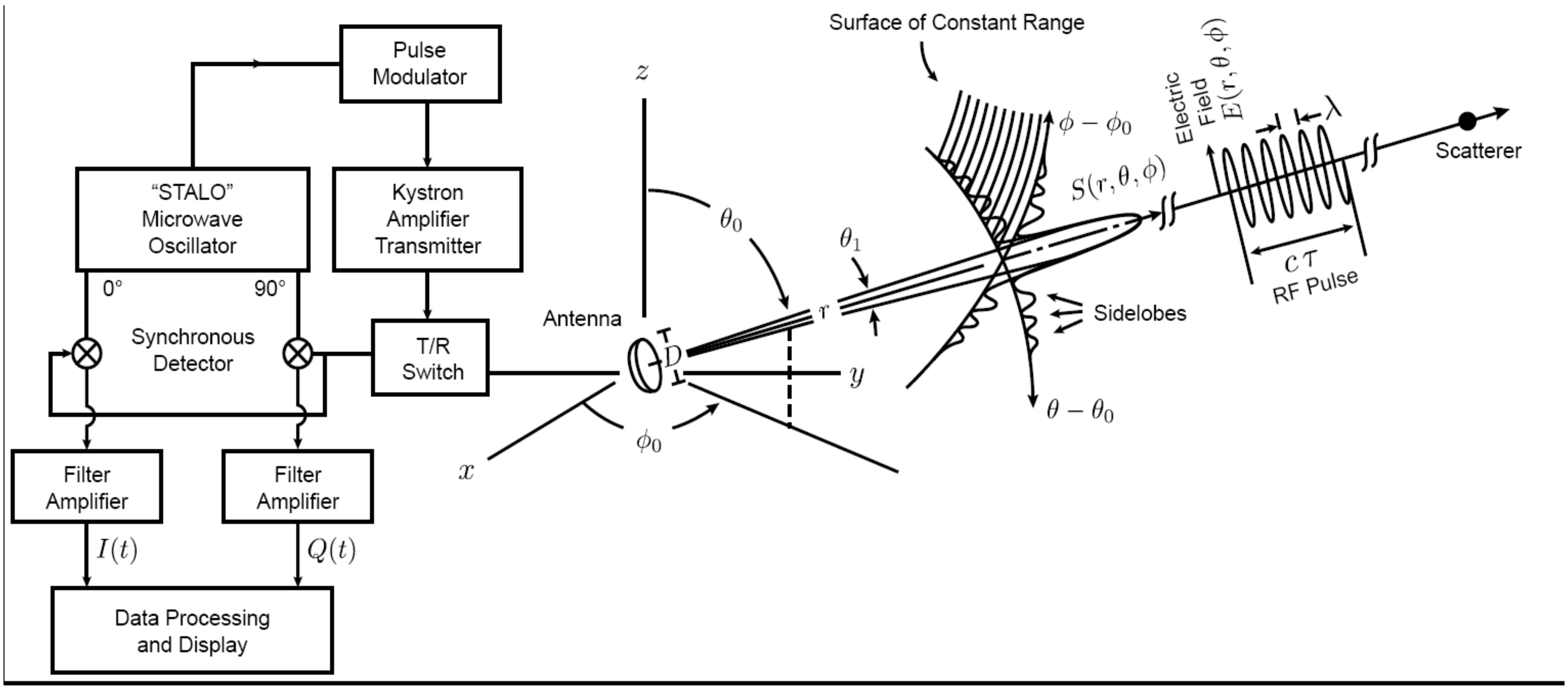
Review
Doppler Radar (Fig. 3.1)
A simplified block diagram



$$E_{\mathrm{i}} = \frac{\boldsymbol{A}_{\mathrm{i}}(\theta,\varphi)}{r} \exp\left[j2\pi f \left(t - \frac{r}{c} \right) + j\psi_t \right]$$

