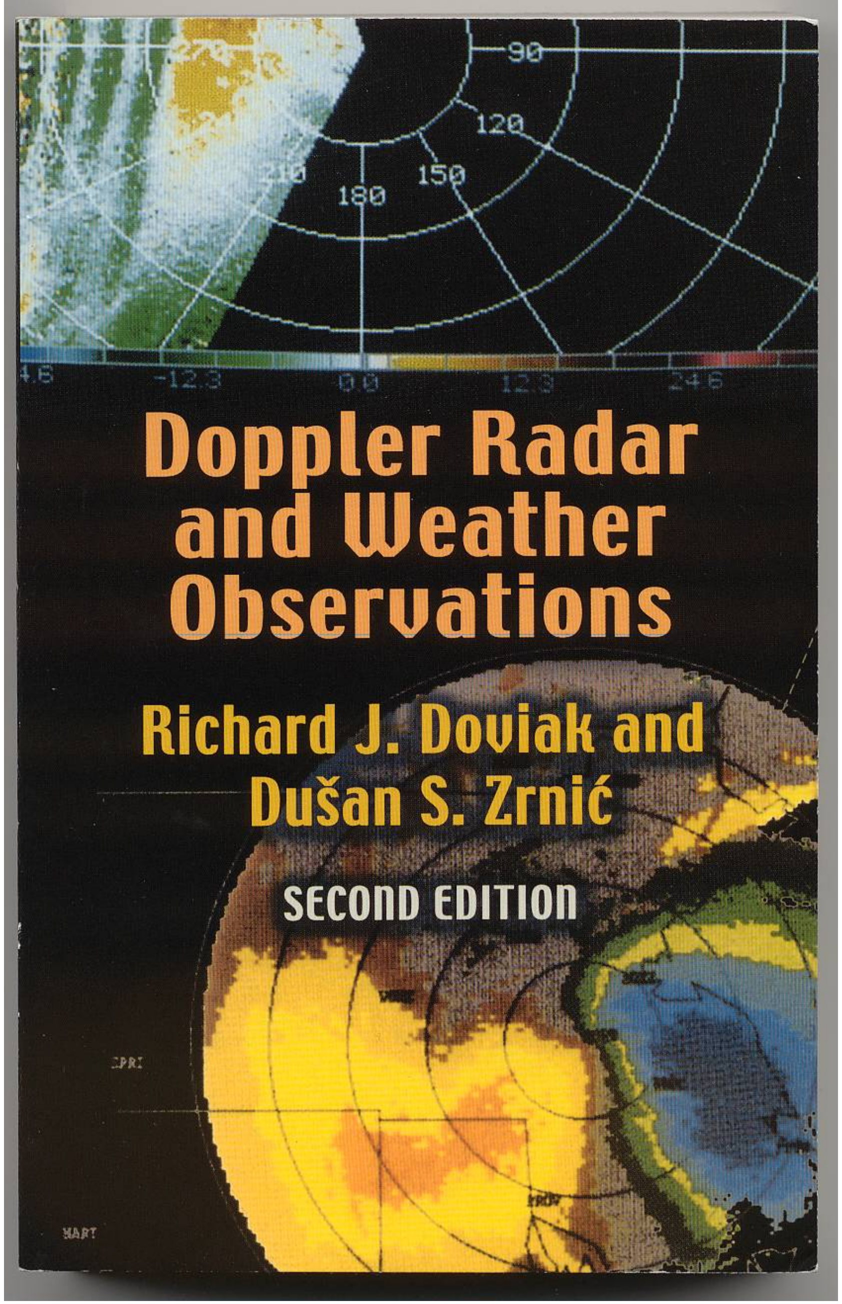


Radar

Radio detecting and ranging of objects

(Taylor and Furth US Navy 1940)

Applications:

- Remote sensing (Air, Sea, and Land)
- Tracking of objects (aircraft, missiles, speeding cars, etc.)
- Astronomy (both active and passive, Doppler measurements)
- Medical, imaging objects in the ground, etc.

Radar Meteorology exposes one to:

- The fundamental aspects of remote sensing using electromagnetic waves
- Random processes (fundamental to weather radar measurements)

Early development of Radar

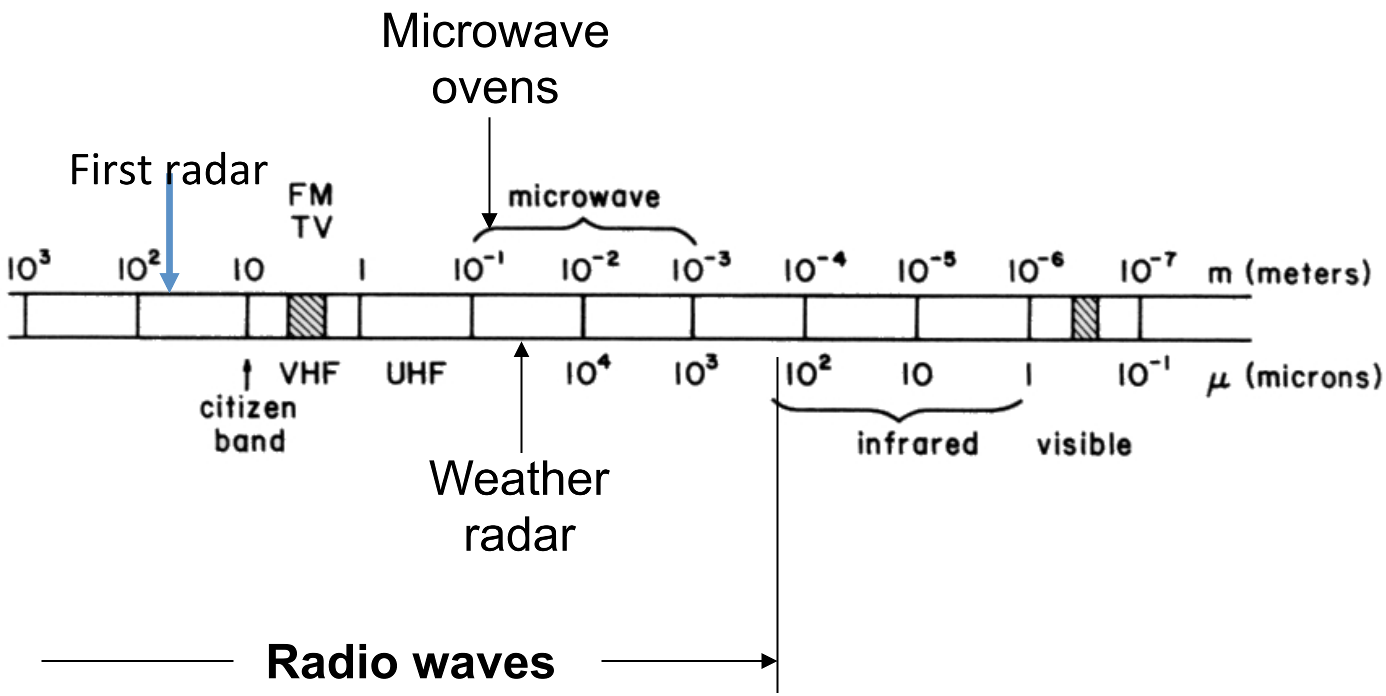
1900: Tesla; Published the concept of radar.

1904: 1st demonstration of radio waves (continuous waves) to detect an object.

1925: 1st successful use of pulsed radio waves or RADAR to detect an object (i.e., an atmospheric layer 150 km AGL) by G. Breit and M. A. Tuve, Dept. of Terrestrial magnetism, Carnegie Institution

Late 1930s and early 1940s: Explosive growth of radar for detecting and ranging aircraft.

The Spectrum of Electromagnetic Waves



Importance of Weather Radars

- **Electromagnetic waves can penetrate clouds and rain regions and thus reveal meteorological features inside storms**!
- Weather radars can provide quantitative and automated real-time information on storms, precipitation, hurricanes, tornadoes, etc.

Properties of Electromagnetic Waves

Chapter 2

The Electric Field Equation

$$\boldsymbol{E} = \frac{\boldsymbol{A}(\theta,\phi)}{r}\exp\left[j2\pi f\left(t-\frac{r}{c}\right)+j\psi_t\right] \qquad (2.2b)$$

Alternately: $\boldsymbol{E} = \frac{\boldsymbol{A}(\theta,\varphi)}{r}\exp\left[j(\omega t - kr) + j\psi_t\right]$ Wavenumber $k = 2\pi f/c = 2\pi/\lambda$

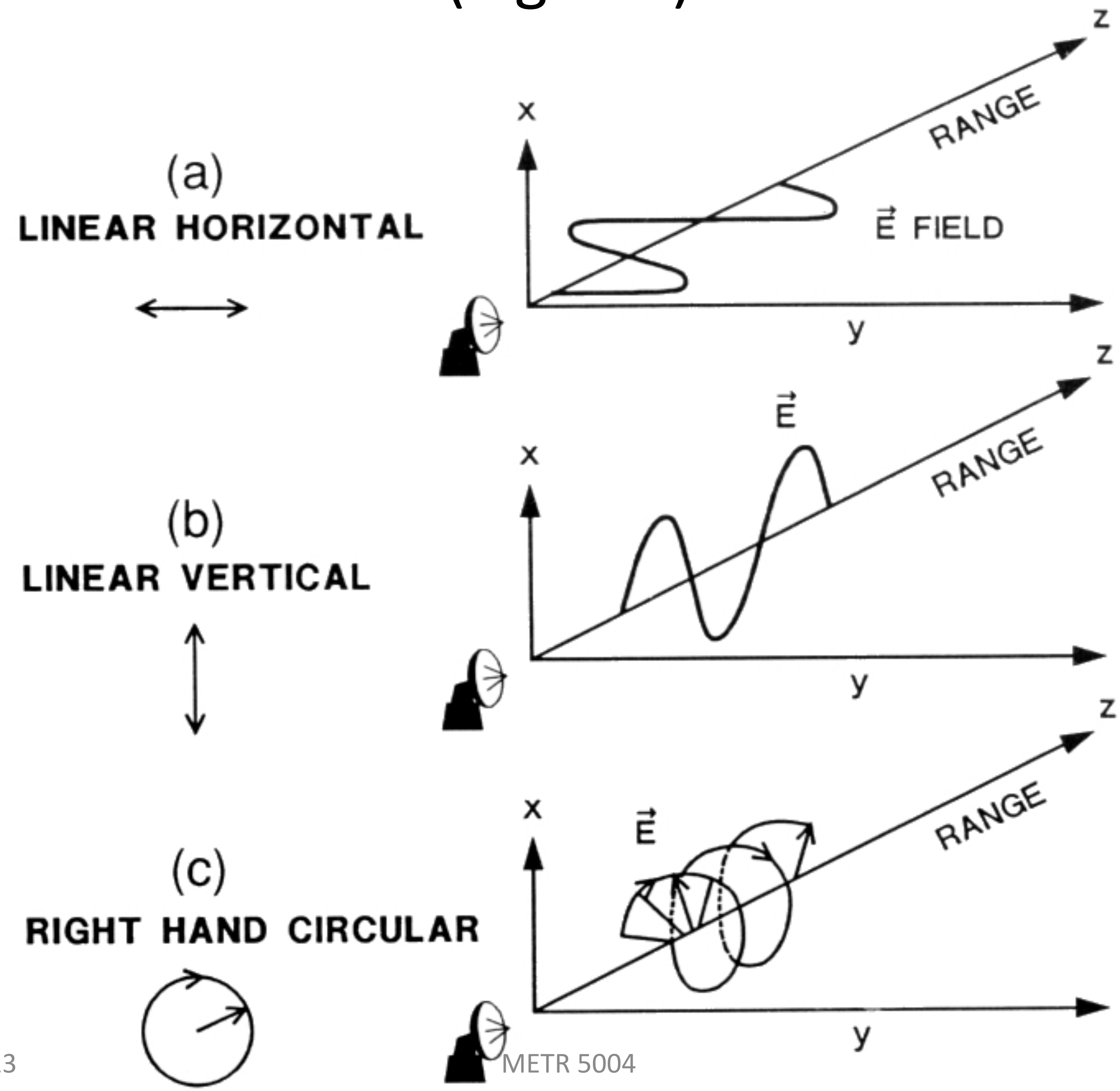
Linear Polarization:

$$\boldsymbol{E} \rightarrow E_x \text{ (or } E_y) = I_x + jQ_x$$

Elliptical Polarization: Transmit both E_x and E_y

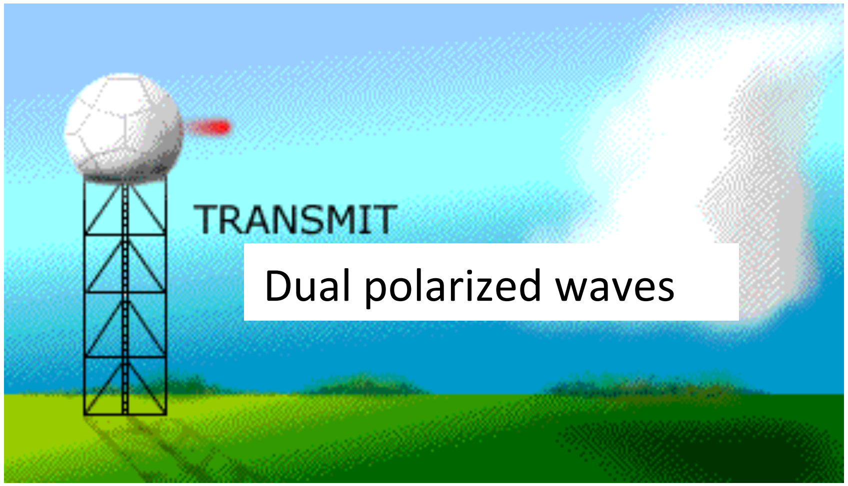
$$E_{x,y} = I_{x,y} + jQ_{x,y} \propto \frac{\mathbf{A}_{x,y}(\theta,\varphi)}{r}\exp\left[j2\pi f\left(t-\frac{r}{c_{x,y}}\right)+j\psi_{t,x,y}\right]$$

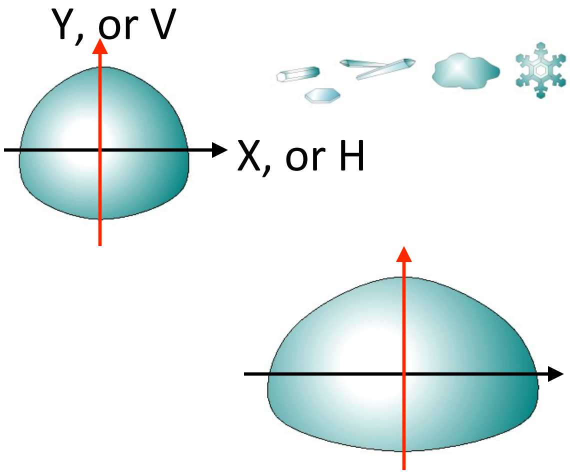
Polarization
(Fig. 2.2)

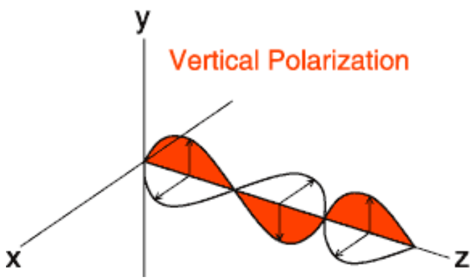


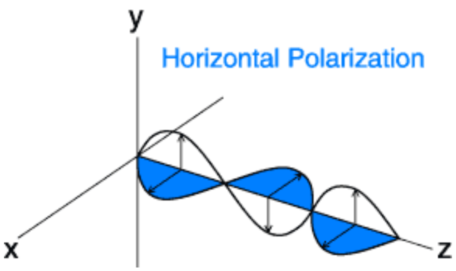
Dual-polarization Radar

S K I P ?





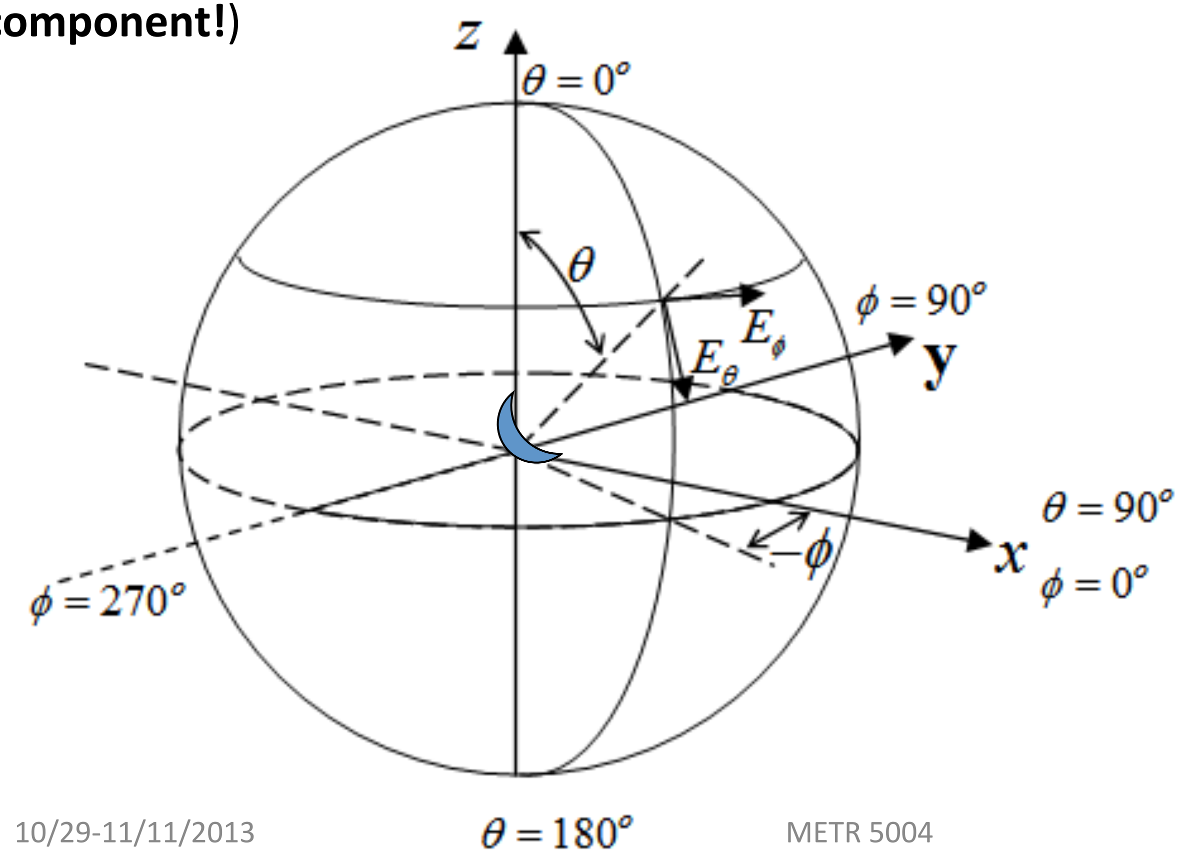




Vertically and Horizontally Polarized Waves

Vertically polarized waves ($V = -E_\theta$):
E vector lies in the **vertical plane, but it has both a vertical and horizontal component!**)

Horizontally polarized waves ($H = E_\phi$)
E vector lies in the horizontal plane!