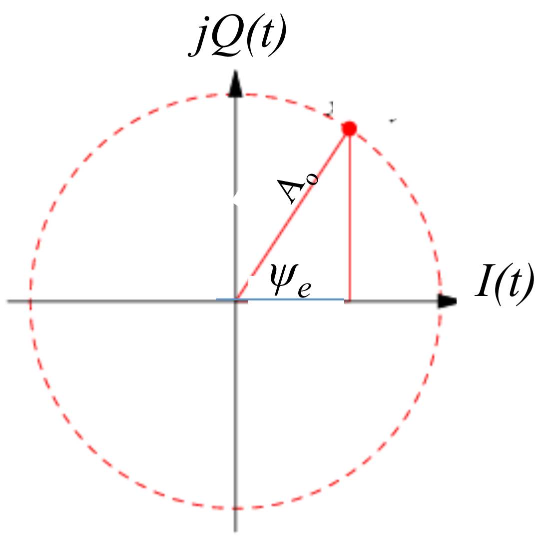
Electric field incident on scatterer

$$\boldsymbol{E}_{\mathrm{r}} = \frac{\boldsymbol{A}_{\mathrm{r}}(\theta,\varphi)}{r^2} \exp\left[j2\pi f \left(t - \frac{2r}{c} \right) + j\psi_t \right]$$

Reflected electric field incident on antenna

$$V_{\mathrm{i}} = A_{\mathrm{i}} \exp\left[j2\pi ft - j\frac{4\pi r}{\lambda} + j\psi_t \right]$$

Voltage input to the synchronous detectors;
This pair of detectors shifts the frequency *f* to 0



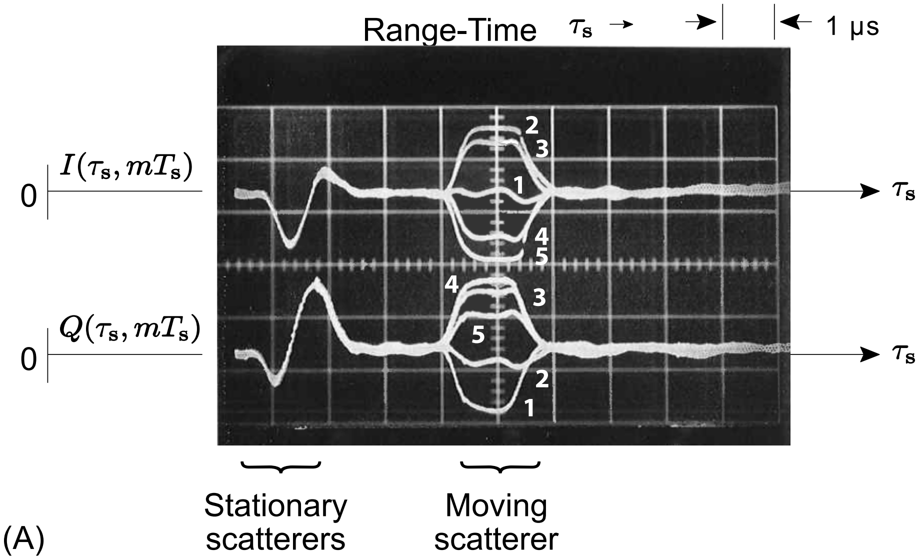
Complex plane
(Phasor diagram)

$$V_{\mathrm{o}} = I + jQ \propto A_{\mathrm{o}} \exp\left[-j4\pi r / \lambda + j\psi_t \right]$$

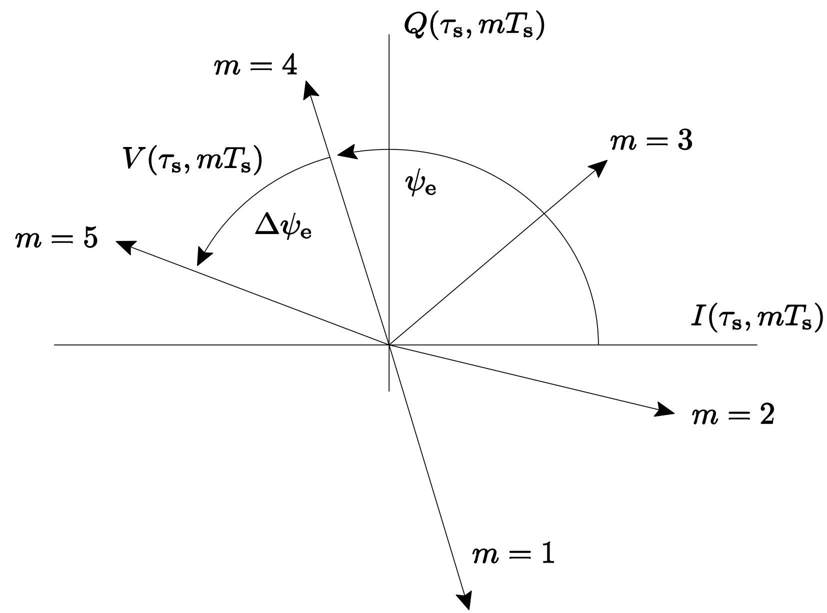
Echo voltage V_o at the output of the detectors and filters . The echo amplitude is A_o and phase is

$$\psi_{\mathrm{e}} = (-4\pi r / \lambda) + \psi_t$$

If the range *r* of the scatterer is fixed, the phasor (A_o, ψ_e) is fixed (i.e., no change in A_o nor ψ_e. But if scatterer has a radial velocity, phasor (A_o, ψ_e) rotates about the origin at the Doppler frequency f_d.

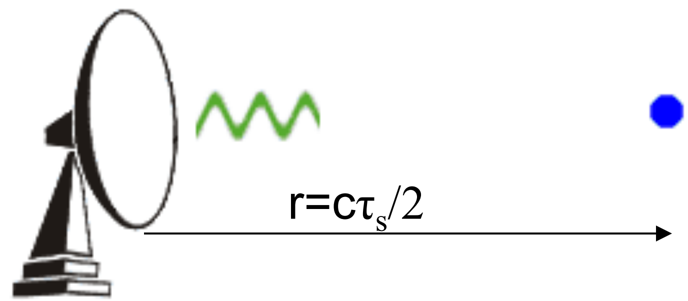

Range-Time
τ_s
1 μs
$I(\tau_s, mT_s)$
0
$Q(\tau_s, mT_s)$
0
τ_s
τ_s
2
3
1
4
5
4
3
5
2
1
Stationary scatterers
Moving scatterer

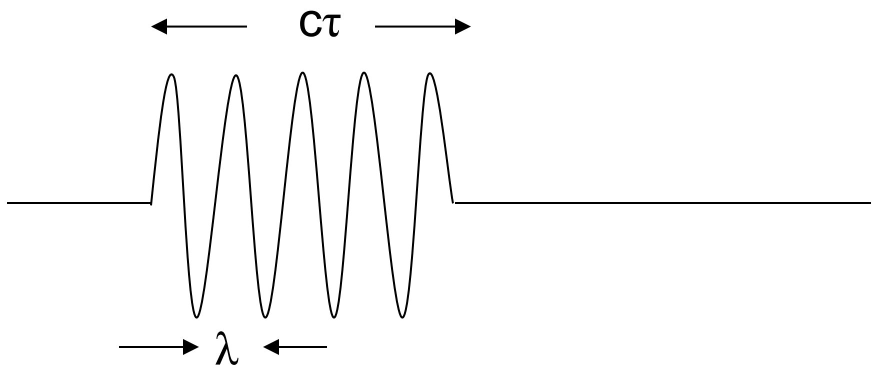
(A)


$Q(\tau_s, mT_s)$
$m = 4$
$m = 3$
$V(\tau_s, mT_s)$
ψ_e
$\Delta\psi_e$
$m = 5$
$I(\tau_s, mT_s)$
$m = 2$
$m = 1$

(B)