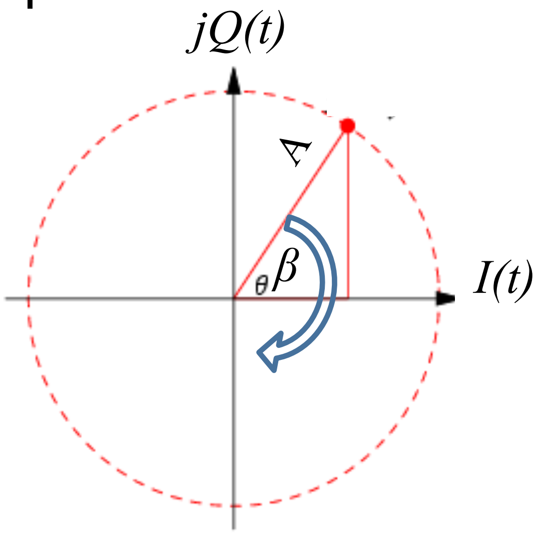
Hydrometeor Properties:

1) Electrical size
2) Apparent canting angle
3) Canting angle dispersion
4) Eccentricity (shape)

Circular polarization provides relatively simple formulas to measure directly these properties. But depolarization during propagation mitigates any advantage of circular polarization. (Doviak et al., JTECH 2000)

Complex Numbers

- Weather signal voltages (i.e., echoes) V are a field of complex numbers of the form $V = I + jQ$, where I and Q are real numbers and j is the imaginary unit: $j = \sqrt{-1}$
- Component notation:
  - $V = (I, Q)$
  - I is the real or
    In-phase part, $I = \mathrm{Re}\{V\}$
  - Q is the imaginary or
    Quadrature part, $Q = \mathrm{Im}\{V\}$
- Polar notation: $V = A(\cos\beta + j\sin\beta)$
  - Using Euler's relation: $V = A\exp[j\beta]$
  - A is the **amplitude:** $A = \sqrt{I^2 + Q^2}$
  - β is the argument or **phase:** $\beta = \arg\{V\} = \tan^{-1}(Q/I)$

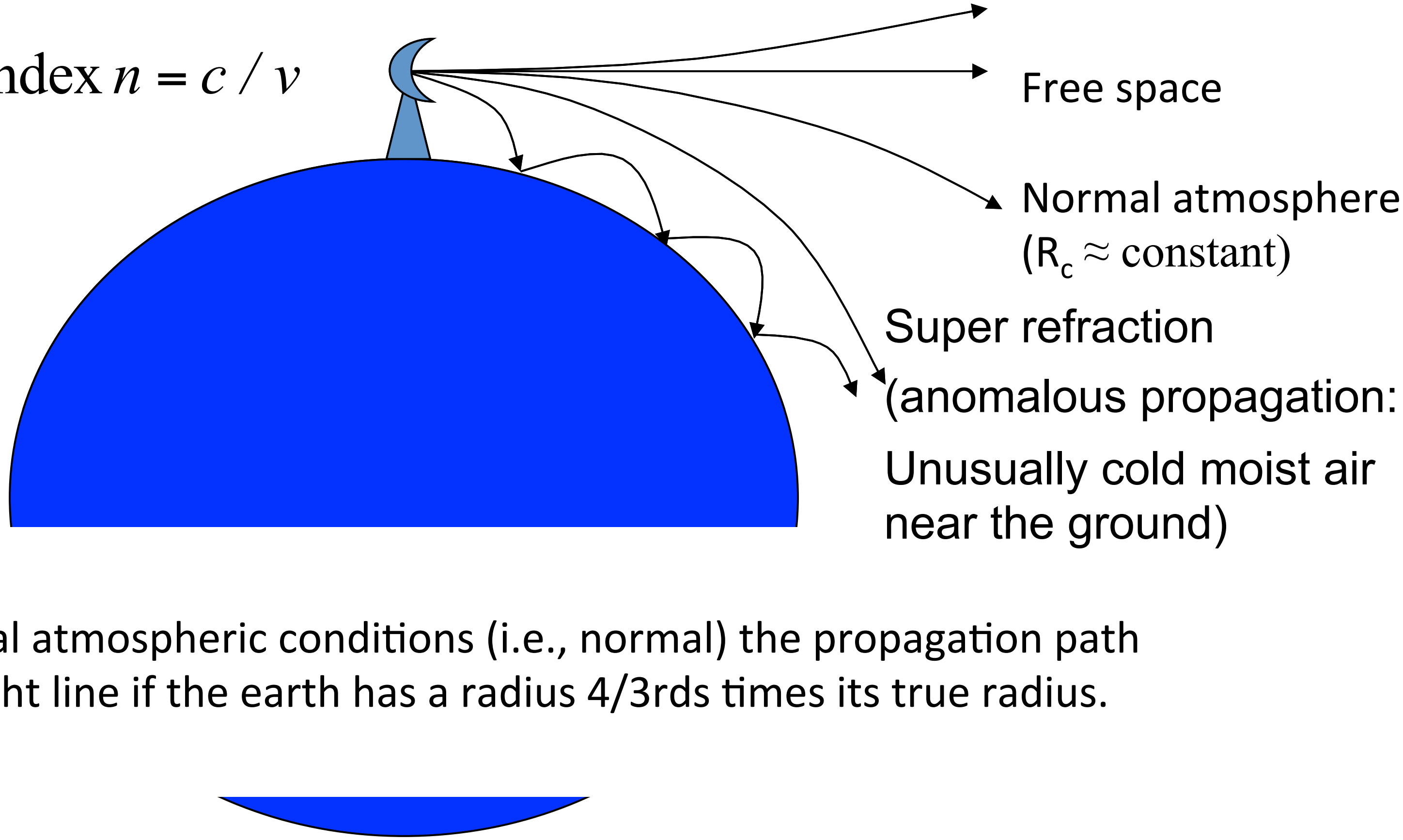


Complex plane
(Phasor diagram)

Normal and Anomalous Propagation

$$n^2 = 1 + C_d P_d / T + C_{w1} P_w / T + C_{w2} P_w / T^2$$

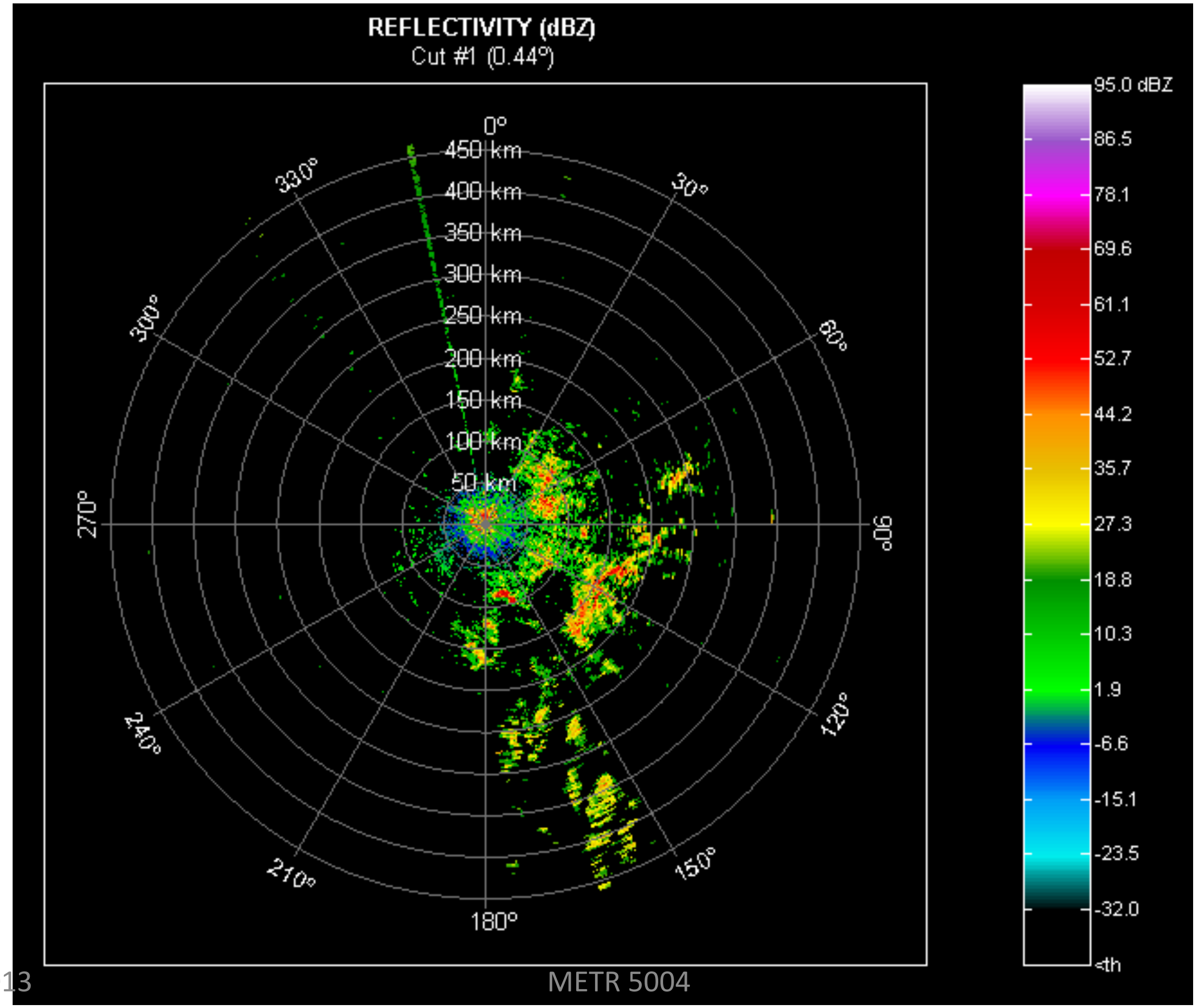
refractive index $n = c / v$



For typical atmospheric conditions (i.e., normal) the propagation path is a straight line if the earth has a radius 4/3rds times its true radius.

AP at KOUN (Norman, OK)

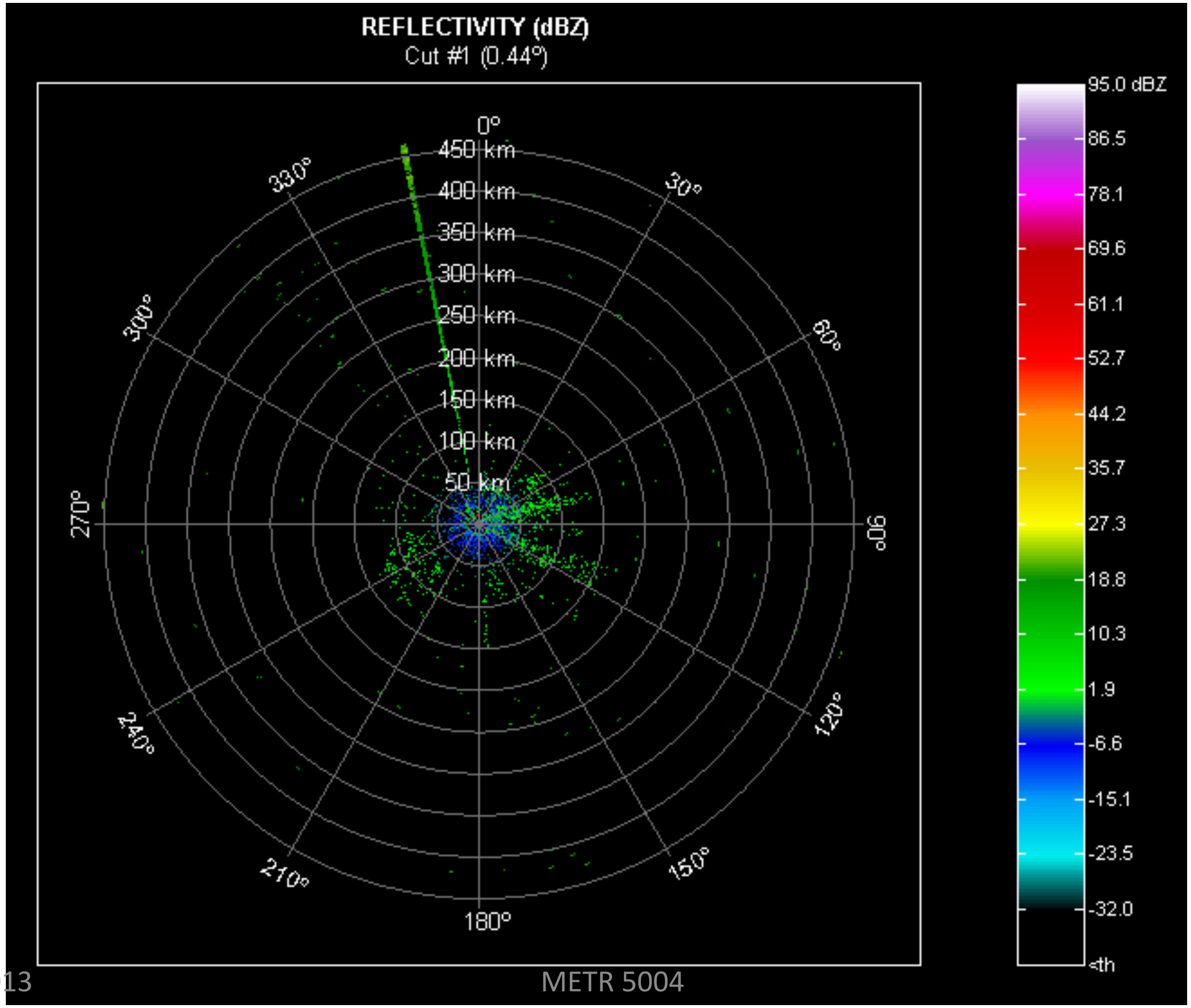
Sept 09, 2004 - 1439 UTC



Without GCF

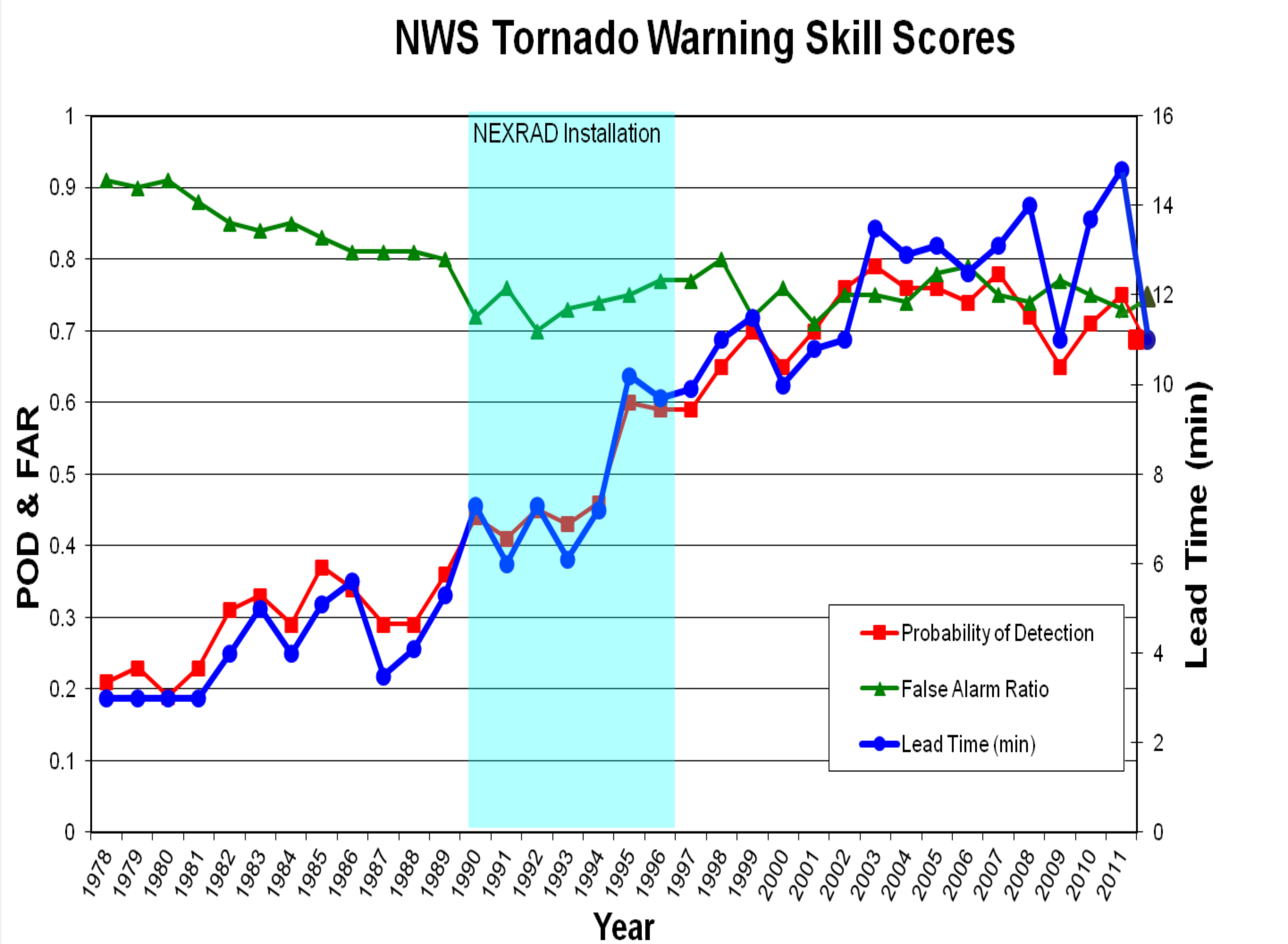
AP at KOUN (Norman, OK)

Sept 09, 2004 - 1439 UTC



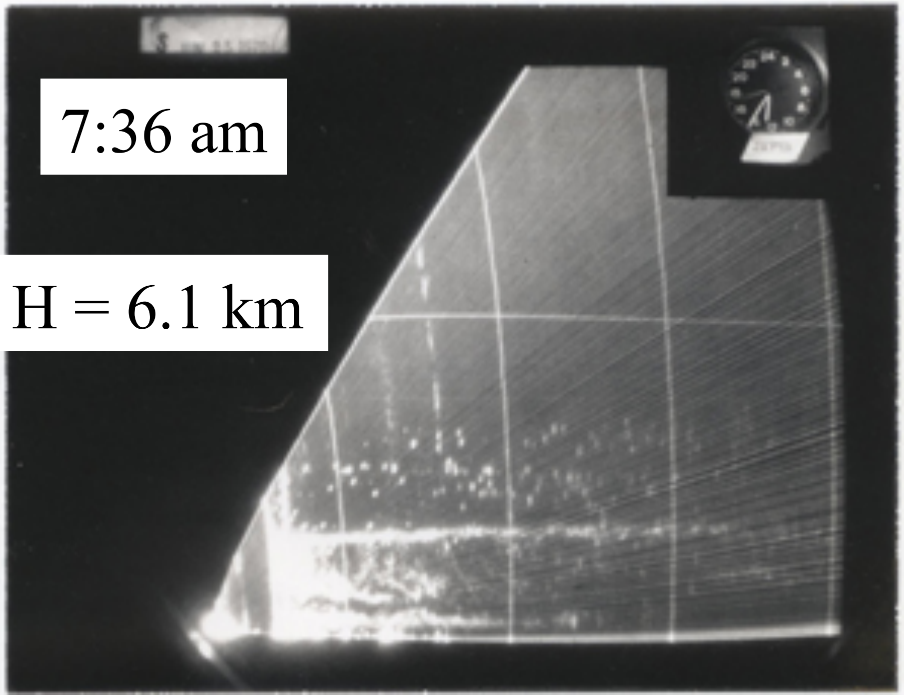
With GCF everywhere

Some improvements in weather warnings and examples of meteorological phenomena observed with Radar

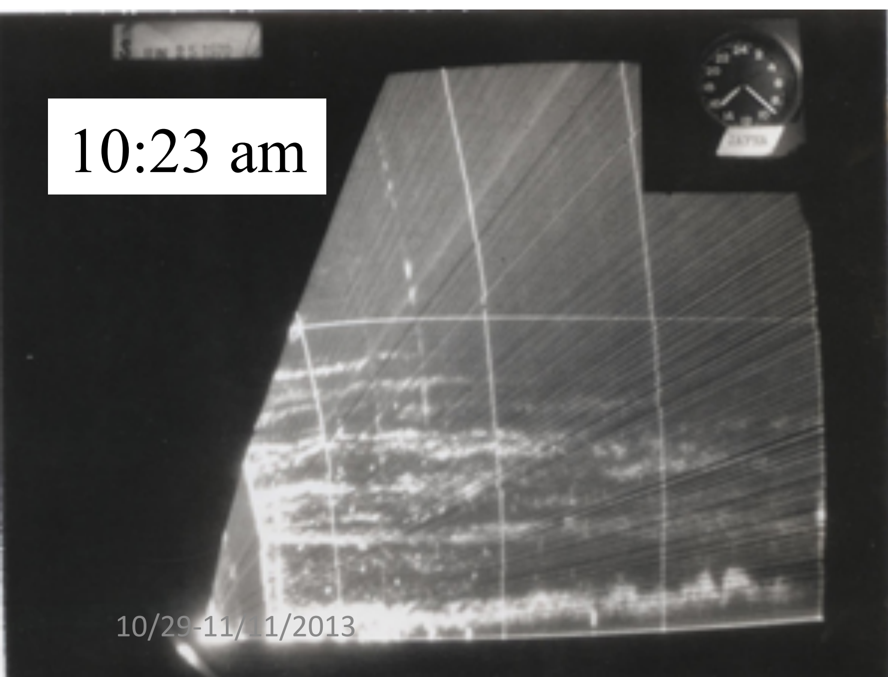


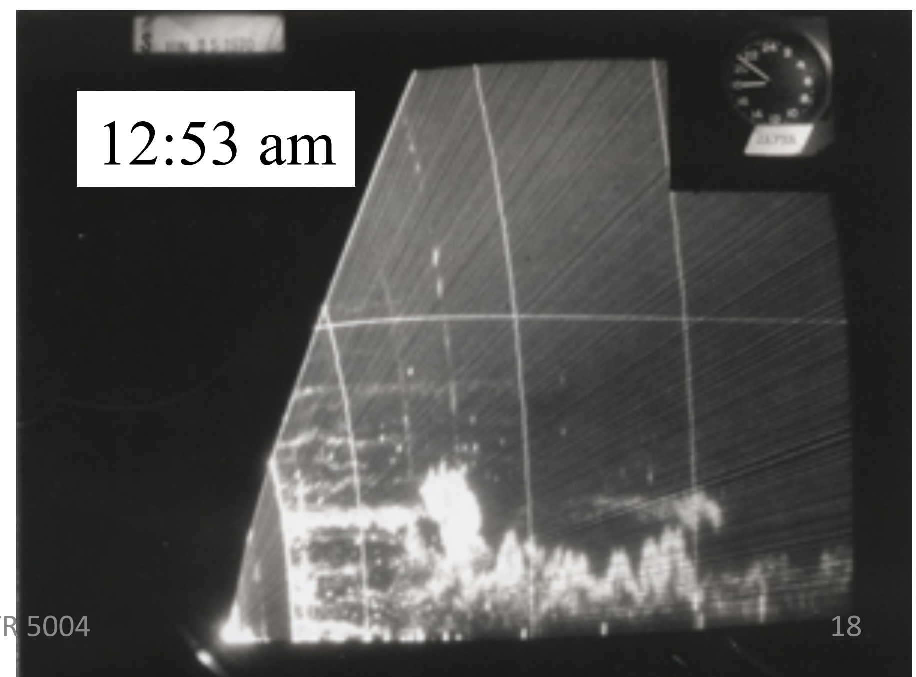
(Thanks to Don Burgess of NSSL)

Evolution of the Boundary Layer (June 25, 1970)

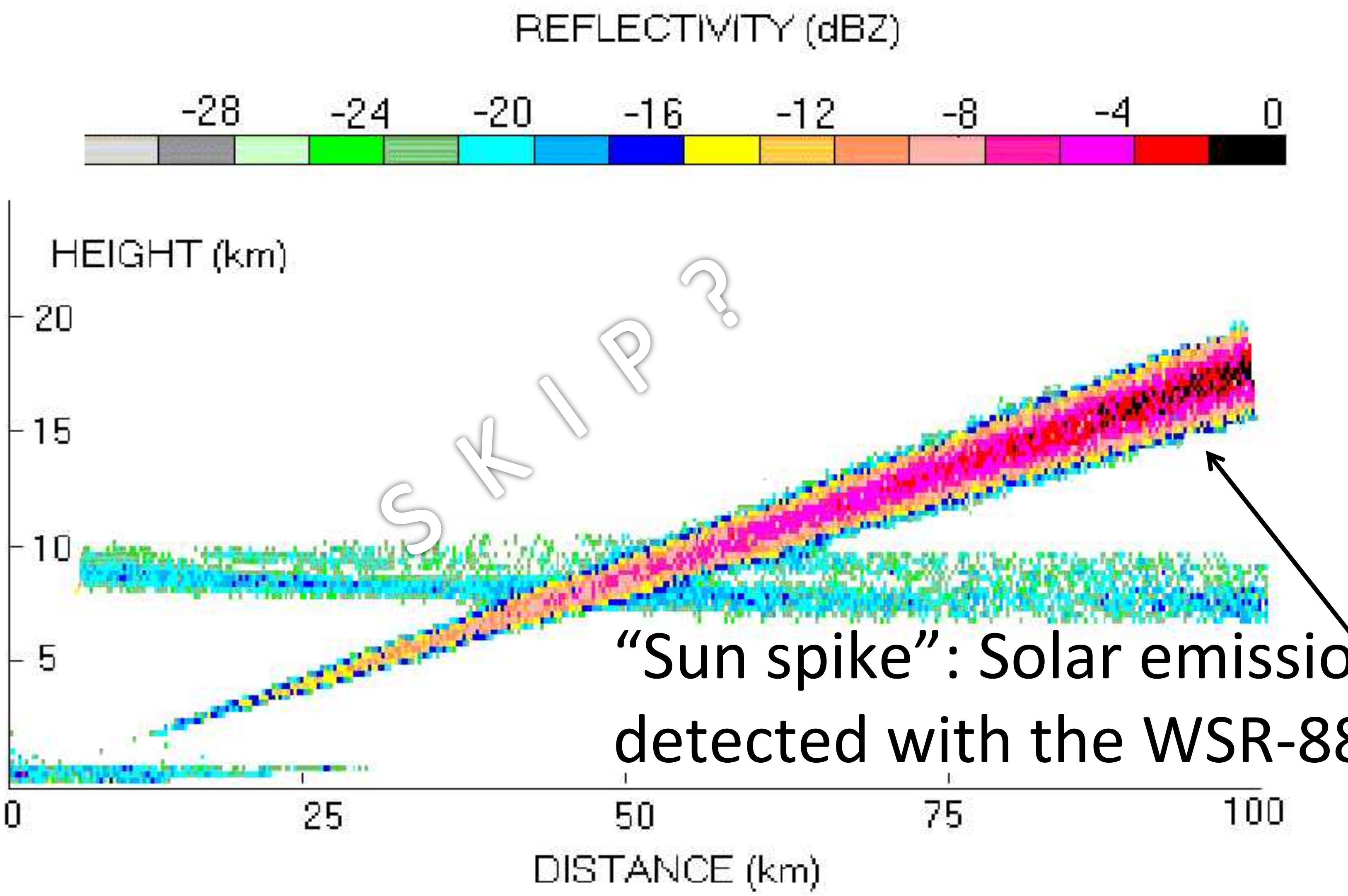


Range Arcs = 9.3 km
λ=10 cm Wallops Is.,VA
(Fig. 11.24)





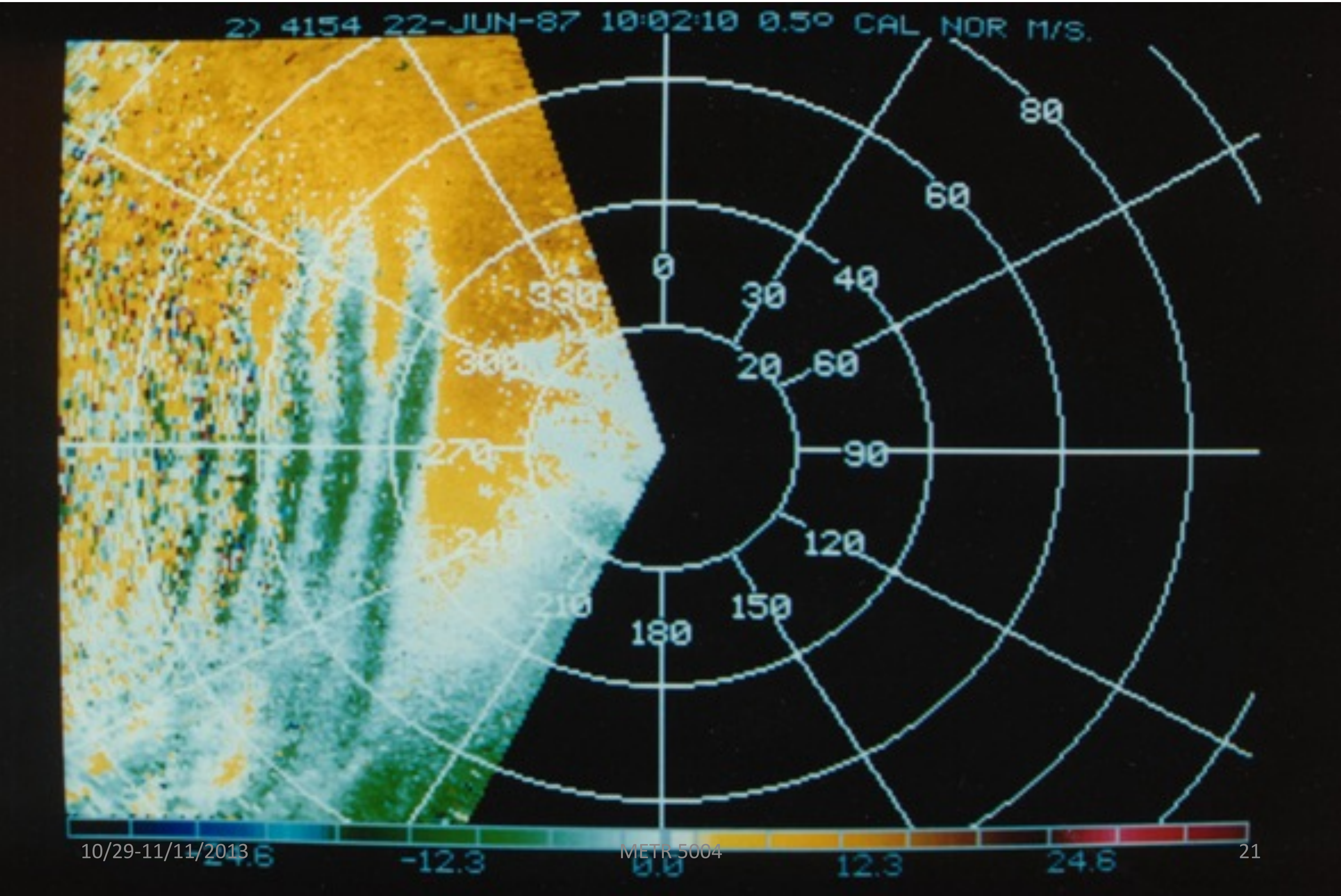
Cirrus Cloud and Solar Emission Detected with the WSR-88D



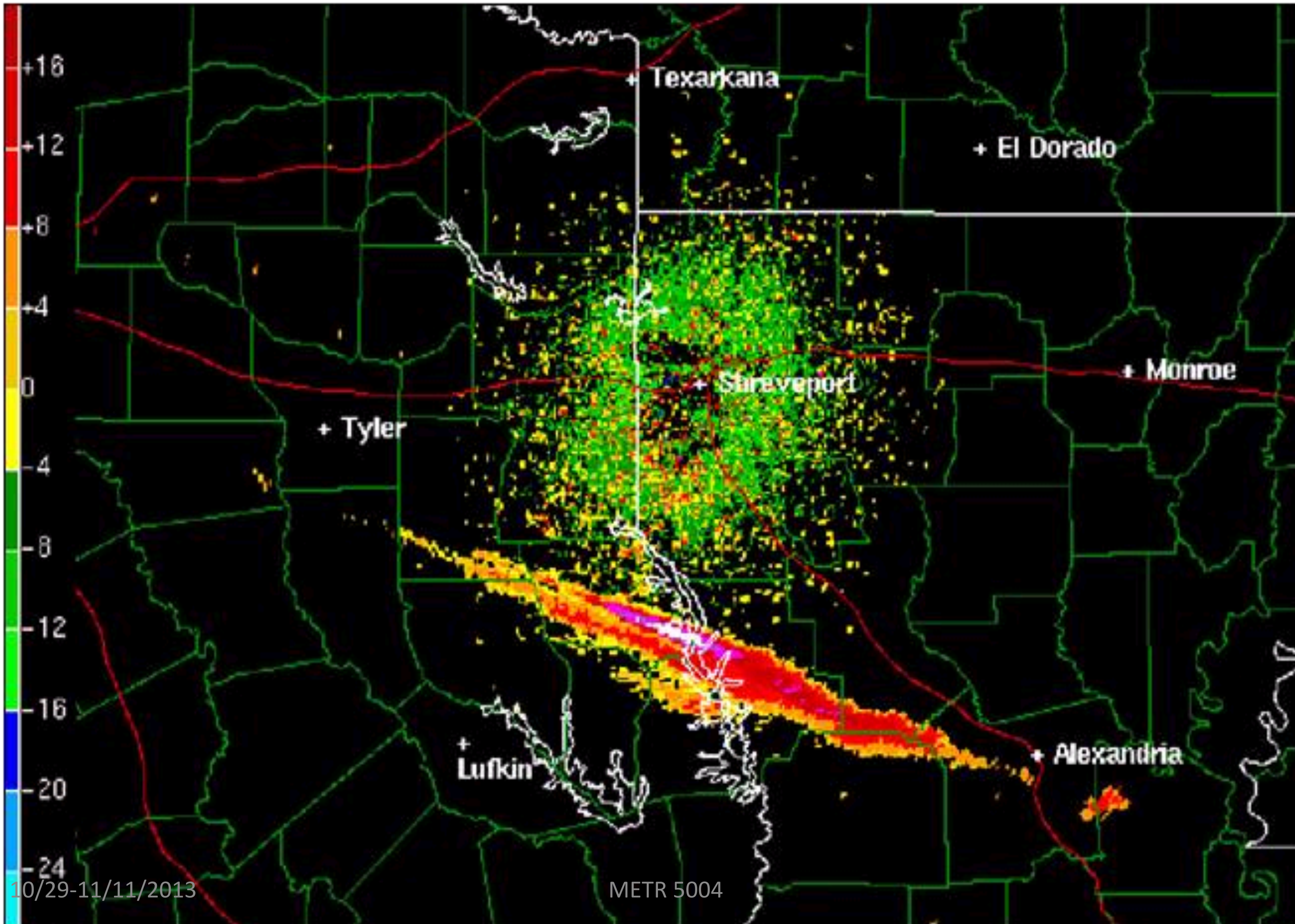
"Sun spike": Solar emission detected with the WSR-88D

Wave Approaching Radar (~10 am)

V_r of the Undular Bore

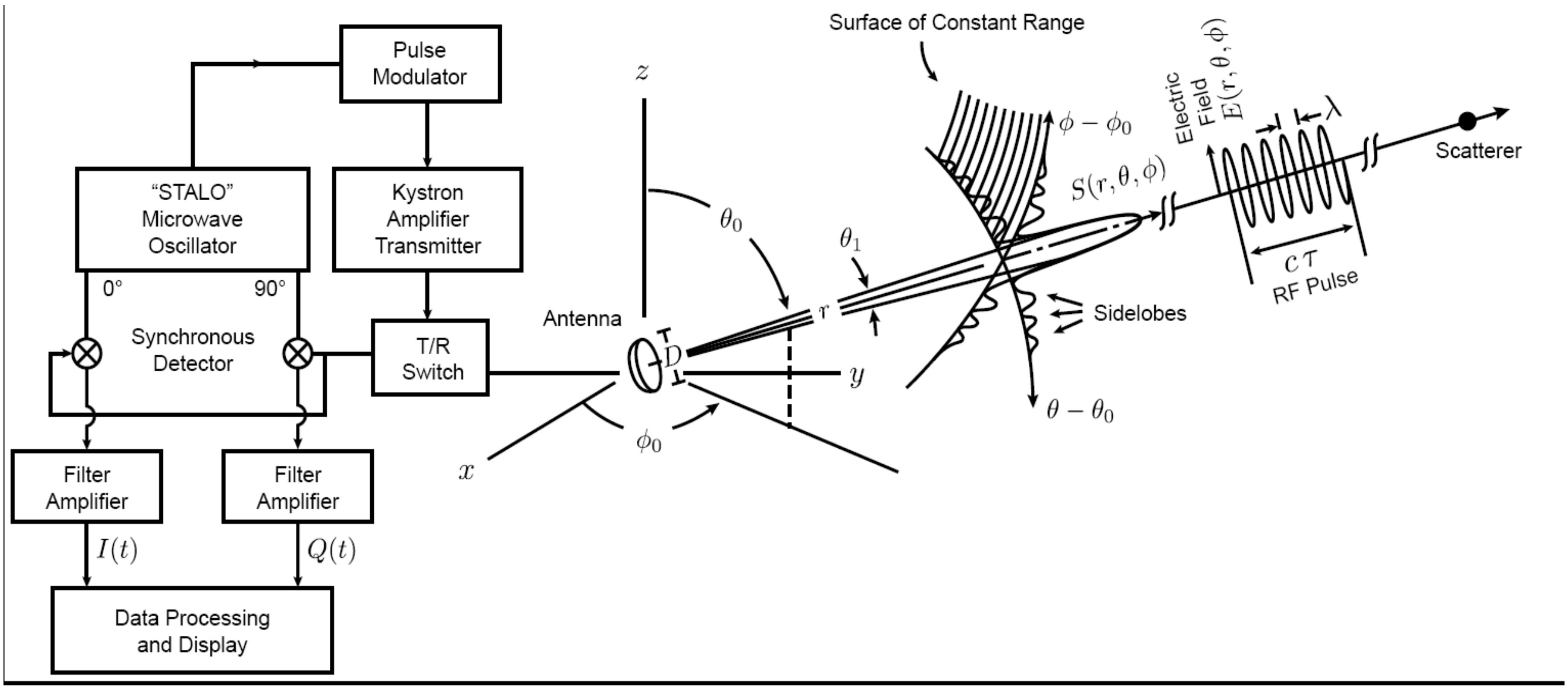


Columbia's debris field and other artifacts
Seen with a WSR-88D weather radar near Shreveport, LA



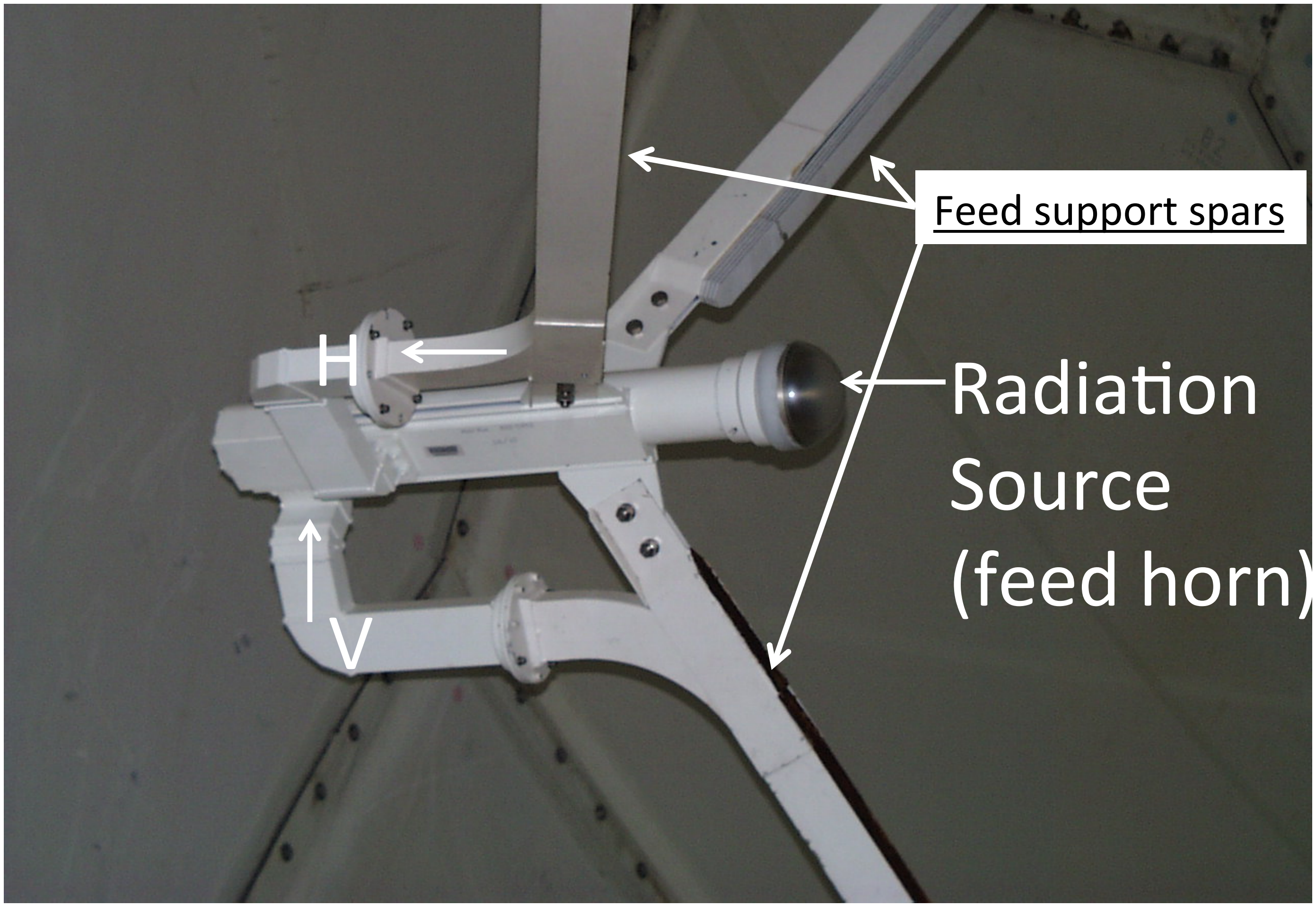
Polarimetric-Doppler radar and its Environment (Chapter Three)