Pulsed Radar Principle

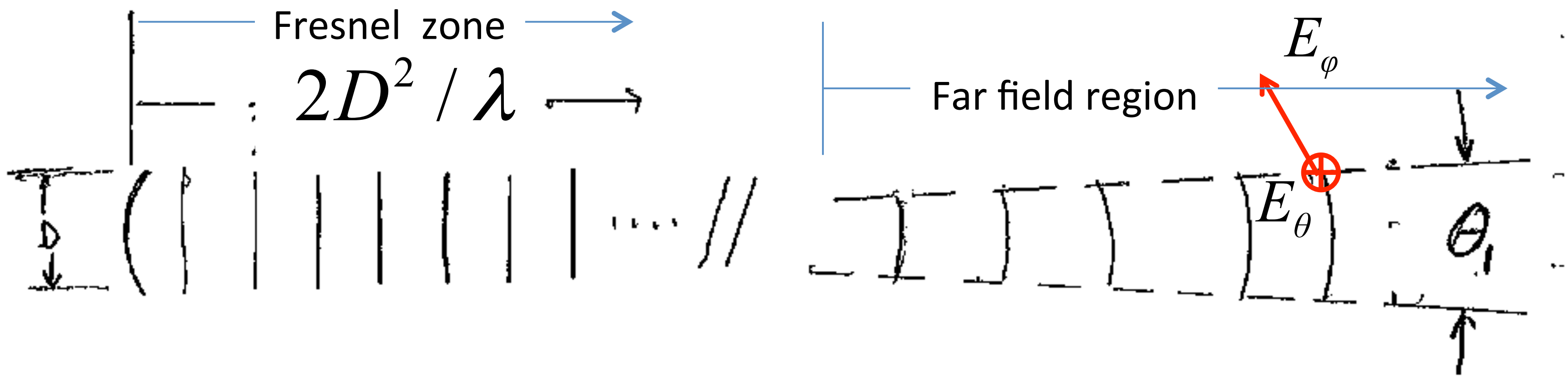




c = speed of microwaves
  = c_h for H and = c_v for V waves
τ = pulse length
λ = wavelength
  = λ_h for H and λ_v for V waves
τ_s = time delay between transmission of a pulse and reception of an echo.

Angular Beam Formation

(the transition from a circular beam of constant diameter to an angular beam of constant angular width)



$$2D^2/\lambda;\ 1.5\ \text{km};$$

$$\text{WSR-88D: } D;\ 8.53\ \text{m}; \lambda=10\ \text{cm}$$

$$\theta_1 = 1.27\ \lambda/D\ \text{(radians)}$$

Antenna (directive) Gain g_t

The defining equation:

$$S_i = \frac{P_t}{4\pi r^2} g_t f^2(\theta,\phi) \qquad \text{Eq. (3.4)}$$

S_i (W m^{-2}) = Power density incident on a scatterer

r = range to measurement (m)

$f^2(\theta,\phi)$ = radiation pattern = 1 on beam axis

P_t = transmitted power (W)

Backscattering Cross Section, σ_b for a Spherical Particle

Rayleigh condition on a spherical particle of diameter D:

$$D < \lambda / 16; \quad \lambda \equiv \text{ wavelength}$$

$$\boldsymbol{\sigma_b = \frac{\pi^5}{\lambda^4} |K_m|^2 D^6};$$

$$K_m \equiv \frac{m^2 - 1}{m^2 + 2}; \text{ Dielectric Factor of the medium filling the sphere; Eq.3.6}$$

$$m = n - jn\kappa = \text{the complex index of refraction}$$

$$|K_m|^2 \rightarrow |K_w{}^2 = 0.93 \quad \text{for water, and}$$

$$|K_m|^2 \rightarrow |K_i|^2 = 0.18 \quad \text{for ice (density} = 0.917 \text{ g m}^{-3})$$

Backscattered Power Density Incident on Receiving Antenna

$$S_r(r,\theta,\phi) = \frac{\overbrace{P_t g_t f^2(\theta,\phi)}^{S_i}}{4\pi r^2 l} \bullet \sigma_b \bullet \frac{1}{4\pi r^2 l} \qquad (3.13a)$$

where l is the loss factor (due to attenuation)

$$l = \exp\left(\int_0^r (k_g + k)\,dr \right) \qquad (3.13b)$$

Echo Power P_r Received

$$P_r = S_r(r,\theta,\phi)A_e(\theta,\phi) \quad (3.20)$$

A_e is the effective area of the receiving antenna for radiation from the θ,φ direction. It is shown that:

$$A_e = g_r f_r^2(\theta,\phi)\lambda^2 / 4\pi \qquad (3.21)$$

If the transmitting antenna is the same as the receiving antenna then:

$$g_r f_r^2(\theta,\phi) = g_t f_t^2(\theta,\phi) \equiv g f^2(\theta,\phi)$$

The Radar Equation
(point scatterer/discrete object)

$$P_r = \frac{P_t g f^2(\theta,\varphi)}{4\pi r^2 l} \bullet \frac{\sigma_b}{4\pi r^2 l} \bullet \frac{g\lambda^2 f^2(\theta,\varphi)}{4\pi} \qquad (3.24)$$