Doppler Radar (Fig. 3.1)
A simplified block diagram



The WSR-88D Antenna

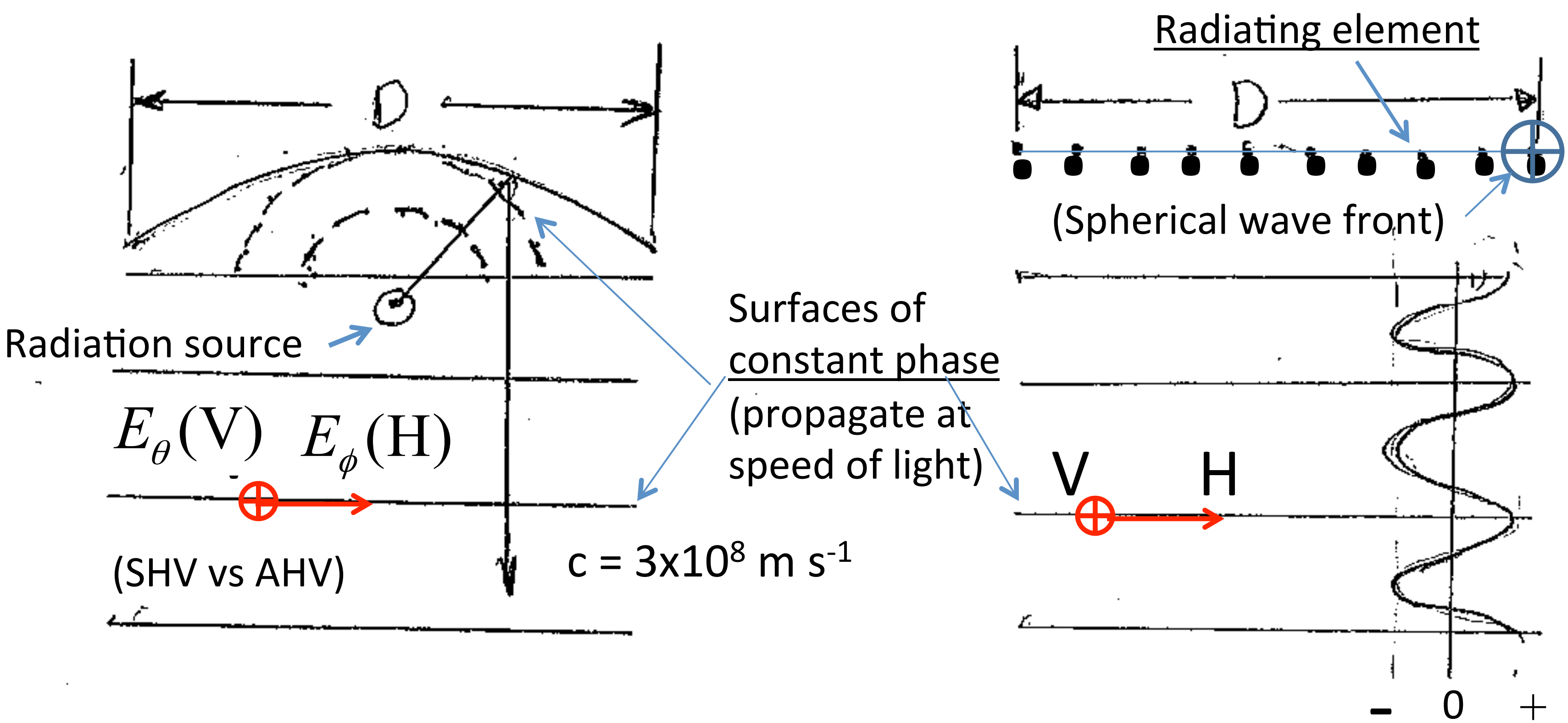
Radiation source (feed) for polarimetric parabolic reflector antenna



Wave Fronts-
Surfaces of Constant Phase ψ

(field near the antenna; broadside PA radiation)

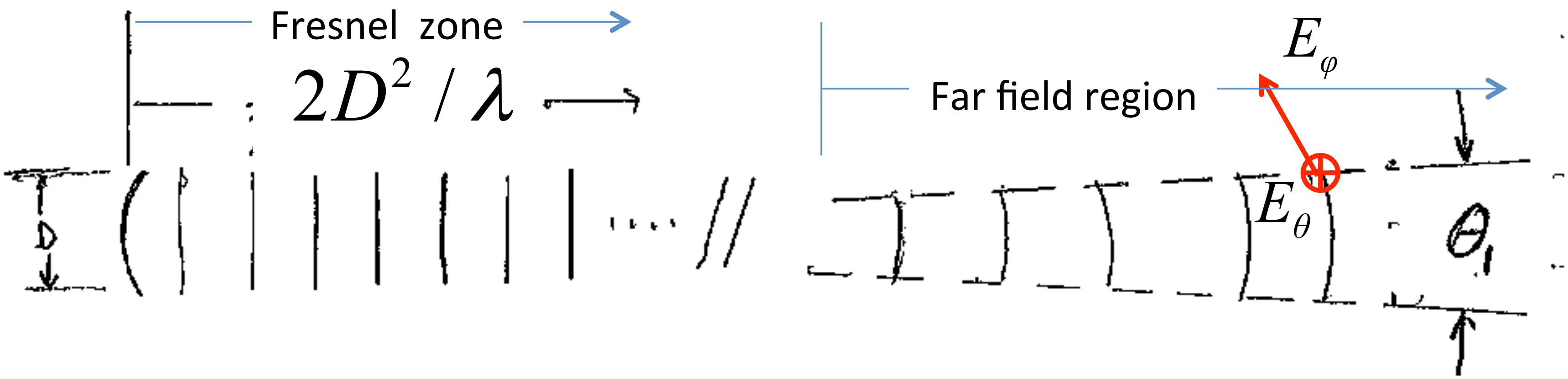
Parabolic Reflector   Planar Phased Array



Huygens Principle: "Each radiating element (or each point of a wave front) can be considered as the source of a secondary wave. The secondary waves then combine to form a new wave front, the new wave front being the **envelope** of the secondary waves".

Angular Beam Formation

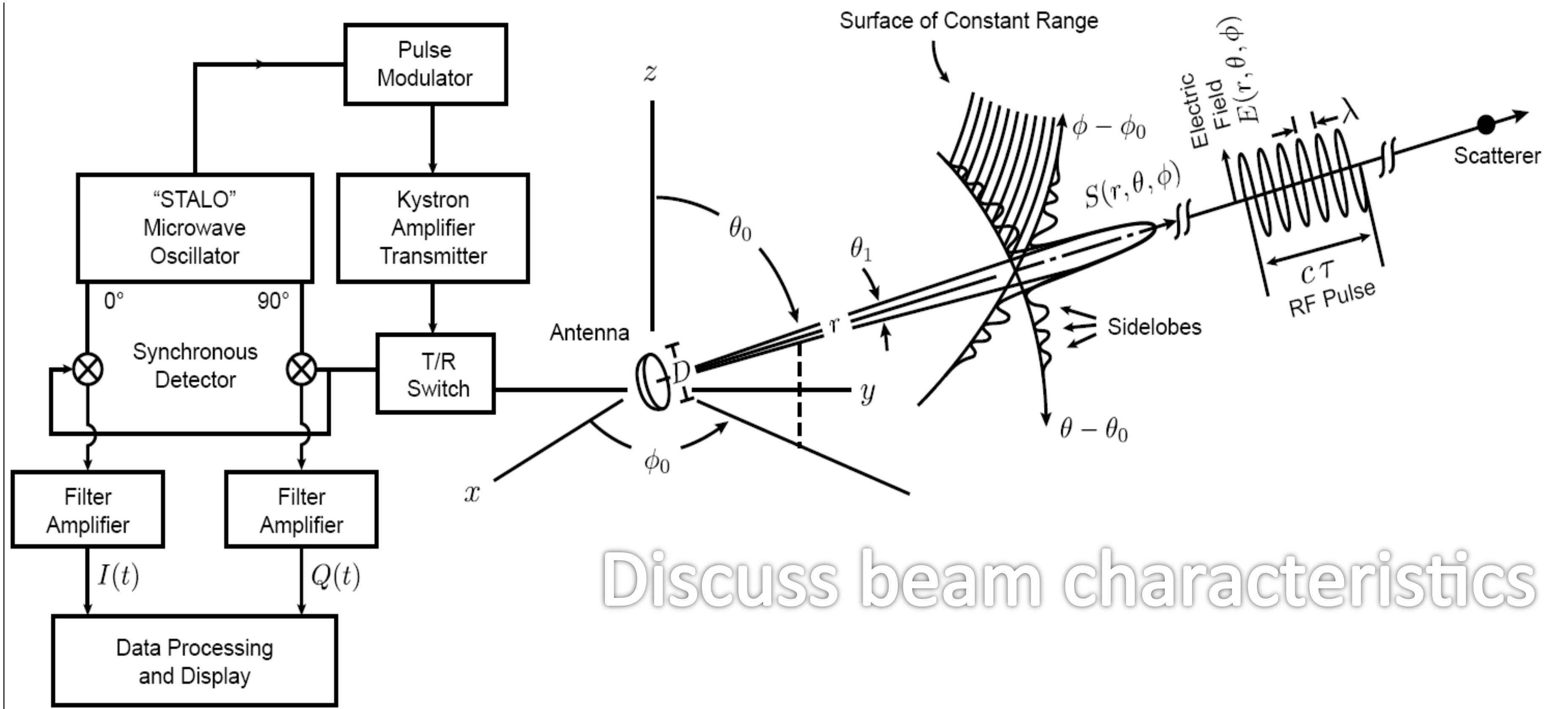
(the transition from a circular beam of constant diameter to an angular beam of constant angular width)



$$2D^2/\lambda\,;\ 1.5\ \text{km};$$

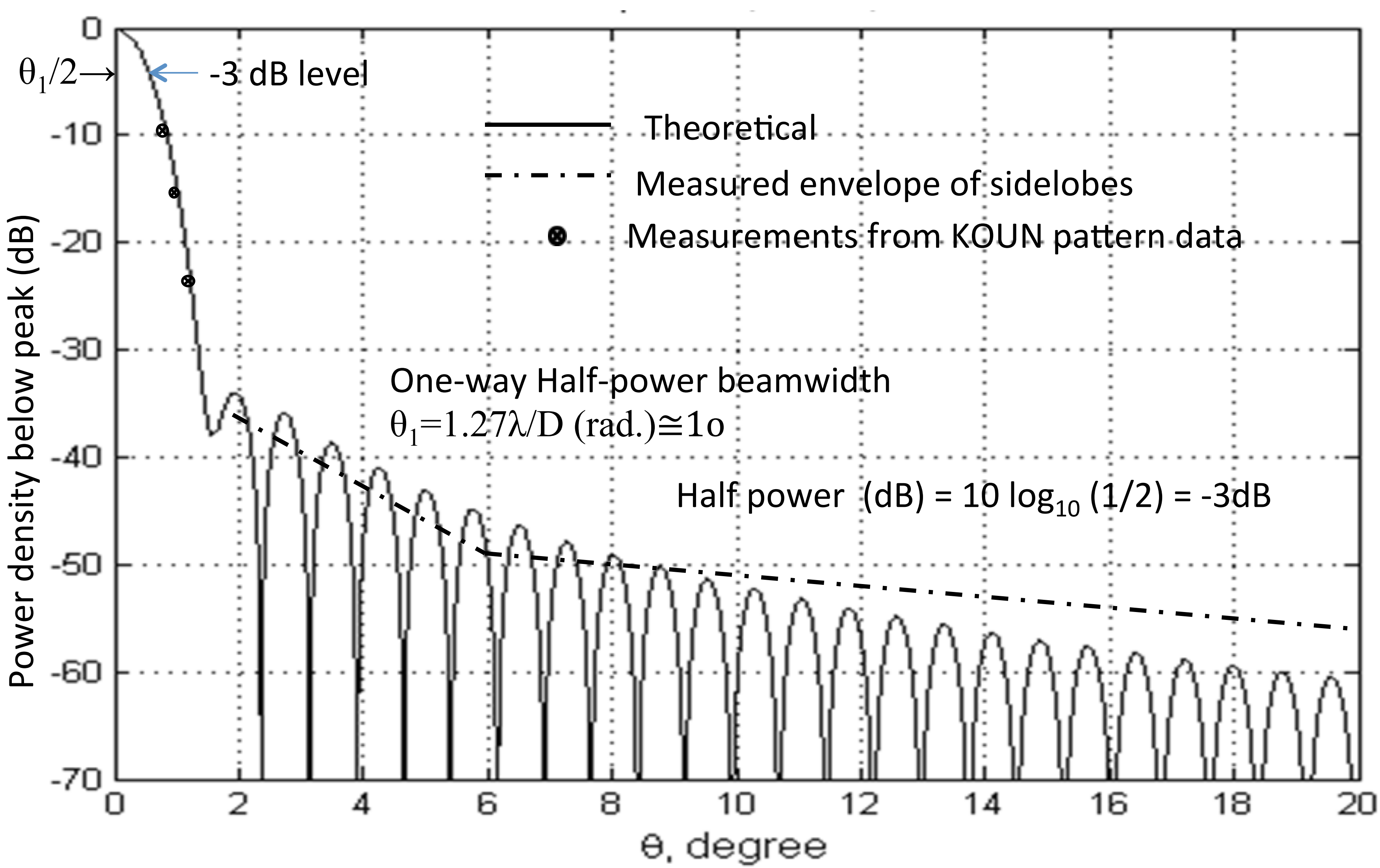
$$\text{WSR-88D: } D\,;\ 8.53\ \text{m};\ \lambda=10\ \text{cm}$$

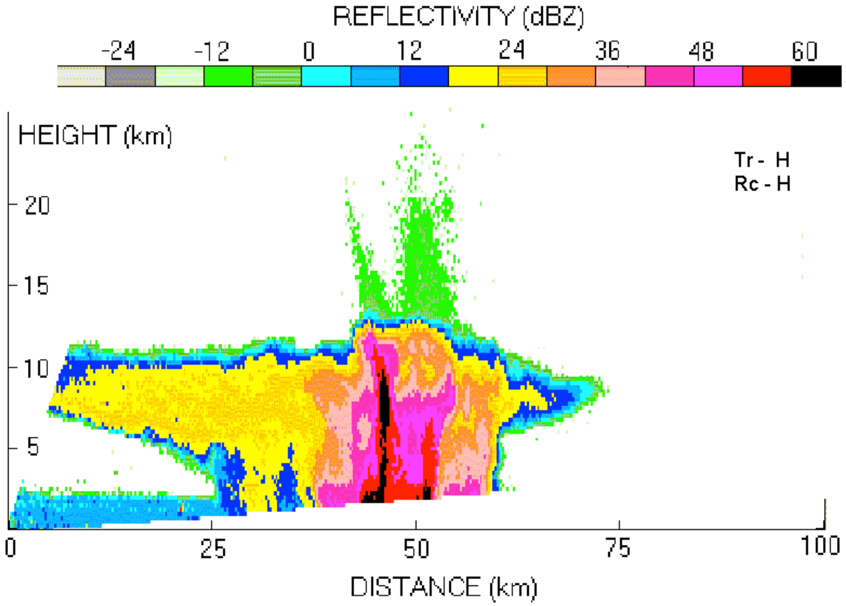
Doppler Radar (Fig. 3.1)
A simplified block diagram



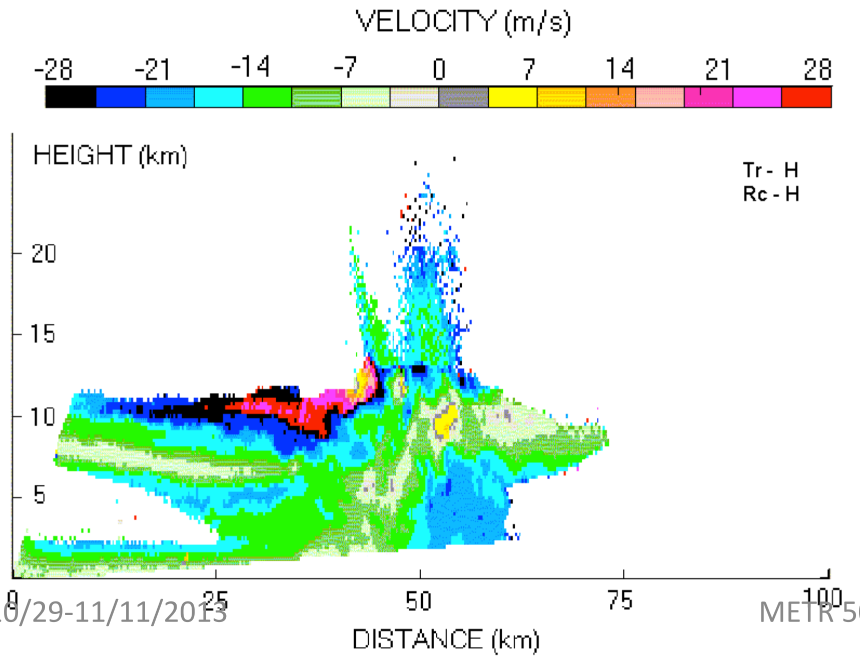
Discuss beam characteristics

Comparison of Theoretical and Measured Copolar One-way Horizontally Polarized Radiation Patterns for a WSR-88D (KOUN)





Effects of WSR-88D Sidelobes on Radar Data (similar to Color plate 2b and Fig. 9.22)



Antenna (directive) Gain g_t

The defining equation:

$$S_i = \frac{P_t}{4\pi r^2} g_t f^2(\theta,\phi) \qquad \text{Eq. (3.4)}$$

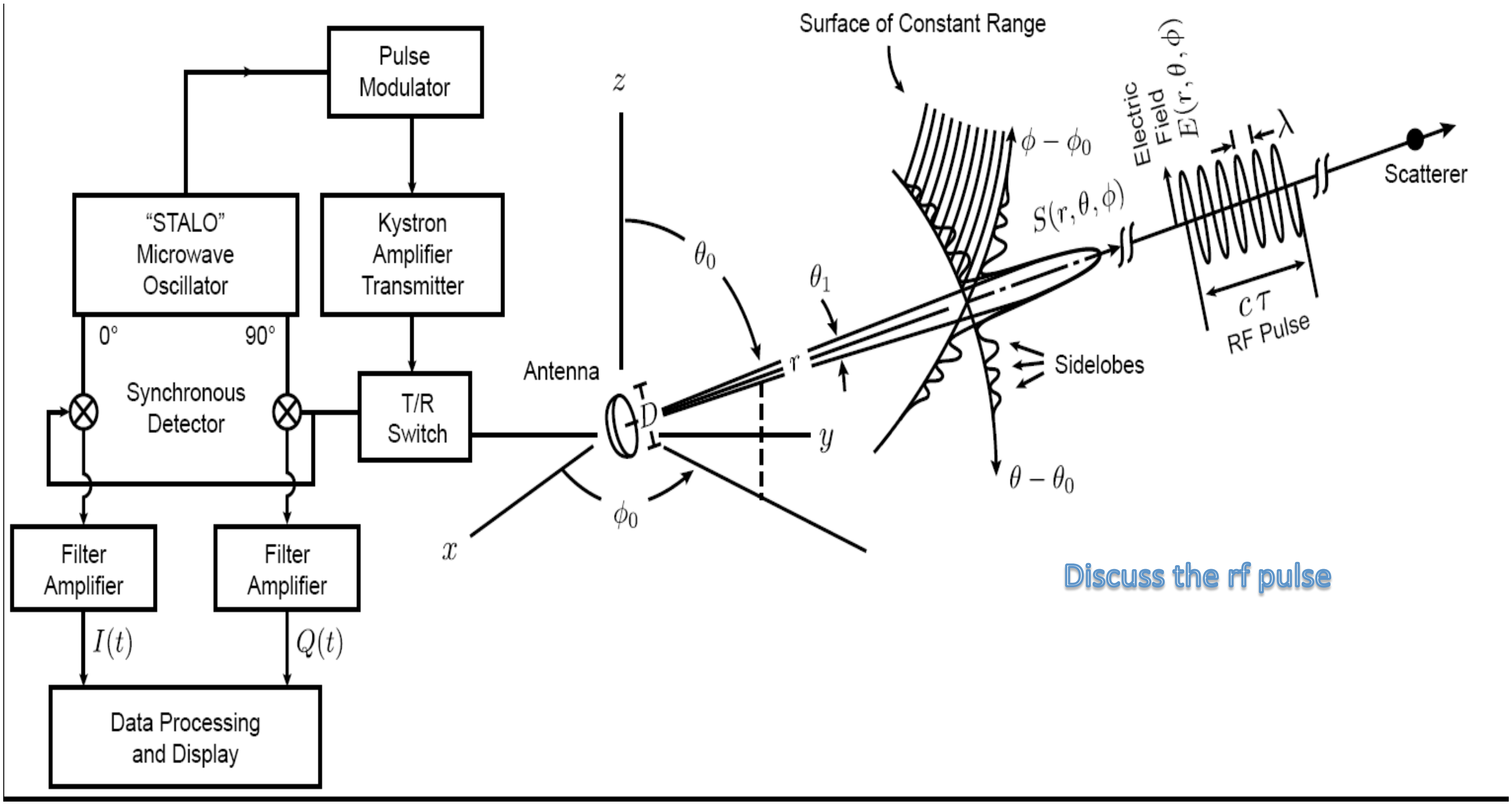
S_i(W m^{-2}) = Incident power density

r = range to measurement

$f^2(\theta,\phi)$= radiation pattern = 1 on beam axis

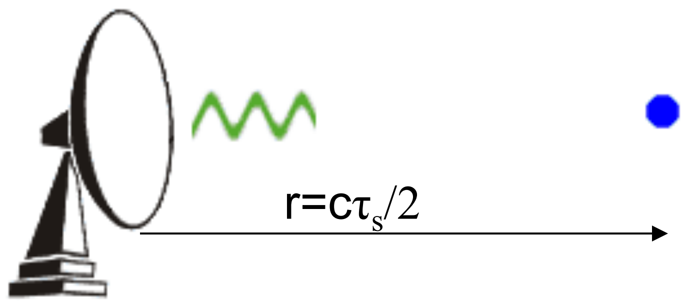
P_t = transmitted power (W)

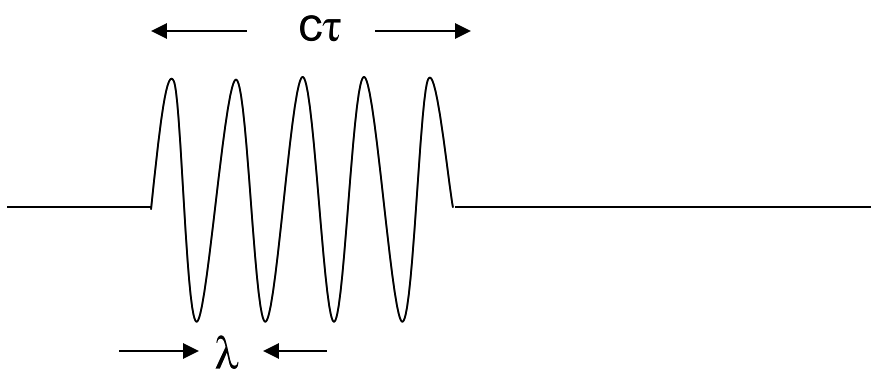
Doppler Radar (Fig. 3.1) A simplified block diagram



Discuss the rf pulse

Pulsed Radar Principle

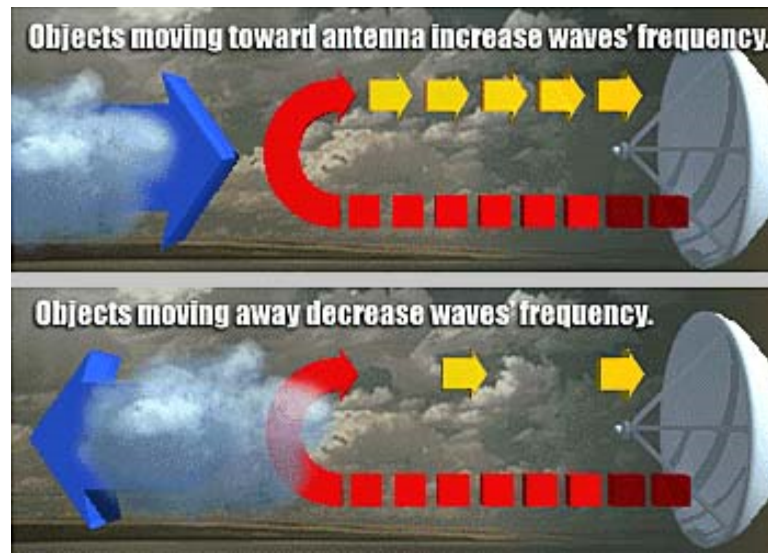


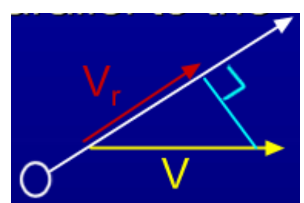


c = speed of microwaves
= c_h for H and = c_v for V waves
τ = pulse length
λ = wavelength
= λ_h for H and λ_v for V waves
τ_s = time delay between transmission of a pulse and reception of an echo.

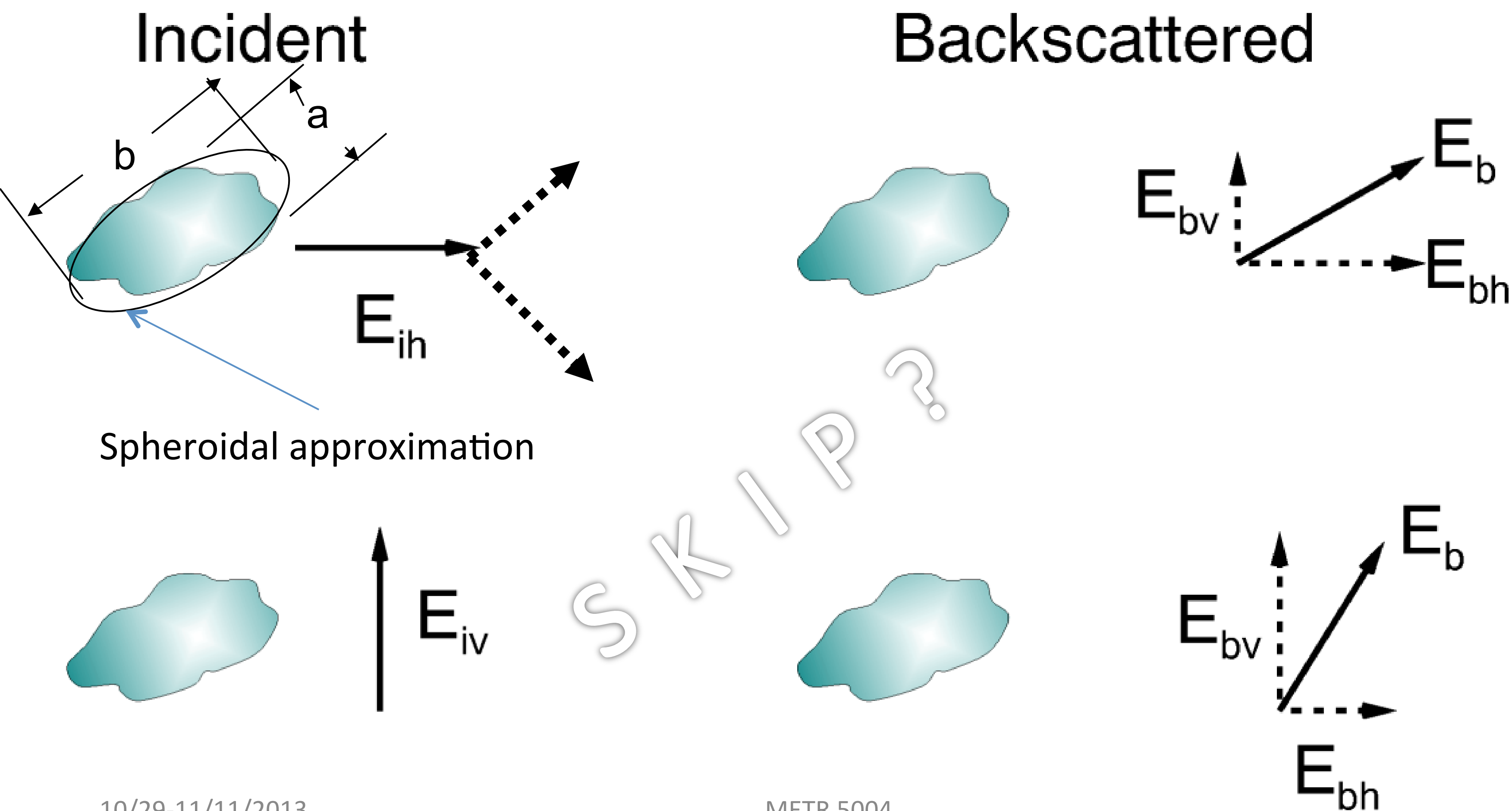
Doppler Radars

- The Doppler effect (Austrian physicist, Christian Johann Doppler,1842) is the apparent change in frequency of a wave that is perceived by an observer moving relative to the source of the waves
- Doppler radars use this phenomenon to measure the radial component of the velocity vector (toward or away from the radar)
  - Note that the radar always measures a velocity that is less than or equal to the true target velocity!





Propagation and backscattering by non spherical precipitation particles



Wavenumber

Phase of a propagating wave: $\omega t - kr$

Wavenumber: $k = 2\pi/\lambda$

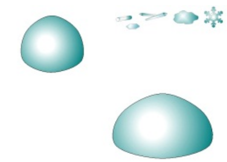
(i.e., $k \equiv$ phase shift per unit length)

In vacuum: $\lambda = c/f$

In rain: $\lambda_r = c_r/f$

$\lambda_r < \lambda$ therefore $k_r > k$

In rain having oblate spheroidal shaped drops:



$\lambda_h = c_h/f$ for H polarized waves

$\lambda_v = c_v/f$ for V polarized waves

Wavenumbers for H, V Waves

Horizontal polarization: $k_{\mathrm{h}} = (k + k'_{\mathrm{h}}) = 2\pi/\lambda_{\mathrm{h}}$

Vertical polarization: $k_{\mathrm{v}} = (k + k'_{\mathrm{v}}) = 2\pi/\lambda_{\mathrm{v}}$

where k = free space wavenumber = 3.6×10^{6} (deg./km)

(e.g., for R= 100 mm h^{-1}, k'_{h} = 24.4^{o}km^{-1}, k'_{v} = 20.7^{o} km^{-1})

Therefore: $c_h < c_v$; $\lambda_h < \lambda_v$; $k_h > k_v$

Specific differential phase:

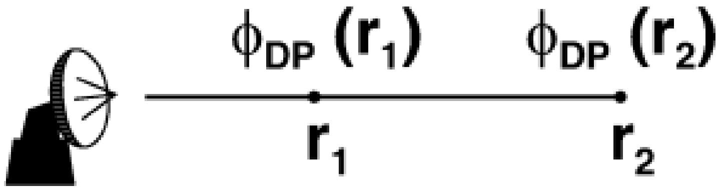
$K_{\mathrm{DP}} = k'_{\mathrm{h}} - k'_{\mathrm{v}} = 3.7(\mathrm{deg.\ km}^{-1})$ (for R = 100 mm h^{-1})

(an important polarimetric variable related to rainrate)

Polarimetric Variables

- Propagation - forward scattering
  - * K_h and K_v - specific attenuations
  - * K_{dif} - specific differential attenuation
  - * Φ_{DP} - differential phase
  - * K_{DP} - specific differential phase

Specific Differential Phase



(Fig.6.17)

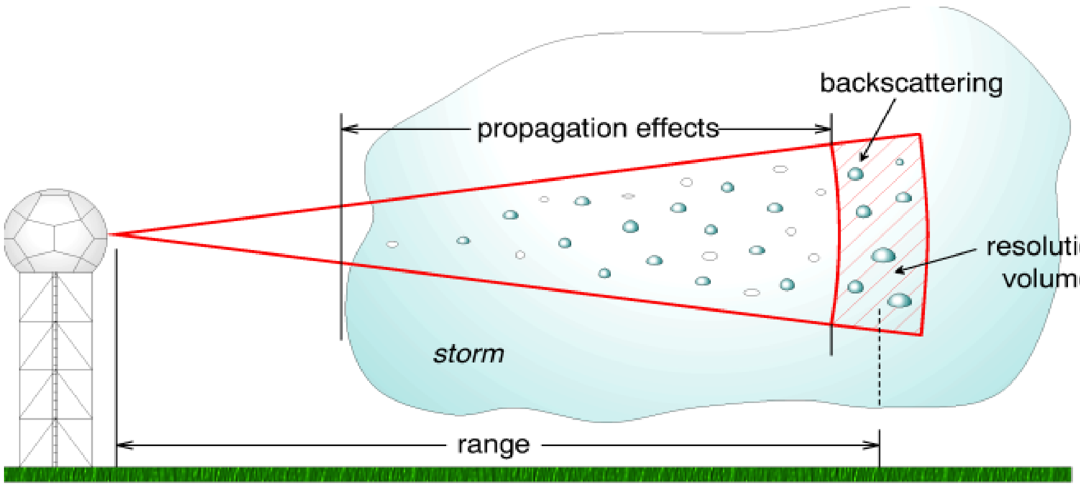
DIFFERENTIAL PHASE SHIFT ϕ_{DP} (two-way) = Phase Lag of H – Phase Lag of V

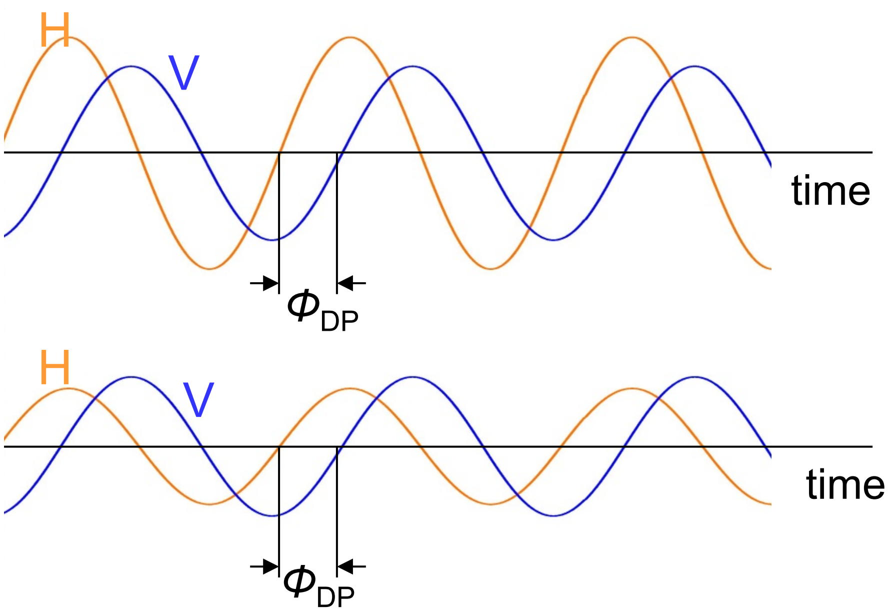
Phase of H = $\phi_h = 2k_h r$

SPECIFIC DIFFERENTIAL PHASE K_{DP} (one-way!)

$$K_{DP} = \frac{d\phi_{DP}}{2\,dr} \approx \frac{\phi_{DP}(r_2) - \phi_{DP}(r_1)}{2\,(r_2 - r_1)}$$ Eq. (6.60)

Differential phase Φ_{DP}





$$H = A_h \exp[j(2\pi f t - \Phi_h)]$$

$$V = A_v \exp[j(2\pi f t - \Phi_v)]$$

$$\Phi_{DP} = \Phi_h - \Phi_v$$

Φ_{DP} is not affected by radar mis-calibration, attenuation, and partial beam blockage

Backscattering Cross Section σ_b

(Echoes from a single discrete scatterer)

Defining equation:

$$S_r(\text{W m}^{-2}) \equiv \frac{S_i \sigma_b}{4\pi r^2} \qquad (3.5)$$

where:

S_i = power density incident on scatterer

S_r = power density at the receiving antenna

Backscattering Cross Section, σ_b for a Spherical Particle

Rayleigh condition on particle diameter D :

$$D < \lambda / 16; \qquad \lambda \equiv \text{ wavelength}$$

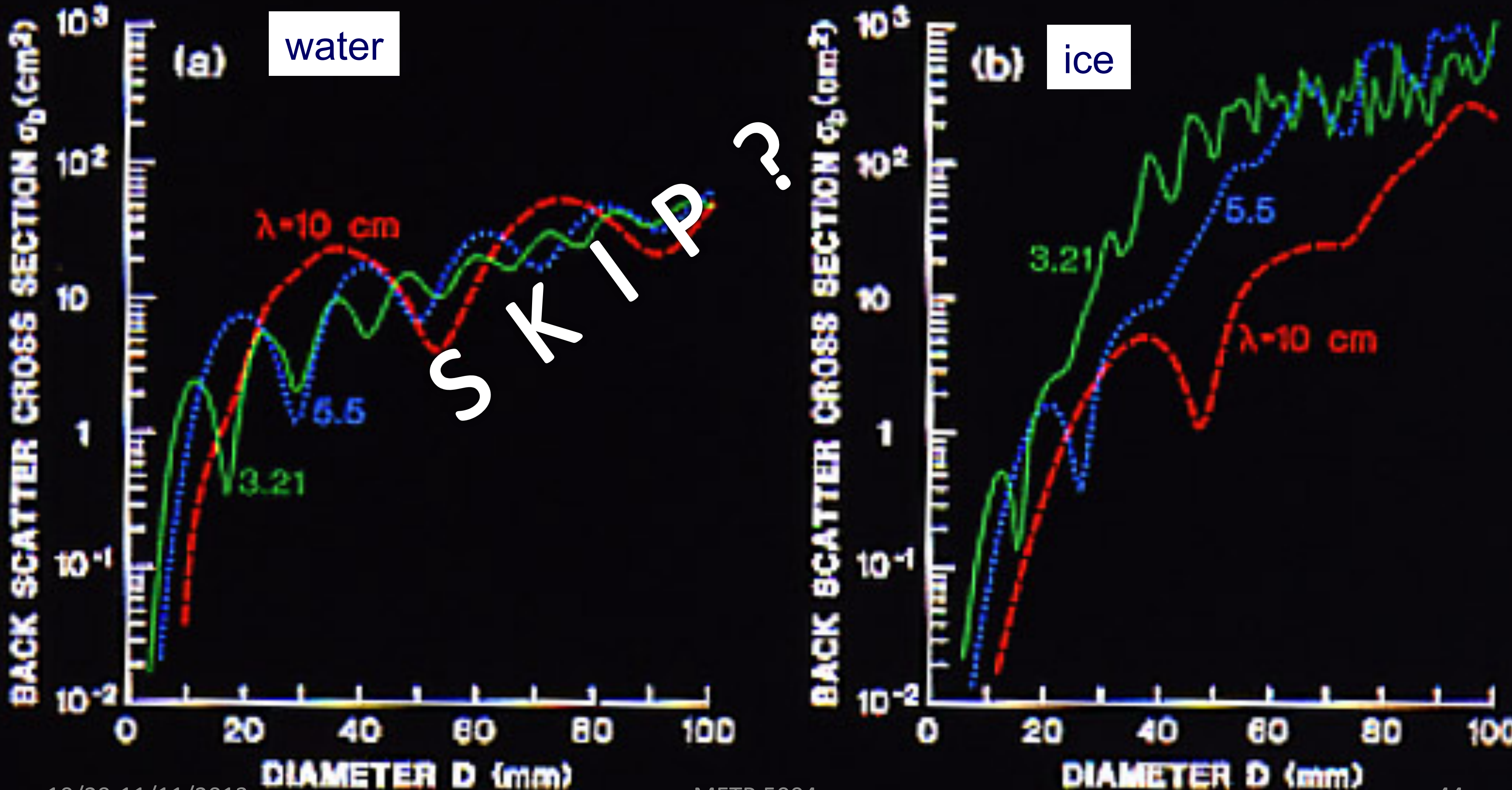
$$\boldsymbol{\sigma_b = \frac{\pi^5}{\lambda^4} | K_m |^2 D^6}; \; K_m \equiv \frac{m^2 - 1}{m^2 + 2}; \text{ Dielectric Factor } (3.6)$$

$m = n - jn\kappa =$ the complex index of refraction

$| K_m |^2 = 0.90 \Rightarrow 0.93$ for water, and

$| K_m |^2 = 0.18$ for ice (density = 0.917 g m^{-3})

Cross Section vs Diameter (Fig. 3.3)



Backscattered Power Density Incident on Receiving Antenna

$$S_r(r,\theta,\phi) = \overbrace{\frac{P_t g_t f^2(\theta,\phi)}{4\pi r^2 l}}^{S_i} \bullet \sigma_b \bullet \frac{1}{4\pi r^2 l} \qquad (3.13a)$$

where l is the loss factor (due to attenuation)

$$l = \exp\left(\int_0^r (k_g + k)\,dr\right) \qquad (3.13b)$$

Echo Power P_r Received

$$P_r = S_r(r,\theta,\phi)A_e(\theta,\phi) \quad (3.20)$$

A_e is the effective area of the receiving antenna for radiation from the θ,φ direction. It is shown that:

$$A_e = g_r f_r^2(\theta,\phi)\lambda^2 / 4\pi \qquad (3.21)$$

If the transmitting antenna is the same as the receiving antenna then:

$$g_r f_r^2(\theta,\phi) = g_t f_t^2(\theta,\phi) \equiv g f^2(\theta,\phi)$$

The Radar Equation
(point scatterer/discrete object)

$$P_r = \frac{P_t g f^2(\theta,\varphi)}{4\pi r^2 l} \bullet \frac{\sigma_b}{4\pi r^2 l} \bullet \frac{g\lambda^2 f^2(\theta,\varphi)}{4\pi} \qquad (3.24)$$