Example:

$$\lambda = 0.1\,\text{m}; \quad r = 20\,\text{km}\,(2\text{x}10^4\text{m}); \quad P_r(\text{min}) = 10^{-14}(\text{W});$$

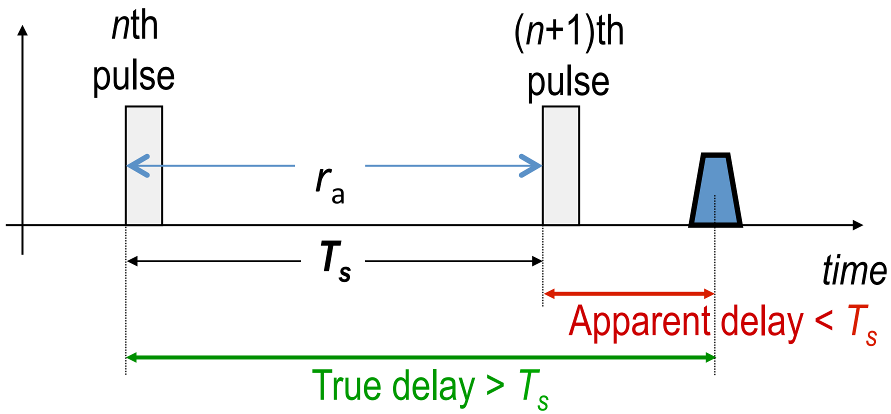
$$P_t = 10^6(\text{W; peak}); \quad g = 3x10^4; \quad l = 1(\text{no path loss})$$

Calculating the minimum detectable backscattering σ_b :

$$\sigma_b(\text{min}) = 2x10^{-7}\,\text{m}^2 = \sigma_b \text{ for a 6.3 mm drop!}$$

Unambiguous Range r_a

- If targets are located beyond $r_a = cT_s/2$, their echoes from the n^{th} transmitted pulse are received after the $(n+1)^{th}$ pulse is transmitted. Thus, they appear to be closer to the radar than they really are!
  - This is known as **range folding**
- T_s = PRT



- **Unambiguous range**: $r_a = cT_s/2$
  - Echoes from scatterers between 0 and r_a are called **1st** trip echoes,
  - Echoes from scatterers between r_a and $2r_a$ are called **2nd trip** echoes, Echoes from scatterers between $2r_a$ and $3r_a$ are called **3rd trip** echoes, etc

Unambiguous Velocity

- A pulsed Doppler radar measures radial Doppler velocity by keeping track of phase changes between samples that are T_s (pulse repetition time) apart
- Recall that echo phase shift is $\psi_e = -4\pi r/\lambda$. Then, the phase change from pulse to pulse is $\Delta\psi_e = -4\pi\Delta r/\lambda = -4\pi v_r T_s/\lambda$
  - Note that only phase changes between $-\pi$ and π can be unambiguously resolved
- Therefore, the unambiguous velocity is:
  - $4\pi v_a T_s/\lambda = \pi \Rightarrow v_a = \lambda/4T_s$
  - This is related to the **Nyquist** sampling theorem:
    Doppler velocities outside the $\pm v_a$ interval will be aliased!

$\Delta r = v_r T_s$ is the change in range of the scatterer between successive transmitted pulses

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Another PRT Trade-Off

- Correlation of pairs: $\rho(T_s) = \exp\left[-8\left(\pi\sigma_v T_s / \lambda\right)^2\right]$
  - This is a measure of signal coherency
- Accurate measurement of power requires long PRTs
  - $\lim_{T_s \to \infty} \rho(T_s) = 0$
  - More **independent samples** (low coherency)
- But accurate measurement of velocity requires short PRTs
  - $\lim_{T_s \to 0} \rho(T_s) = 1$
  - High correlation between sample pairs (high coherency)
  - Yet a large number of **independent sample pairs** are required