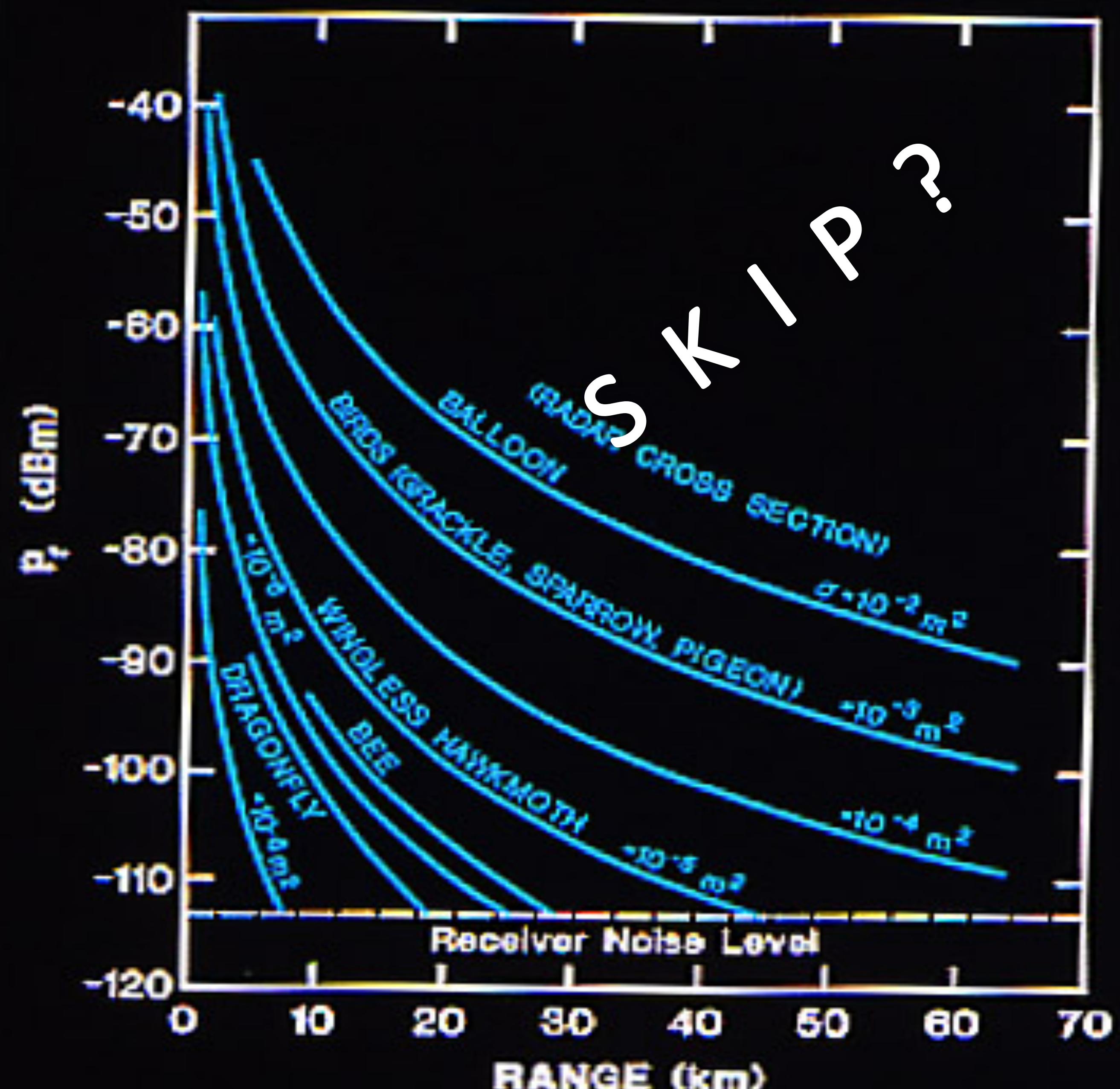
Example:

$\lambda = 0.1\,\mathrm{m};\quad r = 20\,\mathrm{km}\,(2\mathrm{x}10^4\mathrm{m});\quad P_r(\mathrm{min}) = 10^{-14}(\mathrm{W});$

$P_t = 10^6(\mathrm{W;\ peak});\quad g = 3x10^4;\ l = 1(\text{no path loss})$

Calculating the minimum detectable backscattering σ_b :

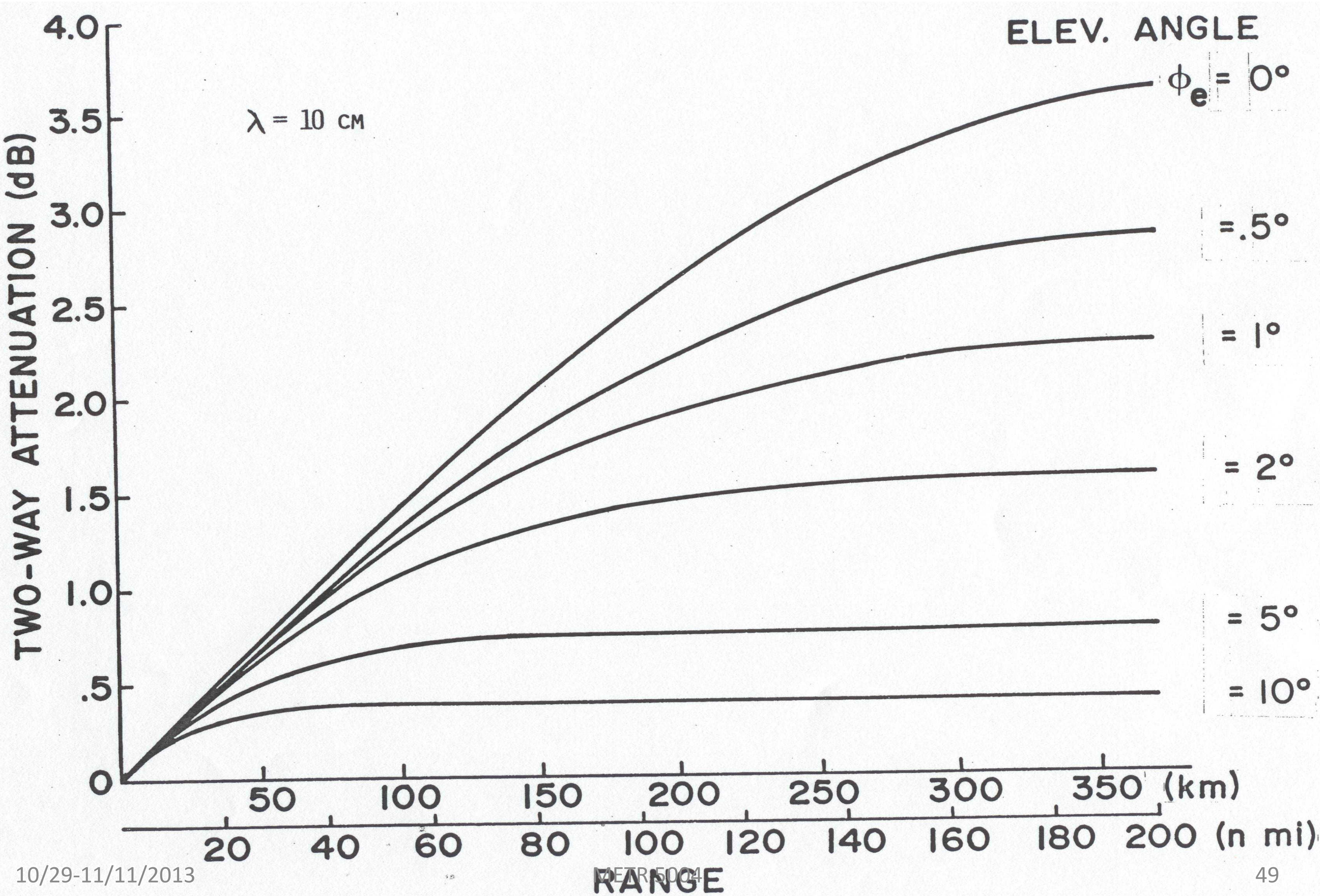
$\sigma_b(\mathrm{min}) = 2x10^{-7}\ \mathrm{m}^2 = \sigma_b$ for a 6.3 mm drop!



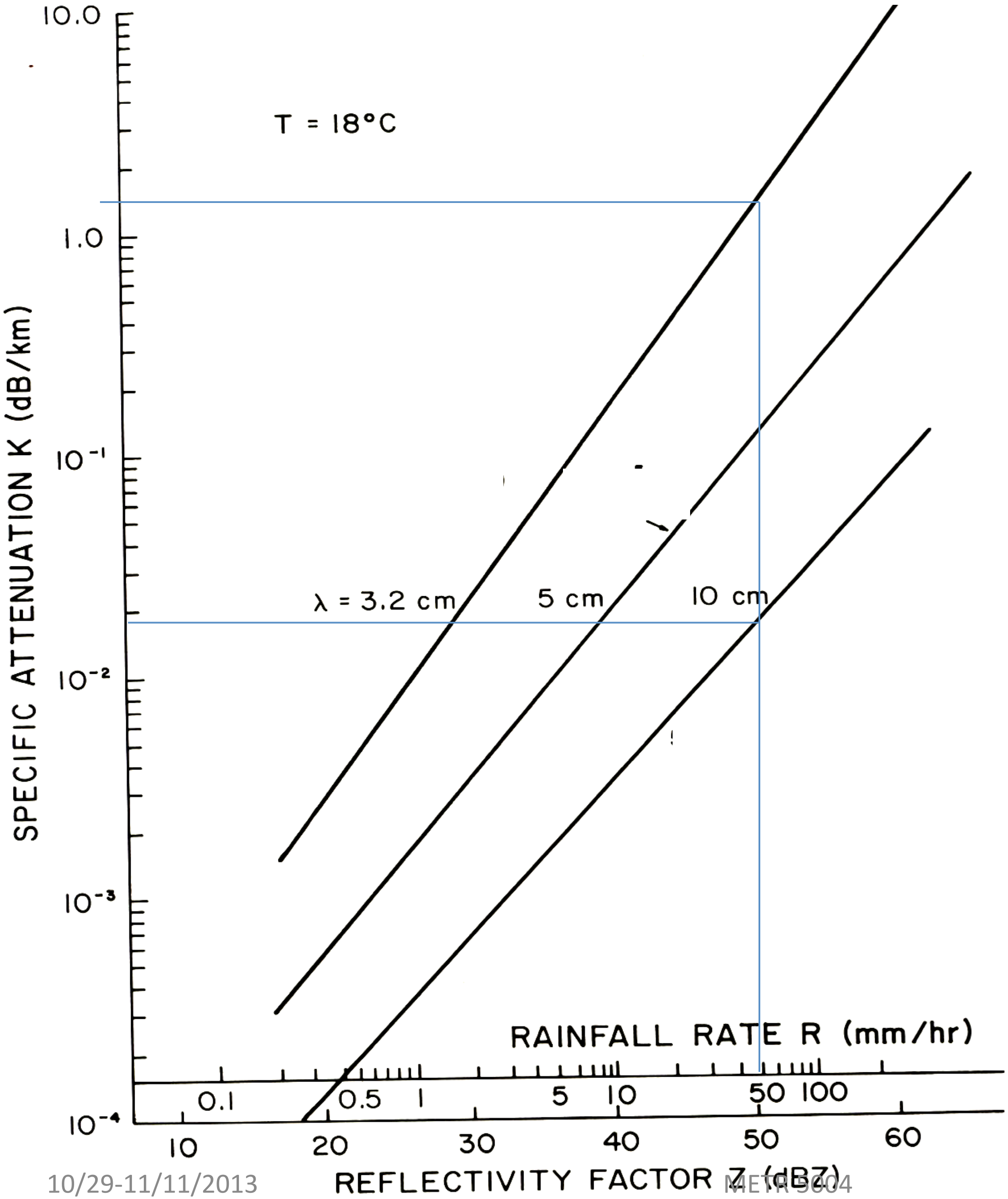
S K I P ?

Echo Power from Point Scatterers

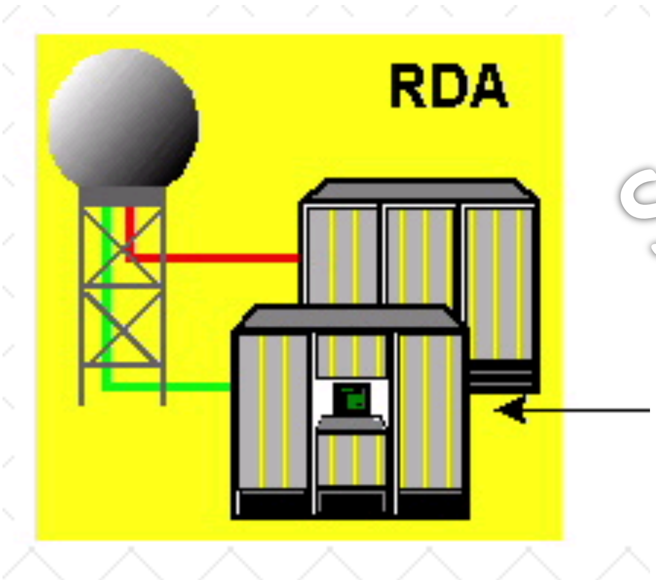
Atmospheric Attenuation (Fig. 3.6)



Attenuation vs Rain Rate (Fig. 3.5)

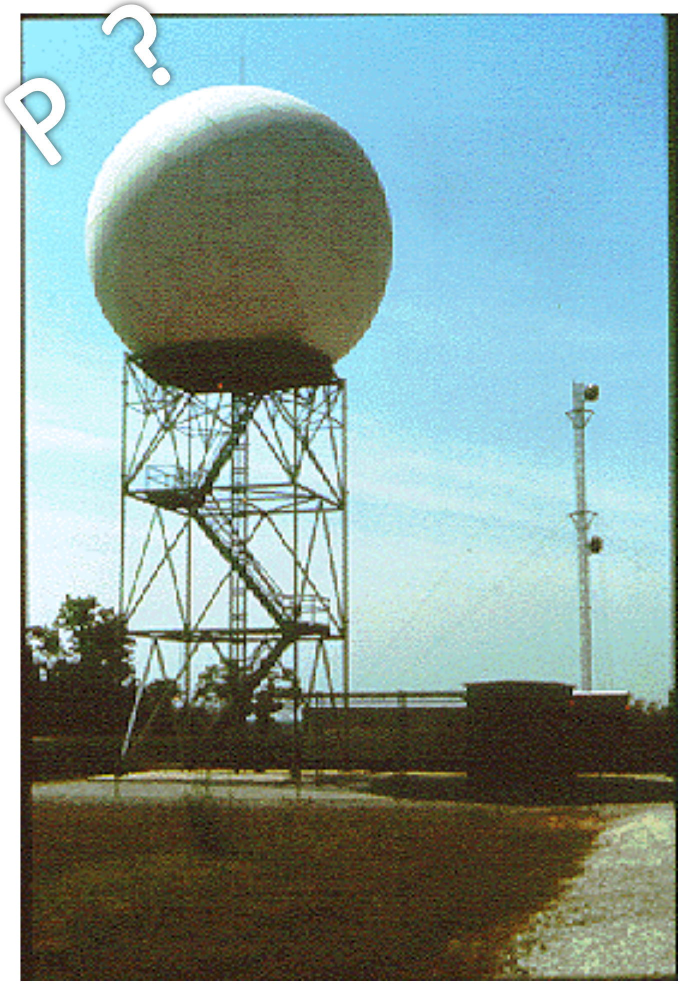


WSR-88D Components



Radar Data
Acquisition

S K I P ?



WSR-88D **Specifications**
(Table 3.1; antenna subsystem)

Radome

Diameter: 11.89 m

- loss: 0.3 dB (two way); 7% of power is lost

Reflector

Diameter: 8.54 m

Polarization: Dual H, V

Gain: 44.5 dB

Beam width: 1^{o}

SKIP ?

Pedestal:

Scanning rate: 30 deg./sec (max.; El. and Az.)

Mechanical limits: -1^{o} to 60^{o} El.

Table 3.1 (cont.)
WSR-88D Transmitter

- Type: Master oscillator power amplifier
- Frequency: 2700 to 3000 MHz
- Pulse power: 475 kW (peak)
- Pulse width: 1.57 and 4.57 microseconds
- Average power: 1 kW
- PRFs:
  - Short pulse: eight selectable 320 to 1300 Hz
  - Long pulse: 320 to 450 Hz

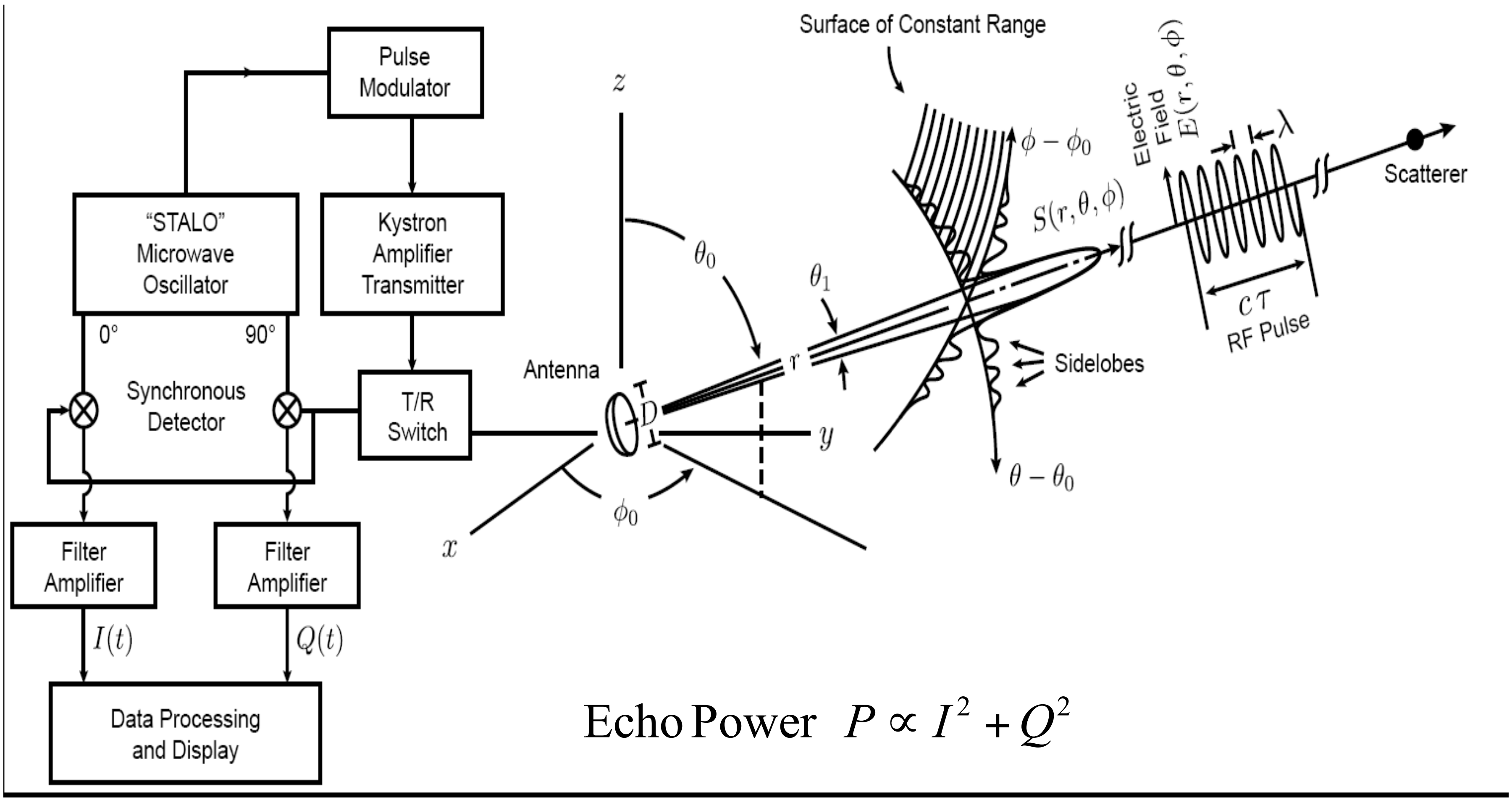
SKIP?

Table 3.1 (cont.)
WSR-88D Receiver

S K I P ?