Signal Coherency

- How large a T_s can we pick?
  - Correlation between *m = 1* pairs of echo samples is:

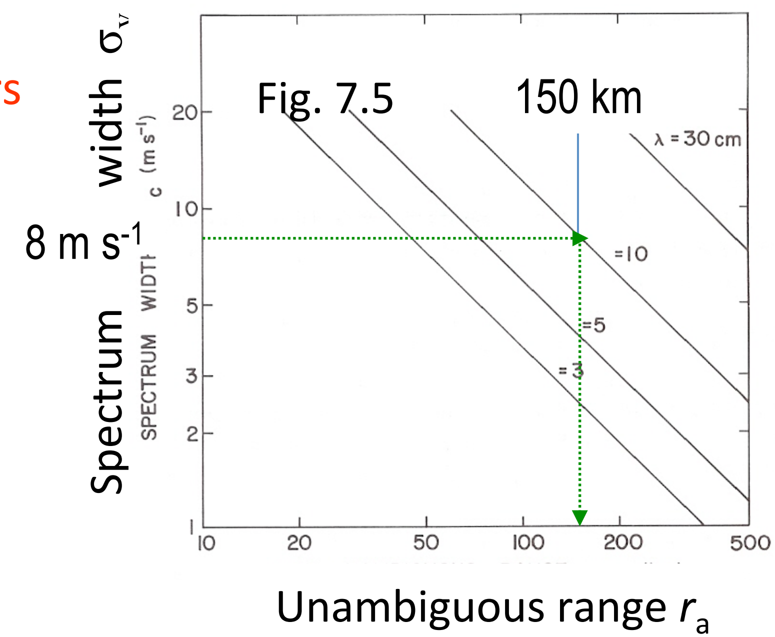
$$\rho(T_s) = \exp\left[-8\left(\pi\,\sigma_v T_s/\lambda\right)^2\right]$$

  - Correlated pairs: $\rho(T_s) \approx 1 \Rightarrow \frac{\pi\sigma_v T_s}{\lambda} << 1 \Rightarrow \frac{\lambda}{\pi T_s} >> \sigma_v$

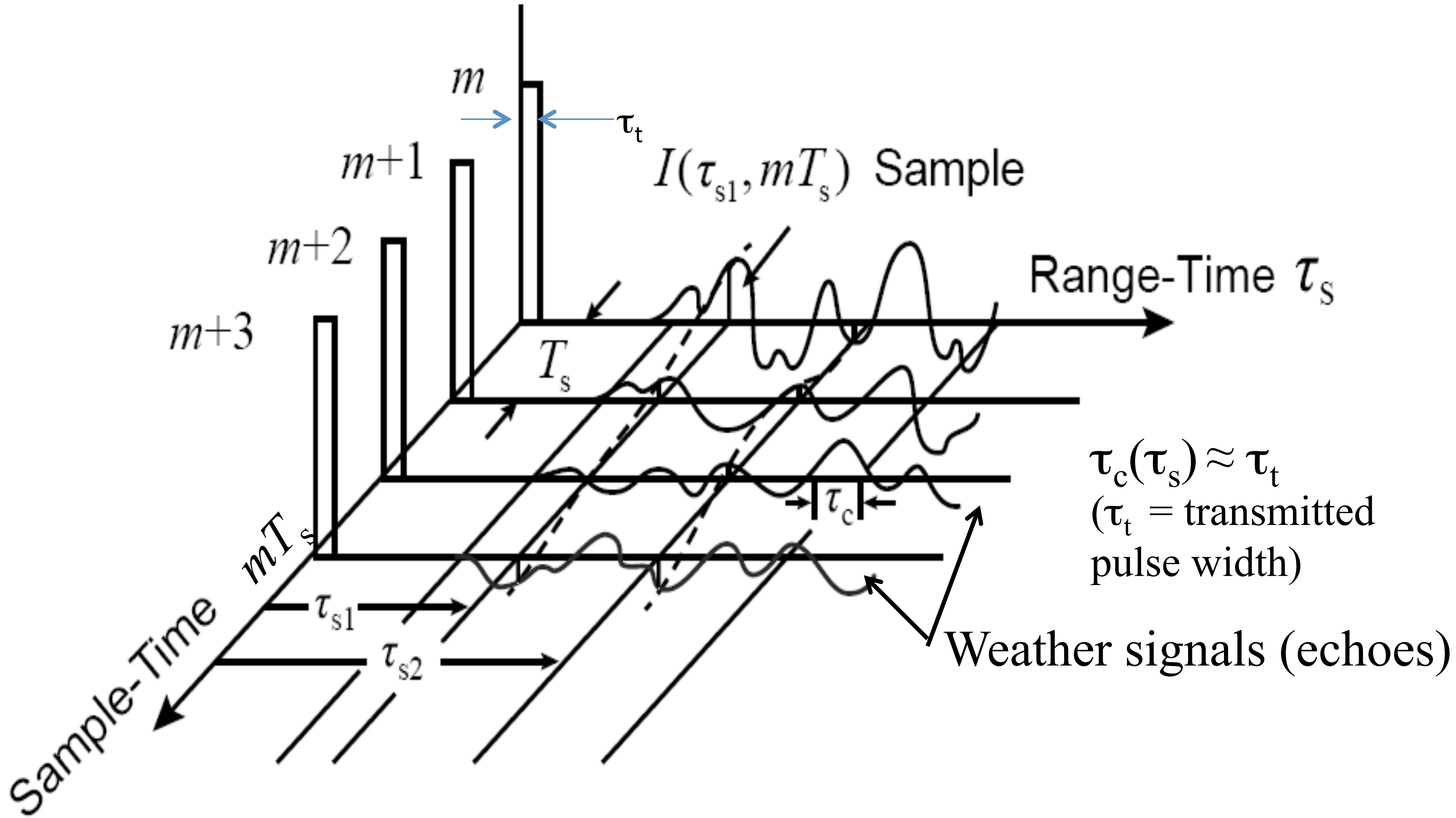
    (i.e., Spectrum width σ_v must be much smaller than unambiguous velocity $v_a = \lambda/4T_s$)
- Increasing T_s decreases correlation exponentially
  - $\text{Var}[\hat{v}]$ and $\text{Var}[\hat{\sigma}_v]$ also increases exponentially!
- Pick a threshold:
  - $\rho(T_s) \geq e^{-0.5} \Rightarrow -8\left(\pi\sigma_v T_s/\lambda\right)^2 = 0.5 \Rightarrow \sigma_v \leq v_a/\pi$
  - Violation of this condition results in very large errors of estimates!