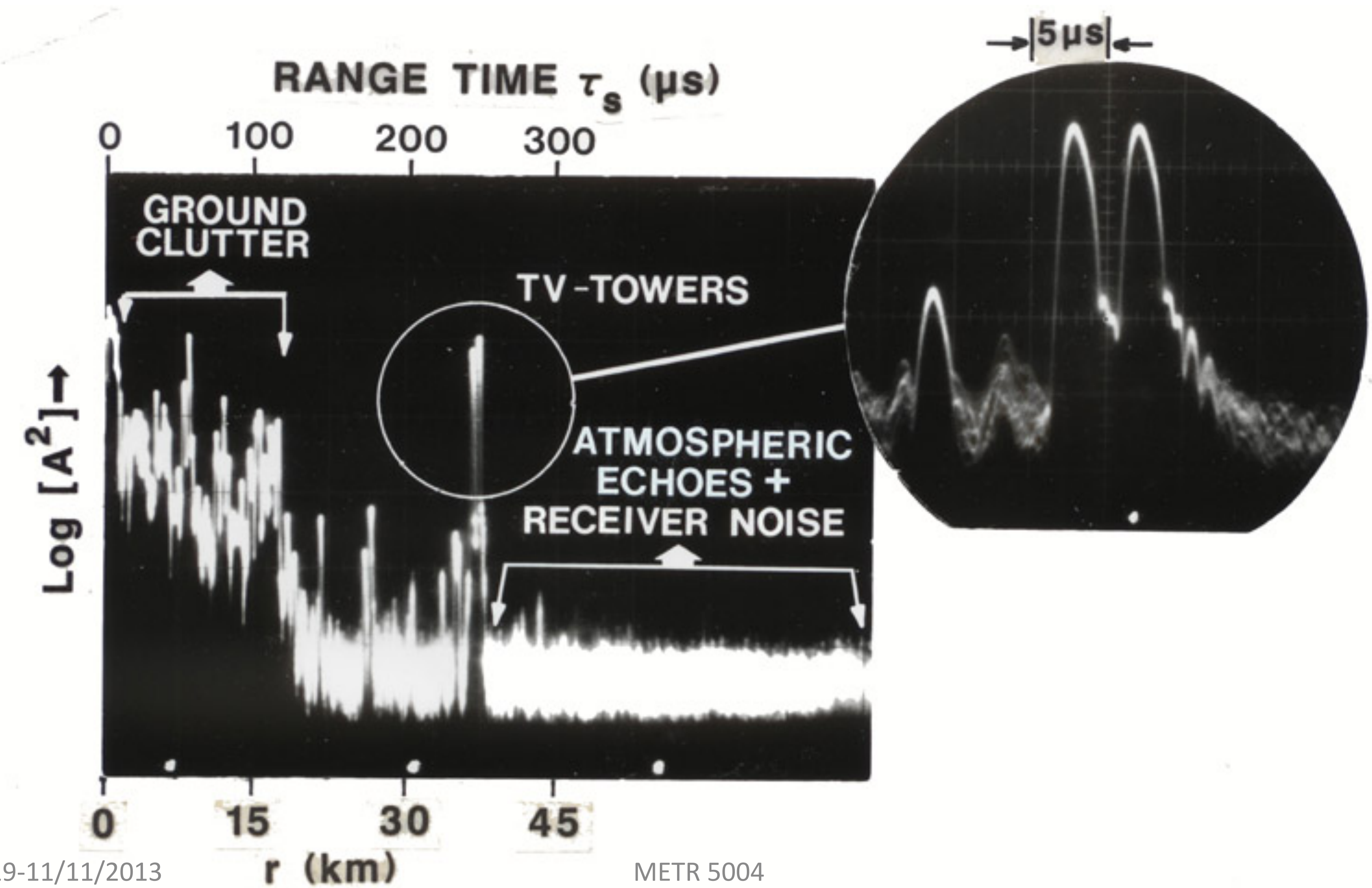
- Type: Linear
- Dynamic Range: 93 dB
- Intermediate frequency: 57.6 MHz
- System noise power: -113 dBm (5×10^{-15} W)
- Bandwidth:
  - Short pulse (matched filter):~ 0.6 MHz (3 dB pts.)---range resolution ~250m
  - Long pulse: (matched filter): ~0.2 MHz;~750m

Doppler Radar Block Diagram (repeat of Fig. 3.1)

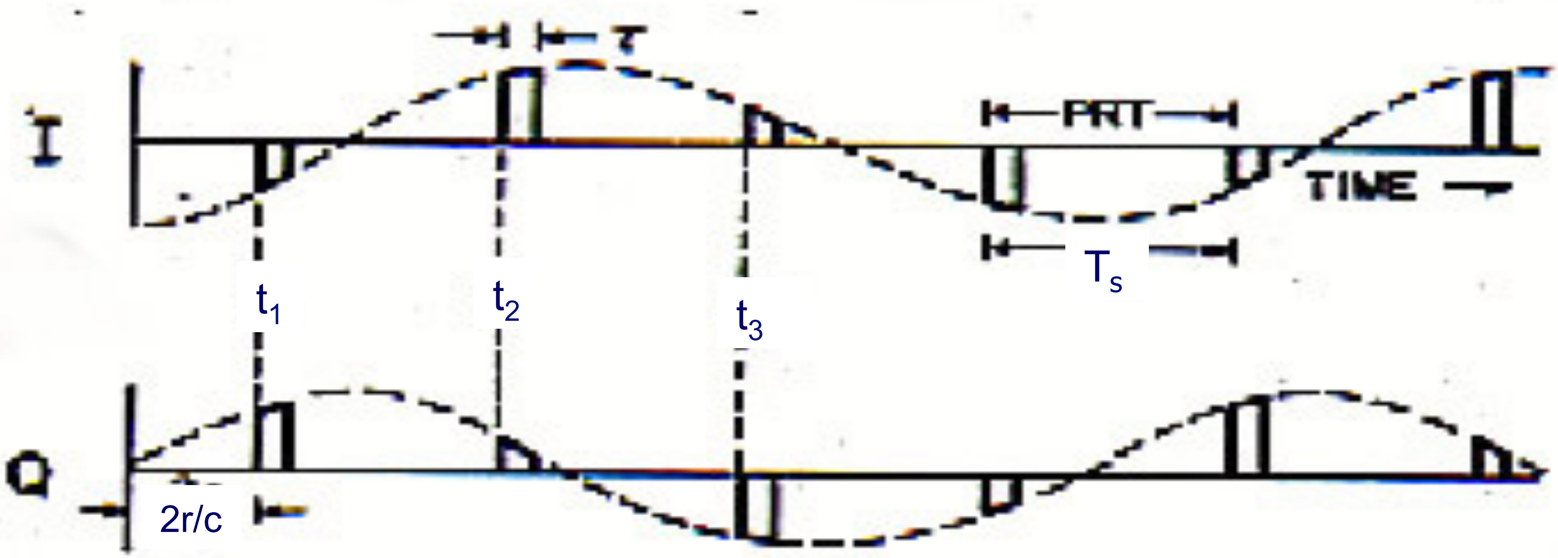


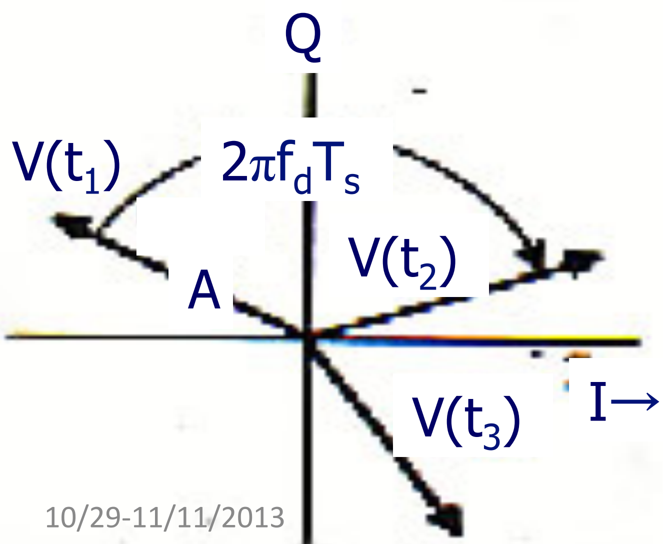
$$\text{Echo Power} \quad P \propto I^2 + Q^2$$

Echo Power vs Range Time (Fig. 3.7)



Echoes from a Moving Scatterer





$A=(I^2+Q^2)^{1/2}$

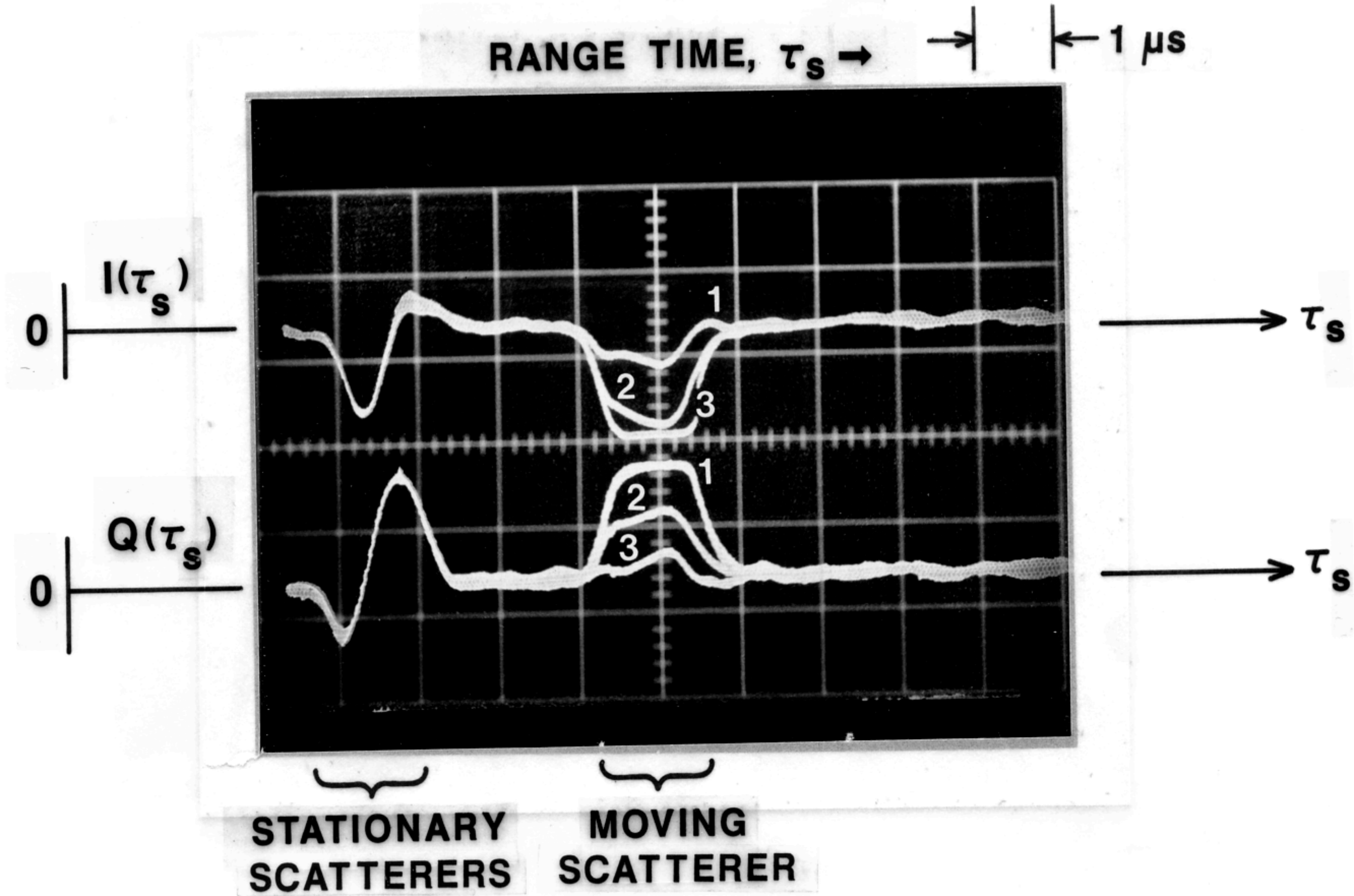
$V(t_n)=Ae^{j2\pi f_d t_n}\,U(t_n-2r/c)$

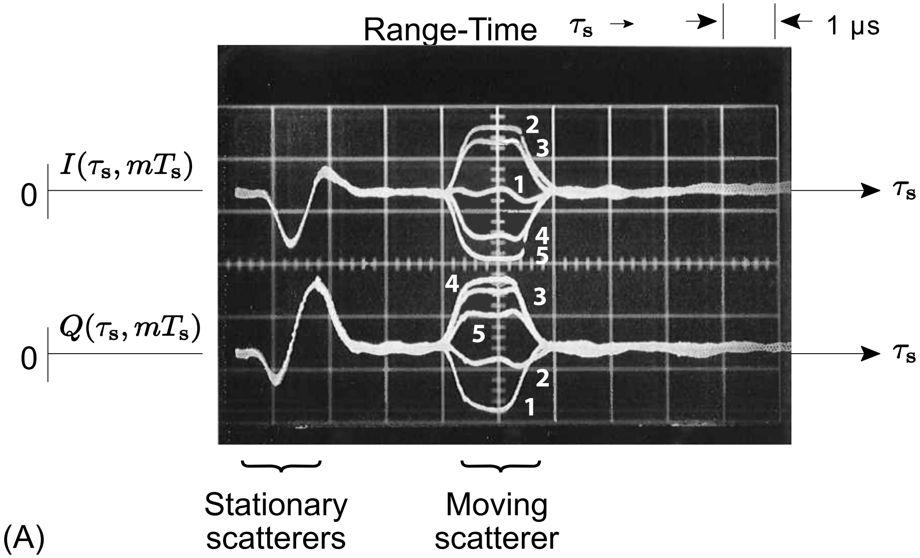
$t_n=n\cdot PRT$

$U(t_n)$=unit pulse function

Negative Doppler shift

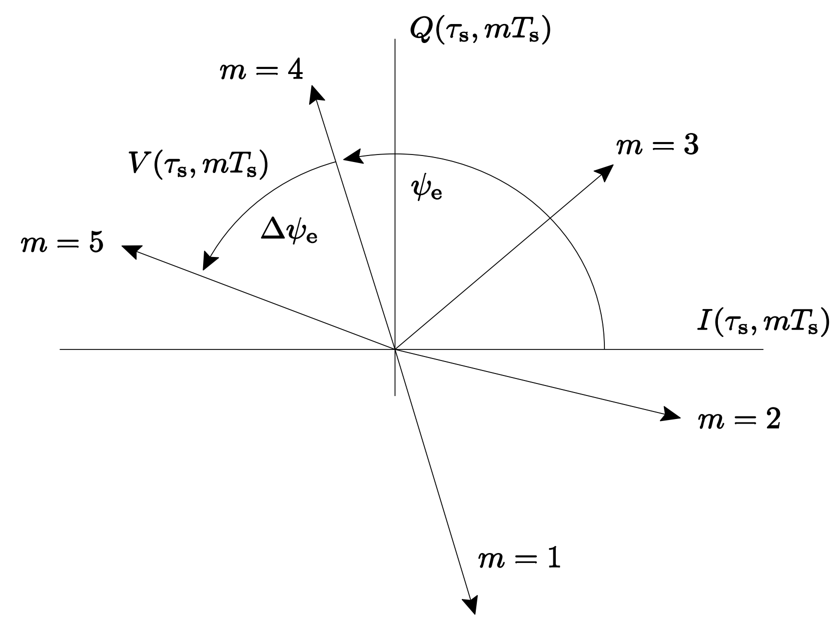
I and Q vs Range Time (Fig. 3.9)





Range-Time
τ_s
1 μs
$I(\tau_s, mT_s)$
0
$Q(\tau_s, mT_s)$
0
τ_s
τ_s
2
3
1
4
5
4
3
5
2
1
Stationary scatterers
Moving scatterer

(A)

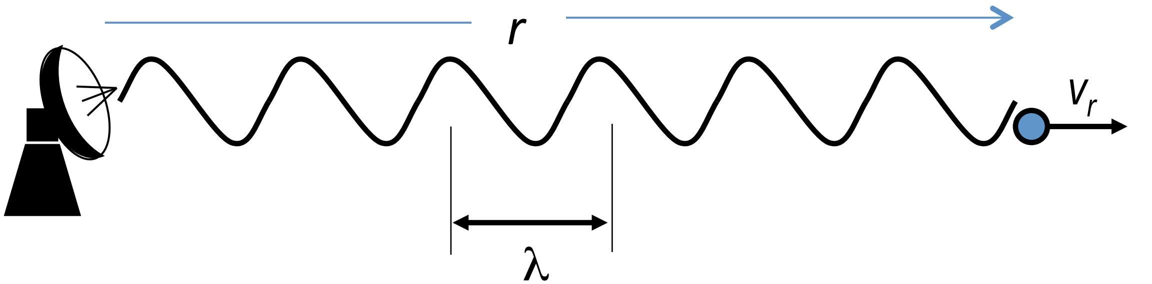


$Q(\tau_s, mT_s)$
$m = 4$
$m = 3$
$V(\tau_s, mT_s)$
ψ_e
$\Delta\psi_e$
$m = 5$
$I(\tau_s, mT_s)$
$m = 2$
$m = 1$

(B)

Doppler Frequency Shift

- If the range to the target is r, the total number of wavelengths in the two-way path is $2r/\lambda$. Since each wavelength corresponds to a phase change of 2π, the total phase change is $\psi_e = 4\pi r/\lambda$

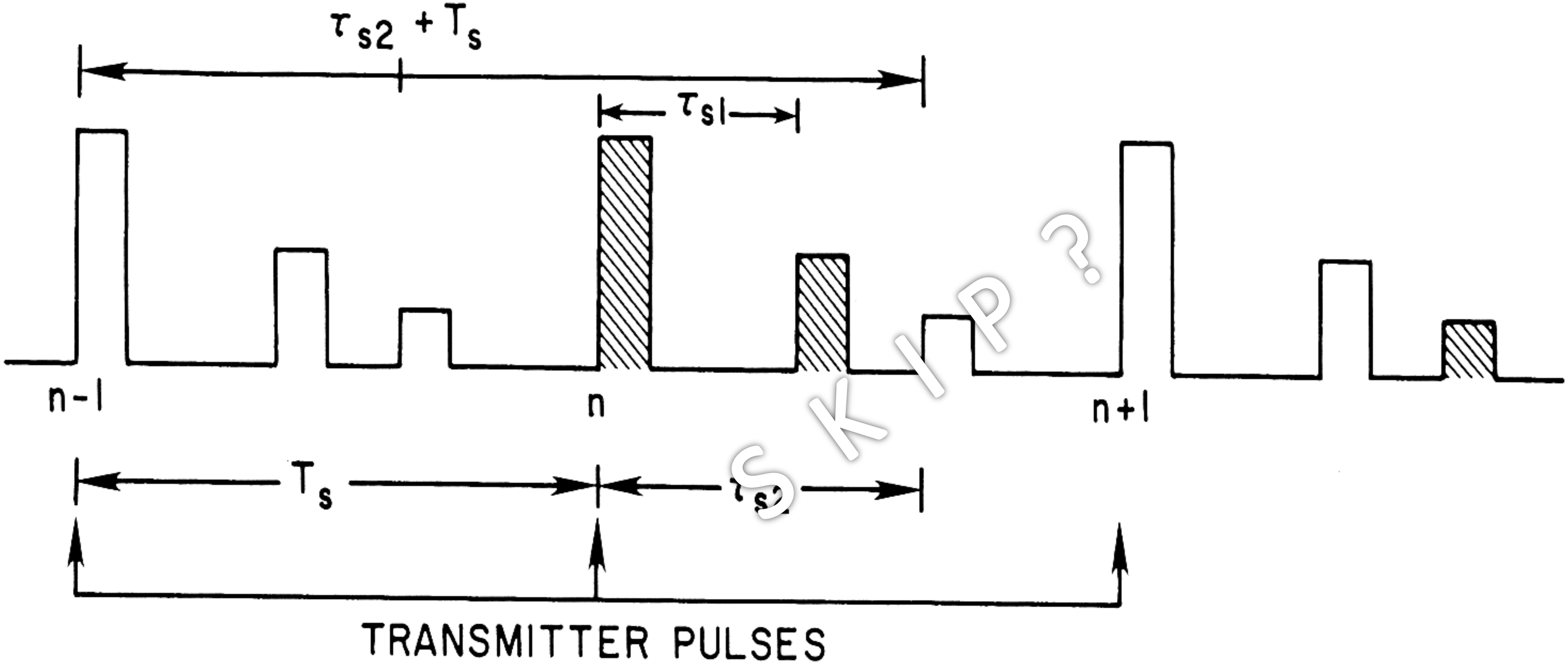


The Doppler frequency is the rate of change of ψ_e

$$\omega_d = \frac{d\psi_e}{dt} = \frac{4\pi}{\lambda}\frac{dr}{dt} = \frac{4\pi}{\lambda}v_r = 2\pi f_d \text{ (radians/sec)} \qquad (3.30)$$

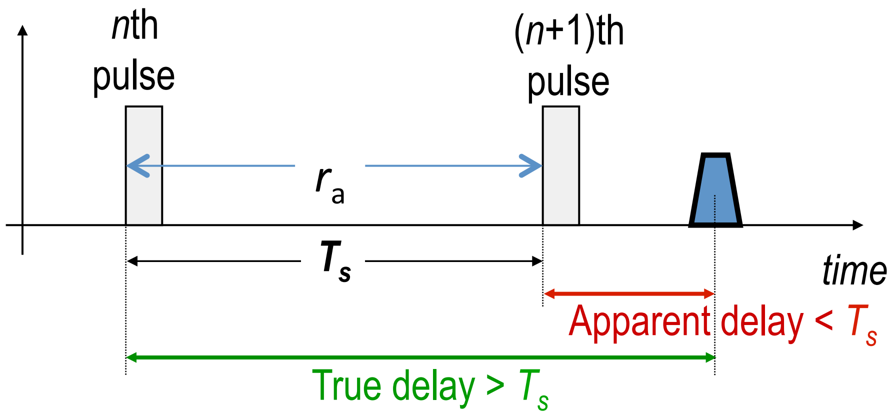
Doppler frequency (cycles/sec) $\equiv f_d = \omega_d / 2\pi = 2v_r / \lambda$