Signal Coherency and Ambiguities

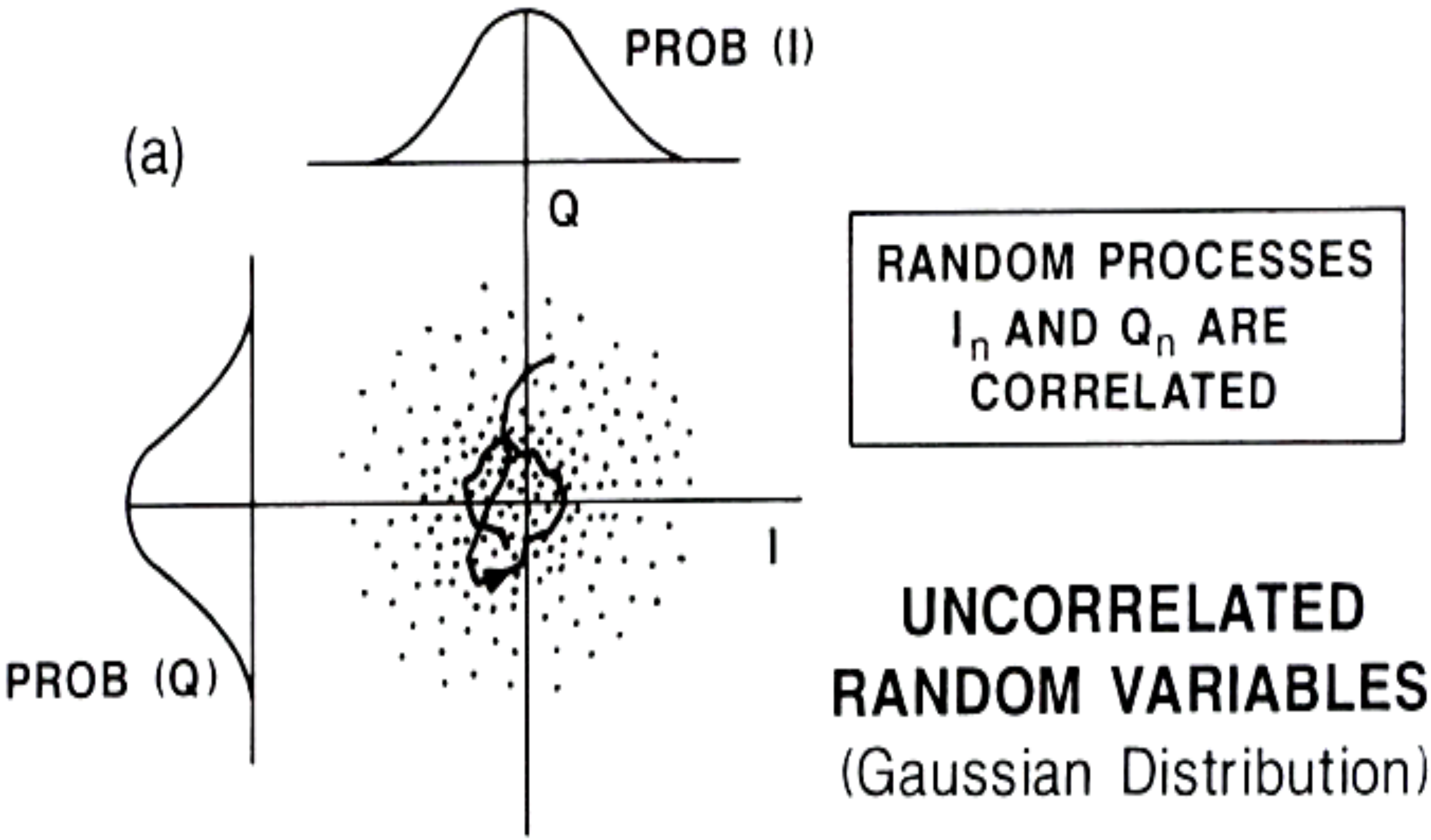
- Range and velocity dilemma: $r_a v_a = c\lambda/8$
- Signal coherency: $\sigma_v \leq v_a/\pi$
- r_a constraint: $$r_a \leq \frac{c\lambda}{8\pi\sigma_v}$$ Eq. (7.2c)
  - This is a more basic constraint on radar parameters than the first equation above
    - Then, σ_v and not v_a imposes a basic limitation on Doppler weather radars
  - Example: **Severe storms have a median σ_v ~ 4 m/s and 10% of the time σ_v > 8 m/s**. If we want accurate **Doppler** estimates 90% of the time with a 10-cm radar (λ = 10 cm); then, $r_a \leq$ 150 km. This will often result in range ambiguities



Echoes (I or Q) from Distributed Scatterers (Fig. 4.1)



Weather Echo Statistics (Fig. 4.4)



Reflectivity Factor Z

(Spherical scatterers; Rayleigh condition: $D \le \lambda/16$)

$$\eta(\boldsymbol{r}) = \frac{\pi^5}{\lambda^4} |K_{\mathrm{m}}|^2 Z(\boldsymbol{r}) \qquad (4.31)$$

where

$$Z(\boldsymbol{r}) \equiv \frac{1}{\Delta V} \sum_i D_i^6 = \int_0^\infty N(D, \boldsymbol{r}) D^6 dD \qquad (4.32)$$

$$\eta(\boldsymbol{r}) = \frac{\pi^5}{\lambda^4} |K_{\mathrm{w}}|^2 Z_{\mathrm{e}}(\boldsymbol{r}) \qquad (4.33)$$

for water drops: $|K_{\mathrm{w}}|^2 \approx 0.93$ independent of $\mathrm{T}(^{\circ}\mathrm{C})$;

for ice particles: $|K_{\mathrm{i}}|^2 \approx 0.16$ dependent on T and ice density.