Range Ambiguous Echoes (Fig. 3.13)



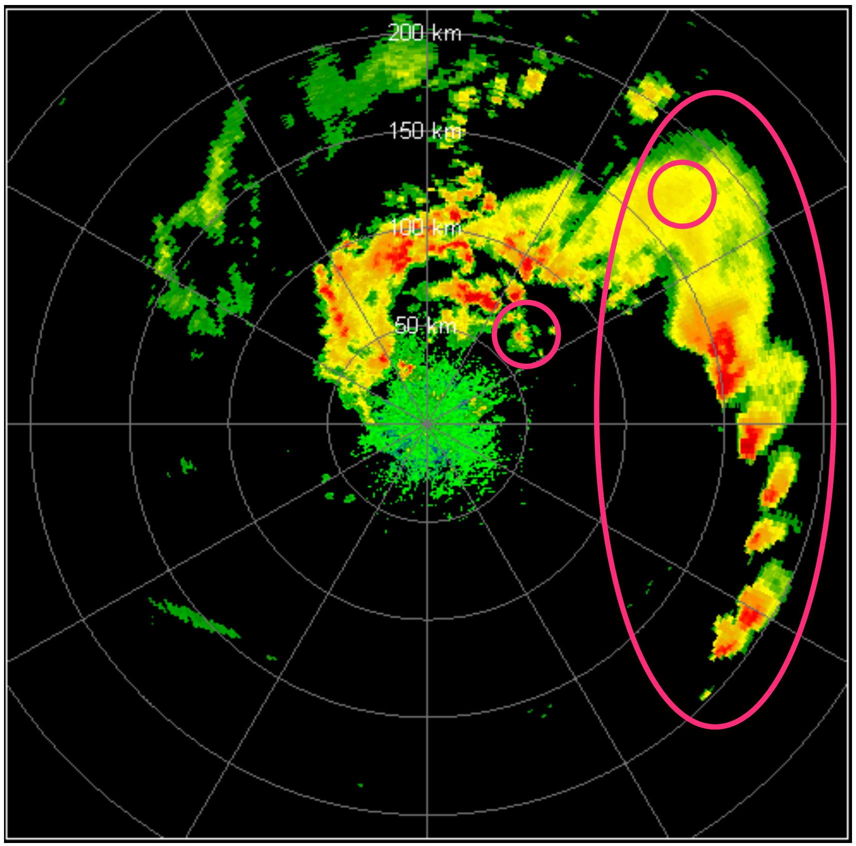
Unambiguous Range r_a

- If targets are located beyond $r_a = cT_s/2$, their echoes from the n^{th} transmitted pulse are received after the $(n+1)^{th}$ pulse is transmitted. Thus, they appear to be closer to the radar than they really are!
  - This is known as **range folding**
- T_s = PRT

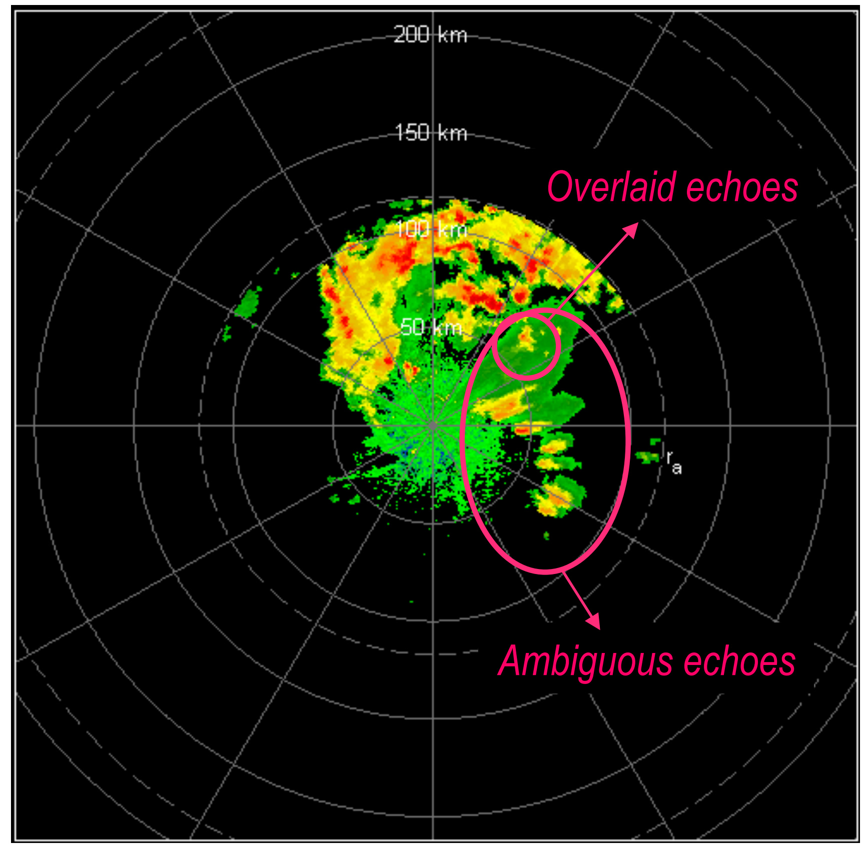


- **Unambiguous range**: $r_a = cT_s/2$
  - Echoes from scatterers between 0 and r_a are called **1st** trip echoes,
  - Echoes from scatterers between r_a and $2r_a$ are called **2nd trip** echoes, Echoes from scatterers between $2r_a$ and $3r_a$ are called **3rd trip** echoes, etc

Range Ambiguities
(range rings spaced at 50 km)

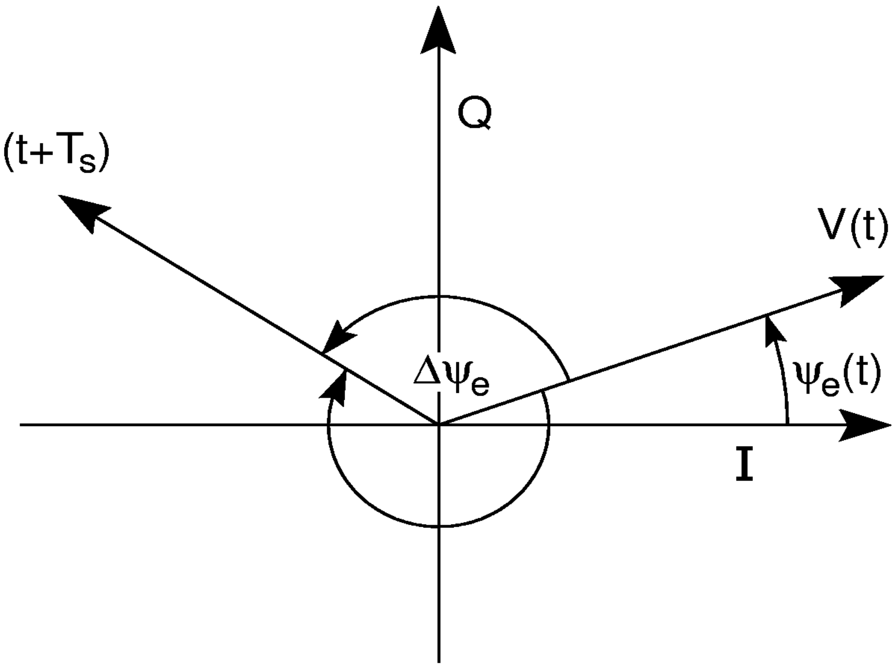


T_s = 3.1 ms and r_a = 466 km



T_s = 780 μs and r_a = 117 km

Ambiguous Doppler Shifted Echoes (Fig. 3.14)



frequency aliases are:

$$\frac{\Delta\psi_e}{2\pi T_s} + \frac{n}{T_s} \quad \frac{\text{cycles}}{\text{sec}}$$

$n = 0, \pm1, \pm2, \ldots..$

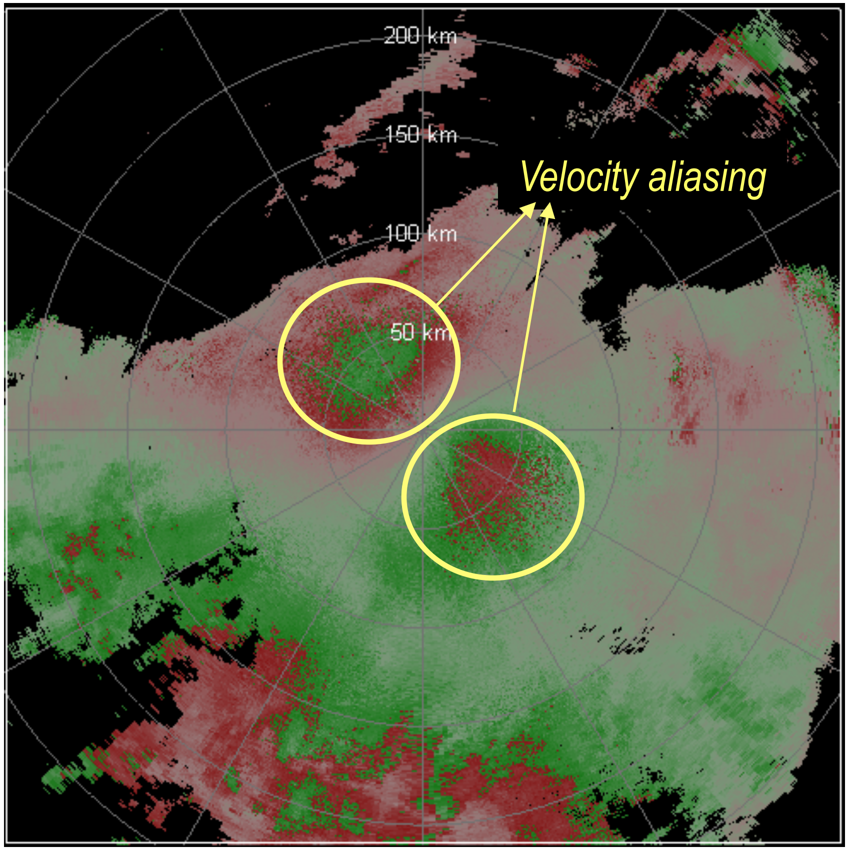
Unambiguous Velocity

- A pulsed Doppler radar measures radial Doppler velocity by keeping track of phase changes between samples that are T_s (pulse repetition time) apart
- Recall that the phase shift is $\psi_e = -4\pi r/\lambda$. Then, the phase change from pulse to pulse is $\Delta\psi_e = -4\pi\Delta r/\lambda = -4\pi v_r T_s/\lambda$
  - Note that only phase changes between $-\pi$ and π can be unambiguously resolved
- Therefore, the unambiguous velocity is:
  - $4\pi v_a T_s/\lambda = \pi \Rightarrow v_a = \lambda/4T_s$
  - This is related to the **Nyquist** sampling theorem:
    Doppler velocities outside the $\pm v_a$ interval will be aliased!

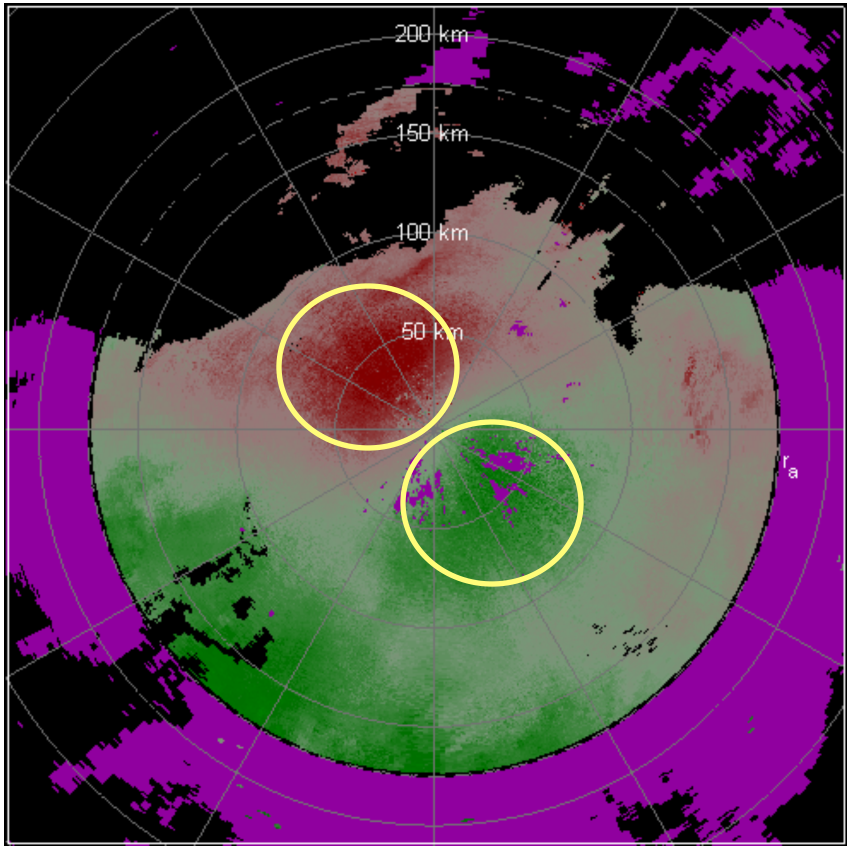
$\Delta r = v_r T_s$ is the change in range of the scatterer between successive transmitted pulses

30

Velocity Ambiguities



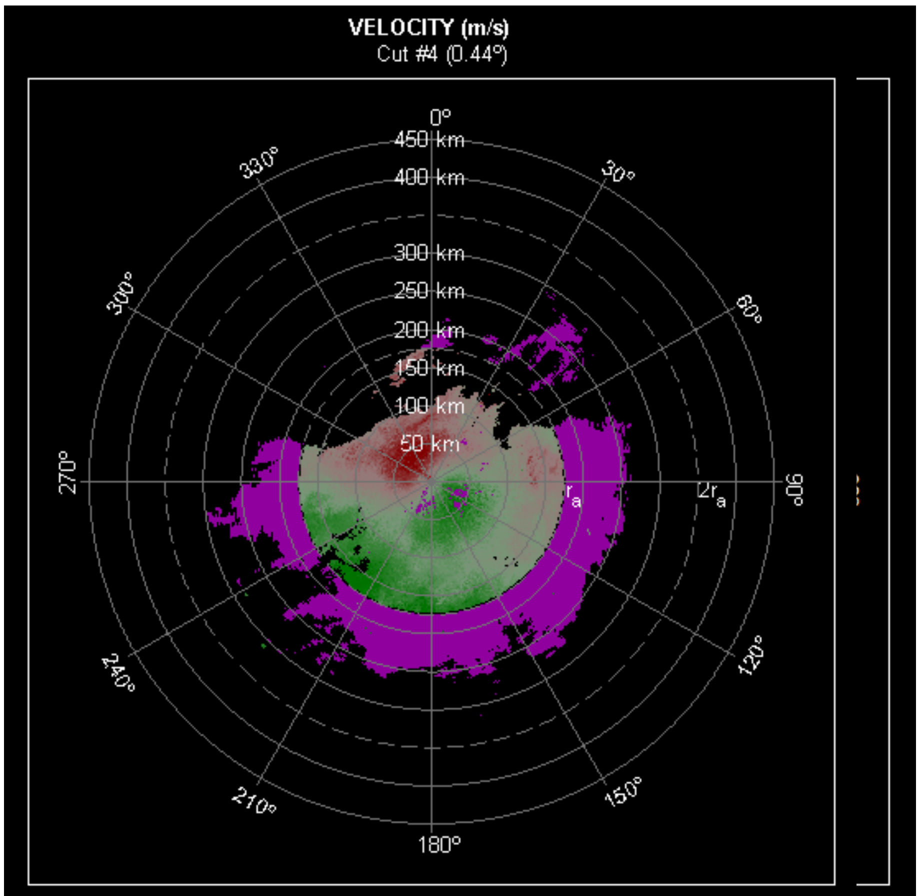
T_s = 3.1 ms and v_a = 8.9 m/s



T_s = 1.167 ms and v_a = 23.75 m/s

Overlaid Echo Censoring

in the WSR-88D network



- Overlaid echoes that have an unrecoverable velocity are censored:
  - Velocity is unrecoverable due to overlaid echoes typically observed in the short PRT
  - Velocity associated with strong trip echoes can be recovered if the ratio of strong to weak overlaid powers is P_s/P_w > 10 dB
  - Velocities associated with weak echoes can not be recovered and are tagged with purple color

Range and Velocity Ambiguities on Pulsed Weather Radars

- Unambiguous range
  - $r_a = cT_s/2$
- Unambiguous Doppler velocity
  - $v_a = \lambda/(4T_s)$
- The Doppler Dilemma: $r_a v_a = c\lambda/8$
  - Insufficient to observe severe convective storms at practical wavelengths
  - NEXRAD specifications:
    λ = 10 cm
    r_a = 230 km ⟹ $v_a \approx 16$ m s^{-1}
  - This problem is worse for smaller wavelengths!

☺ There are techniques (Chap. 7) to mitigate the Doppler dilemma; however there is a more basic constraint (next)

31

thanks to --------

Another PRT Trade-Off

- Correlation of pairs: $\rho(T_s) = \exp\left[-8\left(\pi\sigma_v T_s / \lambda\right)^2\right]$
  - This is a measure of signal coherency
- Accurate measurement of power requires long PRTs
  - $\lim_{T_s \to \infty} \rho(T_s) = 0$
  - More **independent samples** (low coherency)
- But accurate measurement of velocity requires short PRTs
  - $\lim_{T_s \to 0} \rho(T_s) = 1$
  - High correlation between sample pairs (high coherency)
  - Yet a large number of **independent sample pairs** are required

Signal Coherency

- How large a T_s can we pick?
  - Correlation between *m = 1* pairs of echo samples is:

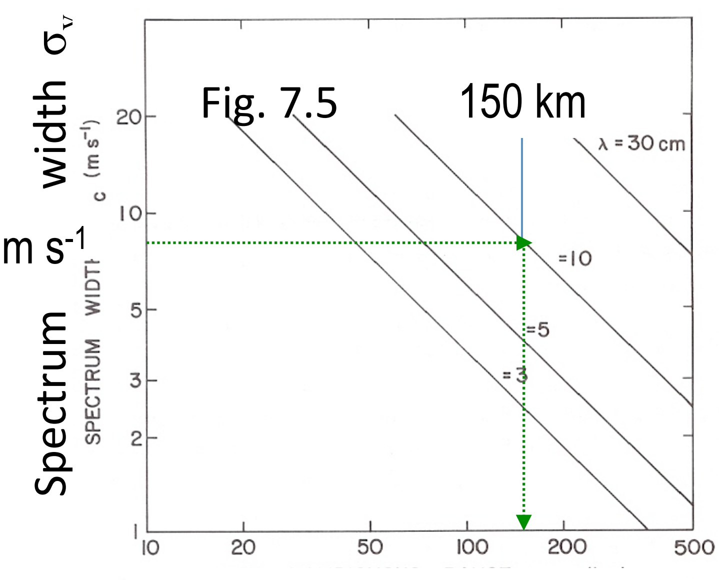
$$\rho(T_s) = \exp\left[-8\left(\pi\,\sigma_v\,T_s/\lambda\right)^2\right]$$

  - Correlated pairs: $\rho(T_s) \approx 1 \Rightarrow \dfrac{\pi\sigma_v T_s}{\lambda} << 1 \Rightarrow \dfrac{\lambda}{\pi T_s} >> \sigma_v$

    (i.e., Spectrum width σ_v must be much smaller than unambiguous velocity $v_a = \lambda/4T_s$)
- Increasing T_s decreases correlation exponentially
  - $\mathrm{Var}[\hat{v}]$ and $\mathrm{Var}[\hat{\sigma}_v]$ also increases exponentially!
- Pick a threshold:
  - $\rho(T_s) \geq e^{-0.5} \Rightarrow -8\left(\pi\sigma_v T_s/\lambda\right)^2 = 0.5 \Rightarrow \sigma_v \leq v_a/\pi$
  - Violation of this condition results in very large errors of estimates!

Signal Coherency and Ambiguities

- Range and velocity dilemma: $r_a v_a = c\lambda/8$
- Signal coherency: $\sigma_v \leq v_a/\pi$
- r_a constraint: $r_a \leq \dfrac{c\lambda}{8\pi\sigma_v}$ Eq. (7.2c)
  - This is a more basic constraint on radar parameters than the first equation above
    - Then, σ_v and not v_a imposes a basic limitation on Doppler weather radars
  - Example: **Severe storms have a median σ_v ~ 4 m/s and 10% of the time σ_v > 8 m/s**. If we want accurate **Doppler** estimates 90% of the time with a 10-cm radar (λ = 10 cm); then, $r_a \leq$ 150 km. This will often result in range ambiguities



Unambiguous range r_a